Division of Labor, Transaction Cost, Emergence of the Firm and Firm Size

Pak-Wai Liu and Xiaokai Yang*

Abstract

In this paper a general equilibrium model is constructed to explain the emergence of firms and change in firm size by the tradeoff between economies of specialization and transaction cost. We show that firms emerge from the development of division of labor if the transaction efficiency for labor is smaller than that for intermediate goods. Given the emergence of firms, change in the average size of firms (average employment) will depend on the change in transaction efficiency for intermediate goods relative to that for labor. If the transaction efficiency is improved in such a way that the transaction efficiency for intermediate goods becomes higher than that for labor, average employment will decrease. We present evidence showing that it is not uncommon that average employment declines as the economy develops. The general equilibrium model provides an explanation for the concurrent increase of productivity and decrease in average employment which is observed in a number of countries. Models based on economies of scale instead of economies of specialization would have yielded the opposite prediction.

JEL Codes: D23, L11, L22, L23, L33

Keywords: irrelevance of size of the firm, division of labor, theory of the firm

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1. Introduction

Various scholars have observed that average employment of labor by firms (average firm size) in many industries in both advanced countries and newly industrialized countries has declined. Data clearly show that in numerous developed countries, such as the U.S., U.K., France, Austria and Belgium, employment share of small and medium enterprises in their manufacturing industries exhibits a U-shape pattern. It declined initially in the 1960s and early 1970s but started to increase in the late 1970s and early 1980s. Among the newly industrialized countries, South Korea’s manufacturing industries exhibit the same pattern as the developed countries, with a turning point in the early 1980s. In Taiwan, average firm size in manufacturing measured in terms of average employment increased from 7.3 persons per firm in 1954 to a peak of 28.2 in 1971 after which it followed a decline trend, reaching 24.2 in 1984. Similarly in the 1980s, manufacturing firms in Singapore were getting smaller in size on the average.

This inverted U-shaped trend is not confined to manufacturing industries. Liu (1992) shows that in Hong Kong, besides manufacturing, industries such as business services, import/export and restaurant exhibit a trend of declining average firm size. Table 1 shows the change in firm size of these four industries measured in two ways, the average number of employees per firm and the percent of employees engaged in firms with employees larger than 50. The trend since the late 1970s when data became available is generally downward. The decline in the average size of firms is associated with an increase in per capita real income and total factor productivity.
Table 1: Declining Firm Size in Hong Kong

A. Average number of employees per firm

<table>
<thead>
<tr>
<th>Year</th>
<th>Manufacturing</th>
<th>Import/Export</th>
<th>Business Services</th>
<th>Restaurants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1978</td>
<td>23.72</td>
<td>6.35</td>
<td>10.67</td>
<td>25.27</td>
</tr>
<tr>
<td>1981</td>
<td>20.61</td>
<td>6.74</td>
<td>11.30</td>
<td>27.42</td>
</tr>
<tr>
<td>1984</td>
<td>19.10</td>
<td>6.07</td>
<td>10.42</td>
<td>28.31</td>
</tr>
<tr>
<td>1987</td>
<td>18.60</td>
<td>6.21</td>
<td>9.78</td>
<td>23.49</td>
</tr>
<tr>
<td>1990</td>
<td>14.47</td>
<td>5.45</td>
<td>8.50</td>
<td>21.30</td>
</tr>
<tr>
<td>1993</td>
<td>13.13</td>
<td>5.10</td>
<td>7.38</td>
<td>21.47</td>
</tr>
</tbody>
</table>

B. Percent of employees in industry engaged in firms with employees larger than 50

<table>
<thead>
<tr>
<th>Year</th>
<th>Manufacturing</th>
<th>Import/Export</th>
<th>Business Services</th>
<th>Restaurants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>58.72</td>
<td>23.45</td>
<td>52.85</td>
<td>59.03</td>
</tr>
<tr>
<td>1984</td>
<td>59.07</td>
<td>20.20</td>
<td>51.20</td>
<td>57.72</td>
</tr>
<tr>
<td>1987</td>
<td>57.77</td>
<td>23.14</td>
<td>50.47</td>
<td>56.25</td>
</tr>
<tr>
<td>1990</td>
<td>53.36</td>
<td>21.15</td>
<td>52.31</td>
<td>51.62</td>
</tr>
<tr>
<td>1993</td>
<td>52.73</td>
<td>18.98</td>
<td>45.45</td>
<td>42.74</td>
</tr>
</tbody>
</table>

Source: Annual Digest of Statistics, Census and Statistics Department, Hong Kong Government, various years.

This trend of declining average firm size is contrary to the common perception that due to technological change and economies of scale, firm size should get larger over time. For instance, Kim (1989) shows from his model that because there are economies of scale arising from increasing returns due to specialization, firm size gets larger with specialization and the extent of the market. Our study provides an explanation of why firm size may get smaller over time on the basis of an analysis of the division of labor and transaction cost. We will show that the institution of the firm will emerge if the transaction efficiency for labor is higher than that for intermediate goods. By transaction efficiency we measures the fraction of the purchased good which disappears in transit due to transaction cost. Transaction competes for time and management. For instance, a producer of an intermediate good can either pay the search cost to find a supplier of the primary good used in its production or the managerial monitoring cost of in-house production of the primary good. We will show that given the emergence of firms, their size (measured by employment) decreases if the transaction efficiency for intermediate goods becomes higher than that for
labor. The decrease in the size of firms is driven by two forces. First, each firm becomes more specialized if division of labor develops among firms, so that each firm's scope of activities is reduced. However, if division of labor and specialization are developed within each firm, the level of specialization of each worker in a firm and the scope of the firm may increase hand in hand. But if the transaction efficiency for intermediate goods is higher than that for labor, then organizing division of labor between more specialized firms will be more efficient than organizing division of labor within a firm since the former involves more transactions of intermediate goods while the latter involves more trade in labor, thus generating a decline in the size of firms.\textsuperscript{4}

Several recent studies including Grossman and Hart (1986), Hart and Moore (1990), Milgrom and Roberts (1990), Kreps (1990), and Lewis and Sappington (1991) explain the size of the firm in various ways. None of these models, however, can predict the negative relationship between an increase in per capita real income and the average size of firms, nor can they explore the interaction between that negative relationship and the level of specialization.

Our model extends the Camacho's (1991, 1996) analysis of the tradeoff between benefits of division of labor and coordination costs as an explanation of the optimal size of the firm. Echoing Jones' empirical works (1995 a,b) which conclusively reject a scale effect, our theory explains economic development on the basis of the division of labor instead of economies of scale.\textsuperscript{5} It in effect formalizes Allyn Young's (1928) criticism of the notion of economies of scale. It is also in keeping with the arguments of Young, Stigler (1951), Cheung (1983), and Langlois (1988) which imply that the size of the firm may increase or decrease, depending on whether division of labor is organized through the labor market or through the market for intermediate goods.

In Section 2 a model of consumer-producers with economies of specialization in production and transaction costs in trading is presented. Section 3 characterizes eight market structures that may emerge from the development in division of labor and identifies the general equilibrium by comparing per capita real incomes in different structures. The main theoretical results of the paper on the emergence of the
institution of the firm and average employment are contained in Section 4. The paper concludes in Section 5.

2. **A Model with Consumer-producers, Economies of Specialization and Transaction Costs**

We consider an economy with $M$ ex ante identical consumer-producers (where $M$ is assumed large). There is a single consumer good $z$, the amount of this good that an individual self-provides is denoted by $z$ and the amounts sold and purchased are respectively $z^s$ and $z^d$. A fraction $1-k$ of $z^d$ disappears in transit because of transaction costs. Hence, the quantity that a person receives from the purchase of $z^d$ is $kz^d$. Total consumption is thus $z + kz^d$. $1-k$ is referred to as the transaction cost coefficient for food and $k$ is referred to as the transaction efficiency coefficient or simply transaction efficiency for the consumer good. An alternative formulation is to introduce an explicit time constraint on the consumer-producers in such a way that producing the good takes a certain amount of time but purchasing it takes more or less time. The individual maximizes the objective function $u$ (which can be interpreted as utility) given by total consumption:

$$ u = z + kz^d $$

(1)

The consumer good is produced using labor $L_z$ and an intermediate good $x$, that is

$$ z + z^d = (x + rx^d)^{x(z^d)}L_z^{(1-\alpha)a}, \quad \alpha \in (0,1) \text{ and } a > 1 $$

(2)

where $x$ and $x^d$ are the respective quantities of the intermediate good self-provided and purchased. $r$ is the transaction efficiency of the intermediate good. $1-r$ is the transaction cost coefficient for a unit of the intermediate good purchased and therefore $rx^d$ is the quantity received by a person who buys $x^d$. The parameter $\alpha$ is the elasticity of output with respect to input of the intermediate good and can be interpreted...
as the relative importance of the roundabout productive sector compared to labor. The degree of economies of specialization is represented by a.

Production of the intermediate good requires labor $L_x$ and a primary good $y$.

$$x + x^s = (y + ty^d)^\alpha L_x^{(1-\alpha)a}, \quad \alpha \in (0, 1) \text{ and } a > 1 \quad (3)$$

where $x^s$ is the quantity of the intermediate good sold and $y$ and $y^d$ are, respectively, the quantities of the primary good self-provided and purchased. $1-t$ is the transaction cost coefficient for the primary good ($t$ is the transaction efficiency) and $ty^d$ is the amount that a person receives from the purchase of $y^d$. It is also assumed that fraction $1-s$ of goods produced by an employee within a firm disappears when it is delivered to the employer because of transaction cost of labor. The production function for the primary good is

$$y + y^s = L_y^a, \quad a > 1 \quad (4)$$

where $y^s$ is the quantity of the primary good sold. The labor endowment constraint for each individual is

$$L_z + L_x + L_y = 1, \quad L_i \in [0, 1], \quad i = z, x, y. \quad (5)$$

where $L_i$ is a person's level of specialization in producing good $i$.

This system of production exhibits economies of specialization. The total factor productivity of the consumer good and the labor productivity of the primary good increase with the level of specialization in producing a good concerned. But economies of specialization are individual specific and do not extend beyond the scale of an individual's working time. Also, they are related to diseconomies of an individual's scope of activities.
There are $2^9 = 512$ combinations of zero and non-zero values of $z, z^s, z^d, x, x^s, x^d, y, y^s, y^d$. Following the approach of Borland and Yang (1995), we can exclude many combinations of zero and nonzero variables from the list of candidates for an individual’s optimal decision.

### 3. Configurations, Market Structures and General Equilibrium

A combination of zero and nonzero variables that is compatible with the Kuhn-Tucker condition and other conditions for the optimum decision is called a configuration. For each configuration, an individual can solve for a corner solution for a given set of relative prices of traded goods. A combination of configurations that satisfies the market clearing conditions is called a market structure, or simply structure. There are eight market structures if the institution of the firm is allowed. We define the firm as contractual arrangements which involve asymmetric relationship between an employer and his employees and the production of intermediate goods using labor. The employer has control rights of his employees' labor and claims the difference between the firm's revenue and wage bill.

The eight market structures include autarky, two structures involving partial division of labor with no firm and two with firm, one structure involving complete division of labor with no firm and two with firms. The eight market structures and the combination of configurations that make up these structures are categorized and characterized as follows:

**Autarky**

1. **Structure A**

   Structure A consists of a configuration with $z^e = x^e = y^e = z^d = x^d = y^d = 0$. In this structure each individual self-provides all goods he needs.
Partial Division of Labor With No Firm

2. Structure P1
   Configuration 1: an individual sells the consumer good, self-provides the intermediate good, and buys the primary good
   Configuration 2: an individual sells the primary good, and buys the consumer good

3. Structure P2
   Configuration 1: an individual sells the intermediate good, self-provides the primary good, and buys the consumer good
   Configuration 2: an individual sells the consumer good, and buys the intermediate good

Partial Division of Labor With Firm

4. Structure PF1
   Configuration 1: an individual sells the consumer good, self-provides the intermediate good, buys labor, and directs workers to produce the primary good in his firm
   Configuration 2: an individual sells labor which is hired to produce the primary good by a firm, and buys the consumer good

5. Structure PF2
   Configuration 1: an individual sells labor which is hired by a firm to produce the intermediate good and the primary good in the firm, and buys the consumer good
   Configuration 2: an individual sells the consumer good, buys labor, and directs each worker to produce both the primary and the intermediate goods.

Complete Division of Labor With No Firm

6. Structure C
   Configuration 1: an individual sells the consumer good, and buys the intermediate good
   Configuration 2: an individual sells the primary good, and buys the consumer good
Configuration 3: an individual sells the intermediate good, and buys the primary good and the consumer good

**Complete Division of Labor With Firm**

7. Structure CF1

Configuration 1: an individual sells the consumer good, buys the primary good, hires workers and directs them to produce the intermediate good using the primary good in his firm

Configuration 2: an individual sells the primary good, and buys the consumer good

Configuration 3: an individual sells labor which is hired by a firm to produce the intermediate good, and buys the consumer good

8. Structure CF2

Configuration 1: an individual sells the consumer good, buys labor, and directs some of the workers to produce the primary good and others to produce the intermediate good using the primary good

Configuration 2: an individual sells labor which is hired by a firm to produce the intermediate good, and buys the consumer good

Configuration 3: an individual sells labor which is hired by a firm to produce the primary good, and buys the consumer good

A corner equilibrium for a certain structure is defined by a set of relative prices of traded goods and a set of relative numbers of individuals choosing different configurations that equilibrate total corner-demand to total corner-supply of each traded goods and equalize all individuals' consumption levels. Here, competition among ex ante identical consumption-maximizing individuals, combined with free entry into each configuration, implies that all configurations in a structure will be chosen only if consumption is
equalized across the configurations. In this section, the corner equilibria for the eight structures are first solved.

A two-step approach developed in Yang and Ng (1993) is used to solve for the corner equilibrium for each of the eight market structures. As an illustration, the algebra for solving for the corner equilibrium for structure PF1 is shown as follows.

**Solving for the Corner Equilibrium for Structure PF1**

Structure PF1 consists of configurations 1 and 2 as summarized earlier. The individual decision problem for configuration 1 is

\[
\begin{align*}
\text{Max:} & \quad u_z = z \\
\text{s.t.} & \quad z + z^* = x^\alpha L_x a^{(1-\alpha)}, \quad x = (sy^d)^\alpha L_x a^{(1-\alpha)} \\
& \quad y^d = ny^s \\
& \quad L_x + L_z = 1 \\
& \quad y^s = L_y a \\
& \quad L_y = 1 \\
& \quad p z^* = wn
\end{align*}
\]

where \(L_x\) and \(L_z\) are the employer's labor allocation to the production of \(x\) and \(z\) respectively. \(n\) is the number of workers hired by an employer. \(L_y\) is an employee's labor which is subject to the employer's control. \(y^s\) is each employee's output of \(y\) within the firm. \(y^d\) is the total output level of \(y\) in the firm or input requirement for the production of \(x\) in the firm. \(w\) is the wage rate, and \(s\) is the transaction efficiency for labor or \(1-s\) is the transaction cost coefficient of labor in terms of the loss of goods produced out of labor in transit. If \(1-s\) is assumed to be in terms of the loss of labor in transit, the essence of the results will not change, but the algebra will be more complicated. The solution of the problem is
\[ L_z = \alpha/(1 + \alpha), \quad L_y = 1/(1 + \alpha), \quad L_y = 1, \]

\[ n = (\alpha^2 A p_z/w)^{1/(1-\alpha)(1+\alpha)}, \quad A = [s^\alpha (1-\alpha)^{\alpha} (1+\alpha)^{(1-\alpha)(1+\alpha)}]^\alpha \]

\[ z^i = wn/p_z, \quad u_z = (1-\alpha^2)A (\alpha^2 A p_z/w) \alpha^2/(1-\alpha^2) \]

where \( n \) is demand for labor by the firm. \( z^i \) is supply of \( z \) by the firm, and \( u_z \) is the indirect objective (utility) function for configuration 1. The decision for configuration 2 is fixed as follows.

\[ u_y = kz^d = kw/p_z \] (objective function)

\[ wL_y = p_z z^d, \quad L_y = 1, \] (budget and endowment constraints)

where \( u_y \) is the indirect objective function (utility) of an employee.

The consumption equalization condition \( u_y = u_z \) yields the corner equilibrium relative price \( w/p_z \) and the market clearing condition for good \( z \), \( M : z^s = M _y z^d \), yields the relative number of individuals choosing the two configurations. Here, \( M _y / M : n \) is the corner equilibrium relative number of individuals choosing to be employees to those choosing to be employers, which happens to be the same as the corner equilibrium value of \( n \), a decision variable for an employer. The solution for the corner equilibrium is summarized in Table 2.

Following this procedure, the corner equilibria in all market structures can be solved. Table 2 summarizes these solutions. A general equilibrium is defined as a fixed point that satisfies the following two conditions: (i) each individual uses inframarginal analysis to maximize his consumption respect to configurations and quantities of each good produced, consumed, and traded for a given set of relative prices of traded goods and a given set of the numbers of individuals selling different goods; and (ii) the set of relative prices of traded goods and the set of individuals selling different goods clear the markets for traded goods and equalize consumption for all individuals selling different goods. Yang and Ng (1993, Chapter 6) have proved that in this kind of models, the general equilibrium is the corner equilibrium that generates the highest per capital real income. The other corner equilibria are not general equilibria.
Table 2: Solutions to Corner Equilibria of Market Structures

<table>
<thead>
<tr>
<th>Market Structure</th>
<th>Relative Price</th>
<th>Labor Allocation</th>
<th>Real Per Capita Income</th>
</tr>
</thead>
</table>
| A                | -              | $L_x = \alpha(1-\alpha)$  
|                  |                | $L_y = \alpha^2$  
|                  |                | $L_z = 1-\alpha$  
|                  |                | $u(A) = \beta^{(1-\alpha)\alpha}$  
| P1               | price of primary good in terms of consumer good $p_x$  
|                  | $= k^{1-\alpha} \alpha^{(1-\alpha)\alpha} \beta^{1-\alpha}$  
|                  | relative number of sellers of primary good to sellers of consumer good  
|                  | $= k\alpha^2/(1-\alpha^2)$  
|                  | $u(P1) = (kt)^{\alpha\beta}$  
| C                | price of primary good in terms of intermediate good $p_y = \beta t^\alpha$  
|                  | price of intermediate good in terms of consumer good $p_z = (k\alpha)^{\alpha-1} r^\alpha \beta^\alpha$  
|                  | relative number of sellers of intermediate good to sellers of consumer good  
|                  | $= \alpha k$  
|                  | $u(C) = (kt)^{\alpha\beta}$  
| CF1              | price of consumer good in terms of primary good $p_y = k^{(\alpha-1)}(st^\alpha)^{\alpha-1}$  
|                  | wage rate in terms of primary good  
|                  | $= 1$  
|                  | number of workers hired by a firm  
|                  | $= k\alpha$  
|                  | $u(CF1) = (kst)^{\alpha\beta}$  
| CF2              | price of intermediate good in terms of consumer good $p_z = (ks)^\alpha t^{\alpha \beta}$  
|                  | wage rate in terms of consumer good  
|                  | $= k^{\alpha-1}(st^\alpha)^{\alpha-1} \beta^{1+\alpha}$  
|                  | number of workers hired by a firm  
|                  | $= k\alpha(1-\alpha)$  
|                  | $u(CF2) = s^{\alpha(1+\alpha)} k^{\alpha-1} \beta^{1+\alpha}$  

Note: $\beta = \alpha^\alpha (1-\alpha)^{1-\alpha}$
A revealed preference argument can be used to show that each corner equilibrium is locally Pareto optimal for a given structure (See Yang and Ng, 1993). Hence, the general equilibrium is globally Pareto optimal. This implies that the competitive market can efficiently coordinate division of labor to fully utilize network division of labor. This result critically depends on the assumption that economies of specialization are individual specific and different from economies of scale.

4. Development in Division of Labor and Average Employment

A comparison between per capita real incomes generated by the eight market structures yields Proposition 1

(1) As transaction efficiency is improved, the general equilibrium shifts from structure A first to structure P1 or PF1, followed by P2 or PF2, and finally to structure C, CF1, or CF2. This process increases per capita real income, productivity, trade dependence, individuals’ level of specialization, and the number of specialized activities.

(2) The institution of the firm will emerge from the development in division of labor if transaction efficiency is higher for labor than for intermediate goods.

(3) Given the emergence of the institution of the firm from the development in division of labor, the average employment increases if the transaction efficiency for labor is higher than that for intermediate goods. It decreases if transaction efficiency is improved in such a way that the transaction efficiency for intermediate goods becomes higher than that for labor.

Proposition 1 is proved in the Appendix. The intuition of Proposition 1 is as follows. There is a tradeoff between economies of specialization and transaction costs. Individuals will choose a low level of specialization and a large scope of production activities if transaction efficiency is low because economies of specialization are outweighed by transaction costs generated by a high level of division of labor. As
transaction efficiency is improved, the equilibrium level of specialization increases and the equilibrium scope of each individual's production activities becomes narrower. As the level of division of labor increases with improvements in transaction efficiency, two types of market structure can be used to organize a higher level of division of labor. The first involves markets for final and intermediate goods and the second involves markets for final goods and for labor. If the transaction efficiency for labor hired to produce an intermediate good is higher than that for the intermediate good, the institution of the firm will emerge from division of labor. If the institution of the firm emerges, but the transaction efficiency for intermediate goods is improved more quickly than that for labor, then the equilibrium size and scope of firms will decrease as each individual increases his level of specialization and narrows down his scope of production activities.

The decrease in the average employment is driven by two forces. First, each firm becomes more specialized if division of labor between firms and individuals develops, so that each firm's scope of activities is reduced. Second, if the transaction efficiency is improved in such a way that the transaction efficiency for intermediate goods becomes higher than that for labor, then organizing division of labor between more specialized firms and individuals will be more efficient than organizing division of labor within a firm since the former involves more transactions of intermediate goods while the latter involves more trade in labor. Hence, an increase in the level of specialization of firms and an increase in division of labor between small scale firms and individuals will occur, generating a decline in average employment of firms.

To illustrate the shifting of market structures. Suppose the initial values of parameters are such that the transaction efficiencies for final and intermediate goods are very low but the transaction efficiency for the intermediate good is higher than for labor. Then the general equilibrium will be autarky, as shown in the Appendix. If the transaction efficiencies for final and intermediate goods are improved while the transaction efficiency for labor is still lower than for intermediate goods, then the general equilibrium will be structure P1 where firms and labor market do not exist and completely specialized producers of
primary good exchange it for final good with non-specialized producers of intermediate and final goods in the market. Suppose the transaction efficiencies for final goods and labor are improved relative to the transaction efficiency of the intermediate good, then the general equilibrium will shift to structure PF2 where a specialist producer of final good hires \(k\alpha/(1-\alpha)\) workers and directs each of them to produce primary and intermediate goods within a firm. In PF2, each firm produces three goods and the level of specialization of workers is not high although it is higher than in autarky. If the transaction efficiency for primary good is sufficiently improved such that it is higher than the transaction efficiency for labor which in turn is higher than that for the intermediate good, then the general equilibrium will shift from PF2 to structure CF1 where a specialist producer of final good hires \(\alpha k\) workers, buys the primary good, and directs the workers to specialize in producing the intermediate good using the primary good within his firm.

For this particular exogenous changing pattern of values of parameters, the general equilibrium will shift along the path represented by the structure sequence \(A \Rightarrow P1 \Rightarrow PF2 \Rightarrow CF1\) in Figure 1.

**Figure 1: Development in Division of Labor and Structure of Firms**

![Diagram showing the development in division of labor and structure of firms](image-url)
The development of specialization in output associated with a decrease in the scope of a firm's activities, an increase in specialization of labor assignment within firms and replacement of self-provision of intermediate goods with market procurement, are common in the economy. For instance, law firms may give up providing a full range of legal services to specialize in only one or two aspects of the legal practice such as conveyancing or family law. Specialization in labor assignment is also common in manufacturing assembling and processing. A manufacturing firm may purchase intermediate services such as advertising, storage and trucking in the market instead of advertising its own products and operating its own warehouses and fleet of trucks.

To summarize, all patterns of changes in division of labor that involve a decrease in average employment of firms are listed in (6a), and those patterns that involve a monotonic increase in average employment are listed in (6b). The values of S are the average employment of firms in the relevant structures.

\[
\begin{align*}
A &\Rightarrow P1 \Rightarrow PF2 \Rightarrow \begin{cases} 
C \\
CF1
\end{cases} \quad S = \alpha k/(1-\alpha) \\
&\Rightarrow \begin{cases} 
CF1 \\
S = \alpha k
\end{cases} \quad (6a)
\end{align*}
\]

\[
\begin{align*}
A &\Rightarrow PF1 \Rightarrow PF2 \Rightarrow \begin{cases} 
C \\
CF1
\end{cases} \quad S = \alpha^2 k/(1-\alpha^2) \\
&\Rightarrow \begin{cases} 
CF1 \\
S = \alpha k
\end{cases} \quad (6b)
\end{align*}
\]

Where the average employment of firms is \(\alpha k/(1-\alpha)\) in structure PF2, \(\alpha k\) in CF1, \(\alpha^2 k/(1-\alpha^2)\) in PF1, and \(\alpha k/(1-\alpha)\) in CF2, and \(\alpha k/(1-\alpha) > \alpha^2 k/(1-\alpha^2)\), \(\alpha k\). (6) is proved in the Appendix.
5. Conclusion

In this paper we use inframarginal comparative static equilibrium analysis to show, after the manner of Coase and North, a change in transaction cost and division of labor leads to organizational changes in production in terms of self-sufficient production and specialization in production. If we assume that each individual maximizes total discounted utility and transaction efficiency changes over time, our model in this paper will become a dynamic general equilibrium model which can predict evolution of firm size over time. Another way to extend the static model is to assume bounded rationality and interactions between dynamic decisions and information of prices. Zhao (1998) has developed such a Walrasian sequential equilibrium model that can predict spontaneous evolution of the institution of the firm and of society's knowledge of institution.

Our model predicts that the average employment of firms will decrease as division of labor changes if transaction efficiency is improved in such a way that the transaction efficiency for intermediate goods becomes higher than that for labor. To test the model one needs to have good measures of the relative transaction conditions of goods to labor. However, these data are difficult to obtain. Nevertheless, empirical work of Murakami, Liu, and Otsuka (1996), and Yang, Wang, and Wills (1992) have provided indirect evidence for this theory. Murakami, Liu, and Otsuka show that in China's machine tool industry the average size of firms declined while division of labor and productivity rose in the 1980s. Yang, Wang, and Wills show that an institution and policy index (a weighted average of 12 subindices of transaction conditions for four types of properties and three components of property rights) that affects transaction efficiency in China increased in the 1980s. The subindex for transaction efficiency of goods was higher than that of labor during that period. These two pieces of empirical work indirectly support the hypothesis based on our theory. More empirical work needs be done to test our theory.
Appendix: Proof of Proposition 1

Since the corner equilibrium that generates the highest per capita real income is the general
equilibrium, the general equilibrium can be identified by comparisons among per capita real incomes in
all structures.

A comparison between \( u(P1) \) and \( u(PF1) \), or between \( u(P2) \) and \( u(PF2) \) yields

\[
\begin{align*}
&u(PF1) > u(P1) \text{ iff } s > t \\
&u(PF2) > u(P2) \text{ iff } s > r.
\end{align*}
\]

This implies that for a given pattern of partial division of labor, the institution of the firm will be used in
the general equilibrium to organize the division of labor iff the transaction efficiency for labor \((s)\) is
higher than the transaction efficiency for intermediate goods \((r \text{ or } t)\). Comparisons among \( u(C) \), \( u(CF1) \)
and \( u(CF2) \) yield the following results.

\[
\begin{align*}
&u(C) > u(CF1) \text{ and } u(CF2) \text{ if } s < \min\{r, (rt^{\alpha})^{1/(1+\alpha)}\} \quad (8a) \\
&u(CF1) > u(C) \text{ and } u(CF2) \text{ if } s \in (r, t) \& t > r. \quad (8b) \\
&u(CF2) > u(C) \text{ and } u(CF1) \text{ if } s > \max\{t, (rt^{\alpha})^{1/(1+\alpha)}\} \quad (8c)
\end{align*}
\]

(8) together with Table 2 imply that the larger the transaction efficiency for labor compared to that for
intermediate goods, the more likely the general equilibrium is a structure involving the institution of the
firm, and that the larger the transaction efficiency for labor compared with that for intermediate goods,
the larger is the employment of a firm.

Since the general equilibrium, which is the corner equilibrium with the highest per capita real
income, will shift between corner equilibria as transaction efficiency parameters reach some critical
values, there are many possible paths of such development in the division of labor, indicated by the
arrows with numbers in Figure 1. The critical values can be identified by comparing per capita real
incomes in all structures. Such comparisons yield the following results.
Shift 1, A $\Rightarrow$ P1, will take place if kt increases to the critical value $kt > \beta_0$  
(9)

Shift 2, A $\Rightarrow$ PF1, will take place if ks increases to the critical value $ks > \beta_0$

Shift 3, P1 $\Rightarrow$ P2, will take place if k and r increase to the critical value $k^{1-\alpha}r/t^\alpha > \beta_1$

Shift 4, P1 $\Rightarrow$ PF2, will take place if k and s increase to the critical value $k^{1-\alpha}s/t^\alpha > \beta_1$

Shift 5, PF1 $\Rightarrow$ P2, will take place if k and r increase to the critical value $k^{1-\alpha}r/s^\alpha > \beta_1$

Shift 6, PF1 $\Rightarrow$ PF2, will take place if ks increases to the critical value $(ks)^{1-\alpha} > \beta_1$

Shift 7, P2 $\Rightarrow$ C, will take place if k increases to the critical value $t/r^{1-\alpha} > \beta_2$

Shift 8, P2 $\Rightarrow$ CF1, will take place if k and t increase to the critical value $st^{1-\alpha}r > \beta_2$

Shift 9, P2 $\Rightarrow$ CF2, will take place if s and r increase to the critical value $s^{1+\alpha}/r > \beta_2$

Shift 10, PF2 $\Rightarrow$ C, will take place if r and t increase to the critical value $rt^{\alpha}/s > \beta_2$

Shift 11, PF2 $\Rightarrow$ CF1, will take place if t increases to the critical value $t^{\alpha}/r > \beta_2$

Shift 12, PF2 $\Rightarrow$ CF2, will take place if s increases to the critical value $s^{\alpha}/r > \beta_2$

where $\beta_0 = \alpha - 2\alpha^2(1-\alpha^2)^{(1+1)/(\alpha+1)}$, $\beta_1 = [\alpha - 2\alpha(1-\alpha)^{\alpha+1}]^{(1+\alpha)}(1+\alpha)^{(1-\alpha^2)/(\alpha^2+1)/\alpha}$, and $\beta_2 = [\alpha^\alpha(1-\alpha)^{1-\alpha}]^{\alpha}$.

It can be shown that the size and scope of firms will decrease if the development in division of labor follows the structure sequences: A $\Rightarrow$ P1 $\Rightarrow$ PF2 $\Rightarrow$ C (paths 1, 4, 10 in Figure 1), A $\Rightarrow$ P1 $\Rightarrow$ PF2 $\Rightarrow$ CF1 (paths 1, 4, 11 in Figure 1), A $\Rightarrow$ P1 $\Rightarrow$ PF2 $\Rightarrow$ C (paths 2, 6, 10 in Figure 1), and A $\Rightarrow$ P1 $\Rightarrow$ PF2 $\Rightarrow$ CF1 (paths 2, 6, 11 in Figure 1). According to (7-9), this will take place if transaction efficiencies for goods and labor increase but the transaction efficiency for labor does not increase too fast.

On the other hand, the employment of firms will increase if the development in division of labor follows the structure sequences A $\Rightarrow$ P1 $\Rightarrow$ P2 $\Rightarrow$ CF1 (paths 1, 3, 8 in Figure 1), A $\Rightarrow$ P1 $\Rightarrow$ P2 $\Rightarrow$ CF2 (paths 1, 3, 9, in Figure 1), and A $\Rightarrow$ P1 $\Rightarrow$ P2 $\Rightarrow$ CF2 (paths 2, 6, 12 in Figure 1). According to (7-9), this will take place if transaction efficiencies for goods and labor increase or the transaction efficiency for labor increases more quickly than the transaction efficiency for intermediate goods. The results are summarized
in (6). Figure 1 or (9), combined with Table 2 imply that as transaction efficiencies for labor, and intermediate and final goods are improved, the level of division of labor increases. Individuals' level of specialization, trade dependence, per capita real income, and the number of specialized activities increase. This change will not involve the institution of the firm if the transaction efficiency for labor is lower than that for intermediate goods. The institution of the firm will emerge from the development in division of labor if the transaction efficiency is higher for labor than for intermediate goods.

(6), (7), (8), and (9) are sufficient to establish Proposition 1.
Footnotes

1. For data on the U.S., U.K. and France, see Loveman and Sengenberger (1991). For data on Austria, see Aiginger and Ticky (1991) and for Belgium, see Storey and Johnson (1987).


4. A firm's decision to increase its level of specialization which leads to the equilibrium phenomenon of disintegration and increasing division of labor between firms is called "outsourcing", "downsizing", "returning to core competencies", and "diverting unrelated businesses" in the literature of management. See for instance, Milgrom and Roberts (1992).

5. Jones (1995a,b) shows that empirical evidence rejects positive effect of scale of the investment and R&D sector on economic growth.

6. The alternative formulation was suggested by the Editor which we gratefully acknowledge. Our specification of iceberg transaction technology is not as realistic as endogenous transaction costs caused by moral hazard and adverse selection. It is not as sophisticated as communication cost based on knowledge in Camacho (1991) but it avoids formidable indices of origins and destinations of deliveries and other technical difficulty.

7. The model of Yang and Ng (1995) of the firm is similar to the model in this paper. But their model has only two goods, so that it is not rich enough to predict declining firm size as division of labor changes. We need a model with at least 3 goods to predict the following concurrent phenomena: change of division of labor, declining employment of the firm, and growth of per capital real income and productivity.

8. The proof of this statement is available from the authors.

9. Change in division of labor in this paper is based on comparative statics. It is exogenous and cannot take place in the absence of exogenous change of transaction efficiency parameters. A dynamic version of the model is developed by Borland and Yang (1995) to generate endogenous evolution of the firm.
References


Census and Statistics Department, Annual Digest of Statistics, Hong Kong Government, various years.


