A Structural Model for Sovereign Credit Risk

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Abstract

This paper provides, and empirically estimates, a structural model of sovereign default risk on external debt. The sovereign endogenously determines its level of foreign debt and default policy. Consistent with default crisis episodes, the sovereign and its lenders bargain at default over a reduction of the debt service. The potential for debt restructuring offers the sovereign greater incentive to default. This model offers theoretical predictions of the relationship between credit spreads and related macro-variables that are consistent with the empirical literature. I also compare estimates of daily credit spreads implied by the structural model with observed EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998-2006. In a panel analysis, the model explains about 92% of the time variation in daily credit spreads. In contrast to some recent studies, there remains limited scope for additional explanation from U.S. Treasury rates and the VIX index.

JEL Codes: F34, G12, G13, G15

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1 Introduction

This paper provides a structural model, and an accompanying empirical analysis, of sovereign default risk. The model helps explain the variation across time in EMBI+ spreads to a degree that had not been offered by prior empirical models. I also generate theoretical predictions of the relationship between credit risk and the macro-variables provided by the model that are consistent with the empirical literature. Sovereign foreign debt has been at the center of a number of international lending crises and now constitutes the largest asset class in emerging markets, representing approximately $5,500bn of principal in 2007 (FT, 2007).

I generate estimates of daily credit spreads implied by the structural model for Brazil, Mexico, Peru, and Russia over the period 1998-2006, and compare these estimates with observed EMBI+ spreads. I use each country’s stock market index as the only time-varying explanatory variable. In a panel analysis with fixed effects, the credit spreads predicted by the model explain about 92% of the variation across time in daily EMBI+ spreads. The explanatory power rises only slightly, to 94%, when accounting for additional time-varying factors such as 5-year U.S. Treasury rates and the VIX option-implied volatility index. This finding may change one’s interpretation of the results of Longstaff, Pan, Pedersen, and Singleton (2007) and Pan and Singleton (2008). These authors show that the VIX index is a key factor in explaining credit risk movements, but do not include the factors that I show to almost eliminate VIX as an additional explanatory variable.

This paper offers the first structural model that explains the dynamics of EMBI+ spreads. Prior studies have considered a reduced-form affine structure model,\(^1\) a reduced-form contingent-claims analysis,\(^2\) and a panel-based approach.\(^3\) An advantage of the proposed structural model is that it provides an intuitive theoretical framework for the determinants of credit spreads that can then be used to motivate empirical specifications, such as those of this paper.

The theoretical predictions of the relationship between credit risk and the macro-variables provided by my model are generally consistent with the empirical literature. First, within the model, credit risk decreases with economic growth because the sovereign is more likely to default in a recession. This is consistent with previous empirical works.\(^4\) In contrast, Kehoe and Levine (1993),

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\(^{1}\)See, for instance, Duffie and Singleton (1999), Duffie, Pederson, and Singleton (2003), Longstaff et al. (2007), and Pan and Singleton (2008).

\(^{2}\)See, for instance, Weigel and Gemmill (2006), and Bodie, Gray, and Merton (2007).

\(^{3}\)See, for instance, Hilscher and Nosbusch (2007) and the references therein.

Kocherlakota (1996), and Alvarez and Jermann (2000) offer models in which default incentives are higher in periods of high output, at odds with the empirical evidence that I have cited. Second, in my model, higher macroeconomic volatility leads to greater credit risk, consistent with prior empirical findings. Third, my model suggests that credit risk increases with risk-free interest rates, which is also observed in the data. This prediction is in contrast with the theoretical results of Gibson and Sundaresan (2001) and Westphalen (2002), whose models predict a negative relation between credit risk and risk-free yields. Fourth, my model implies that sovereign credit risk decreases with the severity of economic sanctions imposed upon default. These sanctions reduce the sovereign’s access to the international market through trade embargoes, and thereby reduce future output growth. If the sovereign does significant trade, the impact of economic sanctions is large, and the sovereign is less inclined to default. This relationship has been confirmed empirically.

Fifth, my model predicts that as domestic investment generates high returns relative to the risk-free rate, sovereigns have incentive to increase public investment. They issue a large amount of foreign debt, which raises credit risk, as has been documented empirically.

In contrast to this paper, existing studies of sovereign debt do not attempt to explain, or have difficulty in explaining, sovereign credit-spread changes. A first strand of literature, launched by the seminal contributions of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), addresses why sovereign lending takes place (costs of future access to credit, trade, and financial markets, and retaliatory actions by way of sanctions). This literature does not provide a clear understanding of why a sovereign defaults, or of when it defaults. A more recent strand of studies, for example Gibson and Sundaresan (2001), Westphalen (2002), and Francois (2006), offers a contingent-claims framework for pricing sovereign debt in the presence of strategic default. These models do not motivate why the sovereign should have debt in the first place. I merge these two strands of literature and derive the endogenous sovereign default policy and its optimal debt level.

The theory that I propose is based on structural models for valuing corporate debt that use contingent-claims analysis. My model assumes that default is triggered when the revenues of the sovereign’s economy fall below an endogenous default boundary that depends on economic conditions.

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fundamentals. Once default occurs, the sovereign and its lenders renegotiate the terms of their debt contracts. The sovereign endogenously determines the amount of debt to issue, based on a trade-off between the benefit of increasing domestic investment and the expected costs of bearing economic sanctions upon default.

The bargaining game at default between lenders and a sovereign determines an endogenous reduction of the contractual debt, rather than a full repudiation of the debt. Within the model, the sovereign benefits from the restructuring process through less severe economic sanctions. Lowering the costs of default increases the incentive of the sovereign to default. Although the potential of a debt restructuring provides lenders some protection against loss, credit risk actually rises if renegotiation is allowed. The importance of incorporating renegotiation upon default in the model is evident in the degree of debt reduction that has been observed in sovereign default episodes (Moody’s, 2006).

The rest of the paper is organized as follows. Section 2 outlines a theory of endogenous default and debt policy for a sovereign. Section 3 offers some theoretical predictions of the model for the relationships between sovereign credit risk and its determinants. An estimation of the structural model based on EMBI+ spread data is provided in Section 4. I conclude in Section 5.

2 A theory of sovereign default

I assume a sovereign that raises external funds only through a single type of foreign debt, which offers limited recourse to the lenders in the case of non-payment. The sovereign chooses the quantity of debt to issue and decides when to default. Upon default, the sovereign and its lenders renegotiate the terms of the debt contract at no cost. Essentially, the renegotiated debt can be considered as a value-redistribution between the sovereign and the foreign debtholders. The sharing rule of the sovereign’s revenues at default results from a Nash bargaining game, as proposed by Fan and Sundaresan (2000), François and Morellec (2004) in the corporate case, or Gibson and Sundaresan (2001), François (2006) in the sovereign case. Lenders anticipate the potential for renegotiation which therefore affects the price of the sovereign debt at issuance. I will solve the model and highlight its predictions in terms of bond pricing and default risk.

I first lay the main assumptions of the model. Second, I price the debt security given a default and financing policy. Third, I determine the default policy that maximizes the value of the economy minus the value of the outstanding debt, and I derive the endogenous level of sovereign
debts. Fourth, I investigate the bargaining game upon default and determine the optimal level of debt reduction. Solving the model backwards, I finally obtain the price of the debt and compute the sovereign credit spread.

2.1 Assumptions of the model

Capital markets are frictionless, and all investors have the same information. All variables in the model are measured or denominated in the currency of the lenders. The default-free term structure is flat with an instantaneous riskless rate \( r \) at which investors may lend and borrow freely.

2.1.1 Dynamics of the economy before defaulting

I first consider the dynamics of the economy before the sovereign defaults on its debt. Until default, the infinitely lived country has revenues \( x_t \), at time \( t \) satisfying

\[
dx_t = \mu x_t dt + \sigma x_t dZ_t, \quad x_0 > 0,
\]

where the process \( Z_t \) is a Brownian motion defined on the probability space \((\Omega, \mathcal{F}, P)\). The standard filtration of \( Z_t \) is \( F = \{\mathcal{F}_t : t \geq 0\} \). The constant \( \mu \) is the mean growth rate of revenues.

2.1.2 Costs of defaulting

Bulow and Rogoff (1989) suggest three reasons why a sovereign makes repayments on its foreign debt and, thus, why investors provide loans. First, lenders may be able to appropriate collateral. However, the sovereign assets that would be accessible to creditors, in the event of repudiation, are worth only a small fraction of the outstanding level of debt (Bulow and Rogoff, 1989). Second, there is a reputation effect for the sovereign, relevant to future borrowing opportunities. Empirical support for the reputation effect is weak (Eichengreen, 1987). Gelos, Sahay, and Sandleris (2004) provide evidence that sovereigns are able to regain market access as quickly as three and half months, on average, after defaulting. Finally, debt repudiation can impede the ability of the sovereign to trade, and the country can be blocked from normal access to trade credits (Cline, 1987). Rose (2005) analyzes a panel data set covering 50 years and more than 150 countries, finding that debt renegotiation is associated with a decline in bilateral trade of 8% a year that persists for around 15 years. Reinhart et al. (2003) and Sturzenegger and Zettelmeyer (2006) also
provide evidence that the costs to defaulting on external debt can be significant for a country’s trade.

I then model the threat of losing trade-market access as the unique motivation for repayment.\textsuperscript{10} Frankel and Romer (1999) have provided persuasive evidence that trade is a significant part of economic growth. Because trade sanctions reduce access to the international market, defaults lower economic growth.

### 2.1.3 Dynamics of the economy after default

A sovereign loan is said to be "in default" when the sovereign fails to repay the debt on schedule. I assume that, in the event of default, the economy incurs a reduction in revenue growth rate. Once default occurs, the dynamics of the economy is driven by changes in revenues $dx_t^A$ with

$$dx_t^A = (\mu - \lambda)x_t^A dt + \sigma x_t^A dZ_t, \quad x_0^A > 0,$$

where the reduction $\lambda$ of the growth rate depends on the reduction in the level of trade. The larger is the sovereign as a trading partner, the more severe are trade sanctions. As in Hayri (2000), lenders can apply economic sanctions that cut off the revenue flow by disrupting international trade, but they cannot receive any revenue from this source. Lenders are indifferent to trade sanctions as they switch to other trading partners at no cost.

Consistent with prior sovereign debt crises, defaults tend to occur in economic downturns (Reinhart et al., 2003). In a recession, the sovereign is likely to repudiate debt to obtain a debt relief, as debt service is a large part of sovereign’s revenues.\textsuperscript{11} The sovereign strategically declares default on its debt obligation when the country’s revenues fall under an endogenous default boundary $x^N < x_0$. Reorganization of the debt contract with lenders is initiated once,\textsuperscript{12} at $T(x^N) = \inf\{t \geq 0 | x_t \leq x^N\}$.

\textsuperscript{10}Gibson and Sunderasan (2001) also link trade sanctions to sovereign debt default. The authors assume (p.14) that “if there are no exports, there is no incentive in our model to engage in borrowing”. Within their model, being prevented from trade is not a sanction in case of default; it is linked to the absence of debt.

\textsuperscript{11}Issuing new debt to finance existing interests would be a poor alternative as investors would lend at an unsustainable credit spread.

\textsuperscript{12}Sovereigns do not tend to default once but several times (Reinhart et al., 2003). Generalizing the framework to account for multiple defaults is left for future research.
2.2 Sovereign debt pricing

The debt policy is determined by an infinite maturity debt contract, characterized by a level $D$ and a continuous debt service $c$ until default. Foreign lenders are risk-neutral and require a rate of return $r$ per unit of time. I first determine the debt value given a default and debt policy. Using Ito’s lemma, the value of the perpetual debt satisfies

$$rD = c + \mu x D_x + \frac{1}{2} \sigma^2 x^2 D_{xx}$$

where $D_x$ and $D_{xx}$ are the first and the second derivatives of the sovereign debt value $D$ with respect to the stochastic revenues $x$. The solution to this ordinary differential equation is determined subject to a number of conditions. First, when $x$ tends to infinity, the value of the sovereign debt tends to the value of a risk-free debt

$$\lim_{x \to \infty} D(x) = E \left[ \int_0^\infty ce^{-rt} dt \right] = \frac{c}{r}$$

Second, lenders must value the foreign debt upon default, which depends on the outcome of the renegotiation. I determine the value matching conditions that impose equality between the value of the sovereign debt and the value of the cash flows accruing to debtholders in default.

Upon default, the sovereign and its lenders agree on a reduction of the debt service. Restructuring the terms of the debt contract is in line with sovereign debt crises. Out of the 69 defaults recorded by Standard and Poor’s in the period 1970-1997, 68 were subject to renegotiation. At default time $T(x^N)$, sovereign debt has value

$$\lim_{x \to x^N} D(x) = \frac{c(1 - \phi)}{r}$$

where $0 \leq \phi \leq 1$ denotes the renegotiated reduction of the debt service, which does not result in wasteful public asset sales. I assume that the sovereign cannot scale up its debt after default.

The value of the sovereign debt, associated with the relevant boundary conditions (Eq. 4 & 13)
\[D(x) = E \left[ \int_0^\infty c e^{-rt} dt \right] - E \left[ \int_T^{\infty} c e^{-rt} dt \right] \]  
(6)

\[= \frac{c}{r} \left( 1 - \phi \left( \frac{x}{x^N} \right)^\beta \right) \]  
(7)

where \( \beta \) is the negative root of the quadratic equation \( \frac{1}{2} \sigma^2 \beta (\beta - 1) + \mu \beta - r = 0 \),

\[\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2rd}{\sigma^2}} < 0 \]  
(8)

The value of the sovereign debt \( D(x) \) is equal to a riskless perpetual debt with continuous coupon \( c \) minus a default premium. This premium corresponds to the present value of the unrecovered value of the debt after default, where the stochastic discount factor is defined as the Arrow-Debreu price of default \( E^{x_0} [e^{-rT(x^N)}] = \left( \frac{x}{x^N} \right)^\beta \). Lenders anticipate the opportunistic behavior of the sovereign and reflect the associated wealth extraction in the pricing of the sovereign debt.

### 2.3 Value of the economy

Issuing foreign debt permits to foster domestic investment, and thereby generates economic value. Through exclusive access to public investment, and due to the presence of financial constraints, the sovereign can obtain high returns on investment. Investing domestically generates a return \( r_g \), larger than the risk-free rate \( r \), and the discounted benefit of issuing one unit of debt is

\[\alpha = E \left[ \int_0^\infty (r_g - r)e^{-rt} dt \right] = \frac{r_g - r}{r}. \]

The value of the economy \( V(x) \) is determined by

\[V(x) = E \left[ \int_0^\infty x_t e^{-rt} dt \right] + (1 - \eta)E \left[ \int_T^{\infty} (x^A_t - x_t) e^{-rt} dt \right] + \alpha D(x) \]  
(9)

\[= \frac{x}{r - \mu} - (1 - \eta) \left( \frac{x^N}{r - \mu} - \frac{x^N}{r - \mu + \lambda} \right) \left( \frac{x}{x^N} \right)^\beta + \frac{\alpha c}{r} \left[ 1 - \phi^\lambda \left( \frac{x}{x^N} \right)^\beta \right] \]  
(10)

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14Sovereigns use debt to invest domestically through public expenditure, as empirically documented by Lora (2007) for seven Latin American countries in the period 1987-2001. Public infrastructure investment is in general exclusively undertaken by state-owned enterprises or local governments. Legal restrictions can also play an important role in preventing foreign investors in capturing these returns.

15Financial constraints, arising from the insufficient liquidity in emerging economies to finance domestic investment, increase returns on investment. This assumption draws from the literature on aggregate liquidity shortage (e.g. Holmstrom and Tirole, 1998, Caballero and Krishnamurthy, 2005).
where $\eta$ is the sovereign bargaining power upon renegotiation. The first term of the right-hand side of Eq. (10) is the value of the economy in the absence of debt. The second term is the costs of the economic sanctions upon default multiplied by the Arrow-Debreu price of this event. I assume the severity of the trade sanctions to decrease with the bargaining power of the sovereign relative to that of the lenders,\(^{16}\) and to increase with the discounted value of the potential trade loss \(\left( \frac{x^N}{r-\mu} - \frac{x^N}{r-\mu+\lambda} \right)\). Finally, the third term is the discounted benefits of issuing debt. Sovereign debt thus directly increases the value of the economy through productive domestic investment and indirectly decreases it by affecting the expected drift of the economy’s growth path.

2.4 Optimal default boundary and debt policy

The sovereign’s default policy is characterized by \(x^N\). It is chosen to maximize the value of the economy minus the value of the sovereign debt, which I define as the net sovereign value \(N(x) = V(x) - D(x)\),\(^{17}\) such that the smooth-pasting condition \(\frac{\partial N(x)}{\partial x} \mid_{x=x^N} = \frac{\eta^2}{r-\mu} + \frac{1-\eta^2}{r-\mu+\lambda}\) is satisfied:

\[
x^N = \frac{c\beta(1-\alpha)}{r \left[ \frac{\beta(1-\eta^2)-1}{r-\mu} - \frac{\beta(1-\eta^2)}{r-\mu+\lambda} \right]}
\]

(11)

The endogenous sovereign debt service \(c\) results from a trade-off between the benefits of issuing debt and the risk of having revenues growing at a slower pace after default. The optimal debt level maximizes the net sovereign value \(N(x)\) after debt has been issued plus the proceeds from the debt issue \(D(x)\). While the default boundary \(x^N\) is selected \textit{ex post} to maximize the net sovereign value \(N(x)\), the optimal debt leverage is determined \textit{ex ante} to maximize the value of the economy \(V(x)\). The optimal debt service is

\[
c^* = \arg \max_{c \in \mathbb{R}^+} V^N (x, x^N)
= \left( \frac{x}{V} \right) \left[ (1-\beta) \left( 1 + \frac{A(1-\eta)\beta}{B \left( \frac{1-\alpha}{\alpha} - \eta \right)} \right) \right]^{\frac{1}{\beta}}
\]

(12)

\(^{16}\)There is a unique bargaining power for all lenders, which necessitates a coordination between creditors, through the London or the Paris Clubs for example. This modeling assumption is in line with Hayri (2000) and François (2006).

\(^{17}\)Prior studies have not settled the debate on what exactly the objective function of the sovereign should be. Here, \(N(x)\) corresponds to the part of the economy controlled by the sovereign on behalf of its citizens, as lenders are foreign investors by assumption. Gibson and Sunderasan (2001) consider that the sovereign has an objective function that is linear in the total value of the country wealth \(V(x)\). In Westphalen (2002) and Bulow and Rogoff (1989), the sovereign seeks to maximize terminal country wealth.
with

\[
A = \frac{1}{r - \mu + \lambda} - \frac{1}{r - \mu} \tag{13}
\]

\[
B = \frac{\beta (1 - \eta^2) - 1 - \beta (1 - \eta^2)}{r - \mu + \lambda} \tag{14}
\]

\[
Y = \frac{\beta (1 - \alpha)}{r B} \tag{15}
\]

### 2.5 Bargaining game

#### 2.5.1 Renegotiation surpluses

I now determine the allocation of the renegotiation surpluses between a sovereign and its lenders.\(^{18}\)

The surpluses represent the benefits of renegotiating the debt contracts compared to a full repudiation. The net sovereign value at default is

\[
N(x) \big|_{x=x^N} = \frac{x^N}{r - \mu} - (1 - \eta) \left( \frac{x^N}{r - \mu} - \frac{x^N}{r - \mu + \lambda} \right) + \frac{(\alpha - 1) c (1 - \phi)}{r} \tag{16}
\]

where \(1 - \eta\) is the degree to which lenders sanction a sovereign upon renegotiation. When the sovereign fully repudiates its debt (\(\phi = 1\)), sanctions are imposed as if the sovereign had no bargaining power (\(\eta = 0\)). The net sovereign value is

\[
N(x) \big|_{x=x^N, \phi=1, \eta=0} = \frac{x^N}{r - \mu + \lambda} \tag{17}
\]

The difference between Eq. (16) and Eq. (17) represents the surplus on the sovereign side and equals

\[
\Delta N(x) \big|_{x=x^N} = \eta \left( \frac{x^N}{r - \mu} - \frac{x^N}{r - \mu + \lambda} \right) + \frac{(\alpha - 1) c (1 - \phi)}{r} \tag{18}
\]

On the lender side, the renegotiated debt value at default \(D(x) \big|_{x=x^N}\) is (see Eq. 5)

\[
D(x) \big|_{x=x^N} = \frac{c (1 - \phi)}{r} = \Delta D(x^N) \tag{19}
\]

whereas, in case of full repudiation, the sovereign debt has no value.\(^{19}\) Because the sovereign is

\(^{18}\)The analysis departs from the bargaining game of François (2006), which is characterized by a renegotiation between the whole country and the lenders. Because lenders are claimholders of the country, the author considers them to be on both sides of the renegotiation. I propose that the renegotiation of the debt should be between the sovereign and its lenders.

\(^{19}\)In the corporate case, the value of the debt at default equals the residual value of the firm, following Merton
certain to be fully sanctioned, it will not freely transfer wealth to existing lenders.  

2.5.2 Optimal renegotiated debt reduction $\phi^*$

I consider a Nash bargaining game to determine the new renegotiated debt service. The sharing rule for the renegotiation surpluses upon default satisfies the Nash solution characterized as

$$
\phi^* = \arg \max_{0 \leq \phi \leq 1} \left[ \Delta N(x^N) \right]^\eta \left[ \Delta D(x^N) \right]^{1-\eta} 
$$

$$
= \arg \max_{0 \leq \phi \leq 1} \left[ \eta \left( \frac{x^N}{r - \mu} - \frac{x^N}{r - \mu + \lambda} \right) + \frac{(\alpha - 1)(1 - \phi)}{r} \right] \left[ \frac{c(1 - \phi)}{r} \right]^{1-\eta} 
$$

(20)

(21)

On one side, the sovereign benefits from higher output growth (compared to defaulting in the absence of renegotiation) as trade sanctions are partially avoided. It must however continue to pay a fraction of the debt service. On the other side, debtholders benefit from receiving this (lower) debt service. The outcome of the renegotiation process allocates both surpluses between the sovereign and its lenders according to their bargaining power.

As long as both parties have some bargaining power ($0 < \eta < 1$), there is a unique reduction $\phi^*$ of the debt service that maximizes the surplus allocation between the two parties:

$$
\phi^* = 1 + \frac{\left( \frac{1}{r - \mu} - \frac{1}{r - \mu + \lambda} \right) x^{N*} r \eta (1 - \eta)}{c^*(\alpha - 1)} 
$$

$$
= 1 - \frac{A \beta \eta (1 - \eta)}{B} \geq \bar{\phi} 
$$

(22)

(23)

The new debt service has value $c^*(1 - \phi^*)$. The solution of $\phi^*$ is independent on the initial level of the debt service $c^*$ and does not have a distribution support between 0 and 1, at least given the assumed specification. There is a lower bound $\bar{\phi}$ in the debt reduction, below which renegotiation is never observed. If the sovereign is not better off with renegotiation (because the costs of bearing the new debt service outweigh the benefits of having reduced economic sanctions), the sovereign will not renegotiate with its lenders. Therefore, the debt reduction $\phi^*$ must be above a threshold

$$
\bar{\phi} = \{ \phi : \Delta N(x)_{x=x^N=0} \}, \text{ at which the sovereign is indifferent between renegotiating or not.}
$$

(1974) and Black and Cox (1976). I depart from this case since there is no formal international bankruptcy court that would allow the debtholders to seize part of the existing assets.

In François (2006), a lump sum payment is still accrued to the debtholders upon default, which implies that some of the assets are seizable. However, Bulow and Rogoff (1989) suggest that the sovereign assets that would be accessible to creditors, in the event of repudiation, are negligible relative to the outstanding level of debt.

Parties would not be able to write a contract to guarantee such a payment. Nothing would prevent the sovereign to deviate at default, thus not honoring the contract.
2.6 Credit spread

The market yield spreads is an important measure of the market’s perception of default risk. Under the risk-neutral measure, the credit spread is obtained for two fundamental reasons: First, there is a risk of default, and second, in the event of default, the lenders receive only a fraction of the promised payments. Credit spread is computed as follows

\[ cs(x, c^*) \equiv \frac{c^*}{D(x, c^*)} - r \]

\[ = \frac{r \left( \frac{x}{x^N} \right)^\beta}{1 - \phi^* \left( \frac{x}{x^N} \right)^\beta} \tag{25} \]

3 Empirical predictions and analysis

Several theoretical papers, such as Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000), obtain an equilibrium in which sovereigns never default. Default premia are unobservable as the probability of defaulting is null. In contrast to these studies, my structural model can be used with scenario and simulation analysis to evaluate the effect of shocks and policies on credit risk. This section explores the predictions of the relationship between macro-variables provided by the model and sovereign credit risk. It disentangles the short-term and the long-term effects. Results are derived analytically but the analysis is based on numerical examples.

3.1 Determinants of credit risk

3.1.1 Economic growth

A fast-growing economy can easily bear the burden of the foreign debt, as debt service is proportionally a small part of sovereign’s revenues. Therefore, the incentive of the sovereign to increase leverage rises with expected economic growth.

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21 The credit spread prevailing in the absence of renegotiation is simply defined by \( cs(x, c^*) = \frac{r \left( \frac{x^g}{x} \right)^\beta}{1 - \phi^* \left( \frac{x}{x^N} \right)^\beta} \), with \( x^D = \frac{c^*(1-\lambda)}{r \left( \frac{x}{x^N} \right)^\beta} \).

22 I select parameter values to reflect the situation of emerging market economies. For each figure, I summarize the parameter choices for the interest rates \( r_g \) and \( r \), the initial revenue of the sovereign’s assets \( x \), the growth rate of the revenues \( \mu \), the volatility of the revenues \( \sigma \), the loss in output growth after default \( \lambda \), and the benefits \( \alpha \) of issuing debt. The base case parameters are \( \lambda = 4\% \), \( x = 100 \), \( r = 6\% \), \( r_g = 9\% \), \( \mu = 0.05\% \), \( \sigma = 30\% \), \( \eta = 70\% \).
The analysis (Figure 1, upper-left panel) suggests that credit spread decreases with economic growth, although growth also raises the level of debt. Empirical analyses\(^23\) obtain a negative effect of GDP growth on credit spreads and confirm the model prediction. My model generates countercyclical bond spreads, in line with the theoretical predictions of Yue (2005), assuming a risk-averse country, and Arellano (2006), using an incomplete market approach. In contrast, Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000) offer models in which default incentives are higher in periods of high output, at odds with the empirical evidence that I have cited.

### 3.1.2 Economic volatility

Economic volatility has two effects on credit spreads. First, for a given level of debt, the probability of defaulting increases with volatility. Second, because sovereign debt becomes costly, the sovereign reduces its level of foreign debt. The first effect dominates and greater macro-volatility in the economy is predicted to raise credit spreads (Figure 1, upper-right panel).

Catao and Sutton (2002) and Catao and Kapur (2004), using a panel of 25 emerging market countries for the period 1970-2001, confirm the positive effect of policy induced volatility (fiscal, monetary, and exchange rate policy) on sovereign credit risk. Westphalen (2001) shows that changes in the stock market volatility, proxying for changes in the volatility of the country wealth process, have a significant and positive effect on credit spreads in a sample of 215 sovereign bonds.

### 3.1.3 Risk-free interest rate

My model predicts that credit risk increases with risk-free interest rates (Figure 1, middle-left panel), consistent with the empirical evidence documented by Catao and Sutton (2002) and Catao and Kapur (2004) for credit spreads and Haque et al. (1998) and Monfort and Mulder (2000) for credit ratings. This prediction is in contrast with the theoretical results of Gibson and Sundaresan (2001) and Westphalen (2001).

### 3.1.4 Economic sanctions upon default

The presence of trade sanctions partially insures lenders against default and lowers credit spreads (Figure 1, middle-right panel). When economic sanctions are potentially severe, sovereigns are less

inclined to default and issue less debt. If a sovereign does significant trade, the impact of economic sanctions is large, suggesting a negative relation between the level of exports and credit risk. This prediction is in line with Reinhart et al. (2003), who analyze the effect of the exports-to-external debt ratio on credit risk in the period 1979-2000 for 16 emerging market economies and show a strong negative correlation. Ades et al. (2000) and Rowland and Torres (2004) also provide empirical evidence that the exports-to-GDP ratio negatively affects sovereign spreads, respectively in a sample of 15 and 16 emerging market economies over the period 1996-2000 and 1998-2002.

3.1.5 Gain of issuing foreign debt

The more productive is domestic investment relative to the risk-free rate, the larger the debt, which thereby increases the credit spread (Figure 1, lower-left panel). This observation is consistent with Edwards (1984) and Cosset and Roy (1991), who highlight the negative role of the propensity to invest in explaining credit spread changes. Their variable captures the country’s perspective for obtaining a large return on domestic investment financed with foreign debt.

3.2 Short-term versus long-term effects

In the long run, sovereigns incorporate the available information into their debt and default policy. The amount of debt is endogenous and maximizes the value of the economy. Matters can be quite different in the short run as the financing policy cannot be adapted as quickly as desired. For instance, if economic conditions deteriorate sharply, sovereigns would optimally respond by reducing their outstanding level of foreign debt. This debt reduction may be infeasible because only a tiny proportion of the debt tends to mature in the very short term.24 In the short run, sovereigns certainly endogenize their default policy but not their level of debt.

3.2.1 Differences in predicted spreads

In Figure 2, I compare credit spreads predicted by the model developed in Section 2 with those predicted by a model assuming a fixed debt policy.25 In comparison to the long-run, short-run credit spreads are more sensitive to changes in both economic growth and risk-free rate, suggesting

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24Bi (2006) shows that the pre-crisis average maturity of foreign debt (and respectively the average maturity excluding pre-crisis period) is 6.5 (9.8) years for Argentina, 9.2 (17.9) for Brazil, 4.9 (11.6) for Mexico, 8.8 (14.7) for Russia, and 4.9 (7.9) for Turkey.

25The debt service \( c \) is fixed at 80% of the initial revenue of the economy \( x \), implying an average debt-to-GDP ratio of 35%.
an overshooting effect in the short term (*Figure 2*, upper and middle panels). Spreads approach their long-term values until the optimal debt level is finally attained. In the long run, the optimal debt level decreases with the risk-free rate and increases with output growth. In contrast, the effect of a change in volatility on credit spreads is enhanced in the long term because of the incentive to raise the level of debt (*Figure 2*, lower panel).

3.3 Renegotiation and credit spreads

Within the model, the sovereign benefits from the restructuring process through less severe economic sanctions. Lowering the costs of defaulting increases the incentive of the sovereign to default. Although the potential for debt restructuring provides the lenders some protection against loss, credit spreads are higher when renegotiation is allowed (*Figure 1*).\(^{26}\) As illustrated by *Figure 1*, the potential for renegotiation has a much stronger effect on the explanation of the credit spreads *in levels* than *in changes*.

When varying the base case parameters, the estimated debt reduction upon default ranges between 70% and 90%, which is relatively consistent with the data. For instance, Moody’s (2006) investigates the recovery rates on defaulted sovereign issuers between 1998 and 2005 and report a value weighted recovery rate of 33%.\(^{27}\) Pan and Singleton (2008) have surveyed credit default swaps traders for developing countries and mention a consensus of expected loss on sovereign debt upon default around 75%. My model also predicts that the greater the sovereign bargaining power, the larger the debt reduction upon renegotiation. Argentina and Russia, two powerful countries at time of default, indeed experienced a much larger debt reduction than other countries. The outcome of the bargaining game seems consistent with sovereign default crises.

\(^{26}\)For the base case parameters, spreads are about 50% larger with than without renegotiation. This result stands in contrast with the analysis of François (2006). Within his model, credit spreads are predicted to be lower when debt restructuring is allowed. Both models differ in many important aspects: I incorporate the endogeneity of the debt policy, reverse the sequence of his model, allow the economic sanctions to depend on the bargaining powers, and finally reformulate the bargaining game to avoid double-counting of the debtholders’ side.

\(^{27}\)The trading prices on the sovereign’s bonds thirty days after default were 65% of par for Pakistan (1998), 18% for Russia (1998), 44% for Ecuador (1999), 18% for Ivory Coast (2000), 69% for Ukraine (2000), 33% for Argentina (2001), 66% for Moldova (2001), 66% for Uruguay (2003), 65% for Grenada (2004), and 92% for Dominican Republic (2005).
4 An application of the structural model on EMBI+ data

The previous section has suggested that my model’s predictions are in line with the empirical literature that I have cited. I now consider the model to explain the dynamics of credit spreads in emerging debt markets at the daily frequency. Traded stock market prices are assumed to infer the market’s expectation on the economy. I generate estimates of daily credit spreads implied by the structural model for Brazil, Mexico, Peru, and Russia over the period 1998-2006, and compare these estimates with observed EMBI+ spreads. While the literature so far has difficulty in understanding credit spread movements, my model explains about 92% of the variation in EMBI+ spreads in a panel data analysis. The explanatory power rises only slightly, to 94%, when accounting for additional time-varying factors such as 5-year U.S. Treasury rates and the VIX option-implied volatility index. The predicted credit spreads also reproduce most of the time-series properties of the EMBI+ spreads in terms of the first four moments.

4.1 Empirical methodology and data

I compare estimated and observed credit spreads for Brazil, Mexico, Peru, and Russia, four important issuers with different geopolitical characteristics and levels of credit risk. As a benchmark, I consider the emerging market bond index (EMBI+) spread. JPMorgan, one of the major dealers in the Brady market, derives the credit spread implied in the price of each Brady bond. The company computes each country’s EMBI+ index, which is a weighted average index on spreads using the country’s most liquid Brady bonds. A focus on Brady bonds rather than other emerging market instruments is likely to improve the analysis of credit risk as they are by far the most liquid and the largest emerging debt market. I assume the state of each economy to be determined by the daily price of each country’s IFCG stock market index (measured in U.S. Dollars). As suggested by Figure 3, stock market indices and credit spreads covary together, although in opposite direction. Establishing a clear causality between these two variables constitutes a challenge as both prices are likely to be priced in the markets simultaneously. I only use stock market prices to infer the best forecast on the state of the economy, not to explain their underlying relation.

Figure 3 [about here]

Regarding the sources, data for this section are taken from Datastream for IFCG stock market indices, the VIX index, and 5Y U.S. Treasuries rate series, and CBonds for EMBI+ spreads. All
series consist of 2329 daily observations from January 1 1998 to December 31 2006 (see Table 1 for the descriptive statistics).

Table 1 [about here]

4.2 Computation of the model implied spreads

Before exploring the econometric specifications and results, I illustrate how the structural model is applied to the data. Throughout the following analysis, the credit spread $CS_{Model}$ predicted by the model, for country $i$ at day $t$, is determined by

$$CS_{Model,i,t} = \frac{r \left( \frac{x_{i,t}}{x_i^N} \right) \beta_i}{1 - \phi_i \left( \frac{x_{i,t}}{x_i^N} \right) \beta_i}$$

(26)

where the debt reduction upon default is

$$\phi_i = 1 - \left( \frac{1}{r-\mu} - \frac{1}{r-\mu+\lambda} \right) x_i^N r \eta (1 - \eta)$$

(27)

and the default boundary is defined by

$$x_i^N = \frac{c_i \beta_i (1 - \alpha)}{r \left( \frac{\beta_i (1 - \eta^2) - 1}{r-\mu} - \frac{\beta_i (1 - \eta^2)}{r-\mu+\lambda} \right)}$$

(28)

with

$$\beta_i = \frac{1}{2} - \frac{\mu}{\sigma_i^2} - \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma_i^2} \right)^2 + \frac{2r}{\sigma_i^2}} < 0$$

(29)

4.2.1 Debt service and indebtedness level

As discussed in Section 3.2, the endogeneity assumption of the debt level is unlikely to hold in the short term. When economic conditions deteriorate sharply, the sovereign may need several months or years to reduce its outstanding level of foreign debt. Financing thus becomes sticky because of the long maturity feature of the external debt contracts. Therefore, the predicted credit spreads are derived from a model that assumes a fixed debt policy.\textsuperscript{28}

\textsuperscript{28}When both the financing and the default policies are endogenous, the credit spread measure becomes independent on the state variable $x$, and thus remains constant over time.
For each country, I adopt a constant level of debt service $c_i$ to reflect the average indebtedness level (measured by the debt-to-GDP ratio) provided the World Bank. They are respectively 21%, 34%, 44%, 31%\textsuperscript{29} for Brazil, Mexico, Peru, and Russia. My model predicts country specific debt-to-GDP ratios $I_{i,t} \equiv \frac{D_i}{V_i}$ that are time-varying, as suggested by Figure 4, reproducing the decreasing trend of the World Bank’s debt-to-GDP ratios.

Figure 4 [about here]

### 4.2.2 State of the economy

I assume that investors’ views on stock market prices reflect the best available forward-looking information about the economic prospects of a country. The level of country $i$’s stock market price index at time $t$ provides information on the state of revenues $x_{i,t}$, although the state of the economy is best captured in my model by $V(x)$. This relationship is made possible because $V(x)$ is almost affine in revenues $x$. In theory, as early suggested by Leland (1994), the relation between cash-flows $x$ and the value of the assets $V(x)$ is non-linear.\textsuperscript{30} As Figure 5 illustrates, the concavity appears to be weak and assuming a linear relation between $x$ and $V(x)$ (middle panel) is certainly not a very restrictive assumption.

Figure 5 [about here]

Another approach would be to account for data on the national GDP (representing $V(x)$ in the model) to directly determine the state of the economy. This possibility would raise a few concerns: GDP data is only available quarterly and with a time reporting lag. In addition, it only provides an \textit{ex post} measure of economic performance. In contrast, financial markets account for higher-frequency forward-looking information on the state of the economy.

\textsuperscript{29}The numbers are estimated by averaging the debt-to-GDP ratios for 1996 and 2005. These indicators are found the World Bank’s website but initially appear in the Development Economics LDG Database.

\textsuperscript{30}For a given coupon payment $c$, we can see from Eq. (10) that $\frac{\partial V(x,c)}{\partial x} = \frac{1}{r_x} + kx^{\beta-1}$, where $k$ is a positive constant, suggesting that the value of the economy $V(x)$ is strictly concave in revenues $x$. This effect only obtains when the coupon payment is constant.

With an endogenous financing policy, the relation is strictly linear, and the slope is given by $\frac{\partial V(x)}{\partial x} = \frac{1}{r_x}$. More formally, when a sovereign optimally selects leverage through the debt coupon $c$ in Eq. (12), the default boundary $x^N$ in Eq. (11) is linear in $x$. Hence, the stochastic factor $(\frac{\partial V(x)}{\partial x})^\beta$ becomes independent on $x$, suggesting a linear relation between revenues $x$ and the value of the economy $V(x)$.  

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4.2.3 Model parameters

Consistent with the theoretical model, the state variable $x$ offers the only time-varying information accounted for in the computation of the credit spreads. Following Eq. (1), changes in sovereign asset value $x$ follows a diffusion process with constant country-specific volatility. The volatility parameter $\sigma_i$ is estimated as the unconditional volatility of the daily IFCG stock market index returns. The remaining model inputs are kept constant over time and across countries. The risk-free rate $r$ is determined by the average of the daily 5-year U.S. Treasury rates. In addition, the repartition of the bargaining power upon renegotiation is assumed to be 70% ($\eta = 0.7$) for the sovereign and 30% for the debtholders, the reduction of the country growth rate upon default $\lambda$ is 2%, and finally the return on domestic investment $r_g$ is 40% higher than the risk-free $r$ rate (for example, if $\alpha = \frac{r_g - r}{r} = 0.4$ and sample average $r = 4.48\%$, then $r_g = 6.27\%$). The credit spreads that are in line with the model assumptions. Information on neither debt prices nor EMBI+ spreads is accounted for in the computation of the predicted credit spreads.

4.3 Predicted versus observed credit spreads

I now examine the ability of the credit spreads predicted by my model in explaining the dynamics of EMBI+ spreads. All the presented estimations rely on a panel data analysis.

4.3.1 Estimation in levels

The first estimated model considers credit spread measures in levels. The specification is as follows

$$\ln (CS_{EMBI,i,t}) = \gamma_1 + \gamma_2, i \ln (CS_{Model,i,t-1}) + \omega_i + \tau_t + \nu_{i,t}$$

(30)

where $CS_{EMBI}$ and $CS_{Model}$ respectively stand for the daily observed EMBI+ spreads and the credit spreads predicted by my model for country $i$ at time $t$ and $t-1$. The lag structure helps in (partially) avoiding the endogeneity issue and determines whether there is potential predictive power in the estimation. I account for country-specific coefficients of elasticity $\gamma_{2,i}$ to allow for heterogenous relations across countries. In addition, $\omega_i$ characterizes country-specific effects and $\tau_t$ controls for time specific effects. Finally, $\nu_{i,t}$ is the error term.

The last two decades have experienced an increasing economic and financial integration, which implies substantial interdependencies between emerging market economies. Such cross-sectional
dependence may arise due to the presence of common shocks and unobserved components that become part of the disturbance term.\textsuperscript{31} There is also substantial serial correlation in credit spreads. I then correct the heteroskedasticity consistent standard errors to account for very general forms of temporal and cross-sectional dependence in the panel data by applying Driscoll and Kraay’s (1998) nonparametric covariance matrix. Reported regressions are estimated with a Fixed Effect model.\textsuperscript{32} Although not reported for space consideration, all the results are robust to sample splits and thus not specific to the period 1998-2006.

Table 2 [about here]

Results in Table 2 (Analysis 1a) exhibit a very high explanatory power. Although the stock market index price is the only time-varying variable, the structural model explains more than 91% of the time variation in daily EMBI+ spreads for Brazil, Mexico, Peru, and Russia. In comparison, the correlation between stock market index levels and EMBI+ spreads is merely -0.35 (see Table 1). As presented in Table 3, the predicted and the EMBI+ spreads have also very comparable sample characteristics - in terms of mean, standard deviation, skewness, and kurtosis. Although the predicted spreads consistently underestimate observed spreads, the model is able to generate the positive asymmetry and the fat tailness present in the data.

Table 3 [about here]

At this point, the closer comparable study is certainly that of Hilscher and Nosbusch (2007). The authors explain up to 48% of the time variation in EMBI+ spreads over the period 1993-2004 using a set of macro variables. They also propose a structural and a reduced-form model, which do not improve the power of the explanation. Their cross-section estimation explains 61% of credit spread variations, when either fundamentals or spreads predicted by their model are used as explanatory variables. A significant large part thus remains unexplained, although the authors consider a large number of variables in their estimation. As a likely explanation for this outcome, the authors assume the variables to linearly affect credit risk, which is probably not a sound

\textsuperscript{31}If common factors are unobserved (and uncorrelated with the included regressors), the standard Fixed (or Random) Effects estimators are consistent. The estimated standard errors are however biased.

\textsuperscript{32}The choice of the appropriate model is determined here using the Hausman specification test. This test compares the Fixed versus Random Effects under the null hypothesis that the individual effects are uncorrelated with the other regressors in the model. Within this specification, the individual effects are correlated (and thus $H_0$ is rejected). A Random Effect model would produce biased estimators, violating one of the Gauss-Markov assumptions; so a Fixed Effect model is preferred here.
assumption. Their study also investigates the credit risk components at the yearly frequency and consider the terms of trade to proxy the economic fundamentals within the model. In contrast, stock market indices would have permitted to increase the frequency of the data and to better account for information related to the state of the economy.

Results in this section also depart from those of Weigel and Gemmill (2006). These authors investigate whether variations in sovereign credit risk can be attributed to changes in common factors across countries rather than to country-specific factors. They find that regional and global factors are far more important than country-specific fundamentals, which explain 8% of changes in creditworthiness for Argentina, Brazil, Mexico, and Venezuela over the period 1994-2001. In contrast, I find that country-specific information explains most of the time variation in EMBI+ credit spreads (see Table 2). The discrepancy in results probably arises because these authors derive a credit risk measure that is computed from the past behavior of observed bond prices and credit spreads. I do not account for such information when computing the credit spreads predicted by the model.

4.3.2 Estimation in differences

The analysis provided so far focuses on the time variation in credit spread levels. I now investigate the time variation in credit spread changes. In the area of corporate bond pricing, Collin-Dufresne, Goldstein, and Martin (2001) show that the variables that should in theory (according to most of the structural models) determine credit spread changes have a limited explanatory power as low as 25%. The authors explore 688 bonds from 261 corporate issuers over the period July 1988 through December 1997. In the case of sovereign bonds, structural models are scarce and such empirical testing has not been carried out yet. This paper is then a first step in this direction. The estimated model is

$$\Delta CS_{EMBI,i,t} = \delta_1 + \delta_2 \Delta CS_{Model,i,t} + \omega_i + \tau_t + \nu_{i,t}$$  \hspace{1cm} (31)$$

Reported regressions are now estimated with a Random Effects model and the heteroskedasticity consistent standard errors are still robust to general forms of spatial and temporal dependence using the Driscoll and Kraay’s (1998) covariance matrix (Table 2, Analysis 2a). Variation in the credit spread changes predicted by the model significantly explain the time variation in EMBI+
spread changes. As in the corporate case, the explanatory power at the within level is limited and only corresponds to 14%. Interestingly, all the coefficients - which do not represent the elasticity any more - are less than unity. This result implies that the predicted credit spread changes tend to overreact to new information incorporated in stock market prices. In other words, EMBI+ spread movements are more sluggish and less noisy.

4.3.3 U.S Treasury rate, stock market volatility, VIX index, and credit spreads

I now consider additional time-varying factors - considered to be constant within the model - that are likely to help better capturing the time variation in EMBI+ spreads. The first one is country stock market volatility, computed as the conditional volatility of the stock market log returns using an AR(3)-GARCH(1,1) process. As credit spreads depend by construction on the risk-free rate, I also include the 5-year U.S. Treasuries rates. The credit spreads are still generated by a model assuming constant risk-free rate and stock market volatility. Finally, I also include the VIX option volatility index as an additional explanatory variable.

Results in Table 2 show that the U.S Treasury rate and the conditional stock market volatility only slightly improve the quality of the estimation, either in levels (Analysis 1b) or in differences (Analysis 2b). The contribution of the U.S. Treasury rate, although significant at the 99% confidence level, appears to be only marginal. This finding departs from Catao and Sutton (2002) and Catao and Kapur (2004), who find that a change in the U.S. Treasury rate is a powerful predictor of the likelihood of default. Regarding stock market volatility, the effect on EMBI+ spreads is only slightly significant in levels and not significant in differences. This result is in line with Weigel and Gemmill (2006), confirming the constant volatility assumption of my model.

The VIX option volatility index, which captures a source of risk premium in the U.S. equity market, has recently attracted much interest. Pan and Singleton (2008) view the VIX, "a widely watched measure of event risk in credit markets, as a central ingredient in investors’ appetite for exposure to the high-yield bond credit class". In their results, the VIX is strongly correlated with sovereign spreads, suggesting that this index is a key factor in explaining credit risk movements. A similar finding is emphasized in Longstaff et al. (2007) in a reduced-form affine structure model based on Duffie and Singleton (1999) and Pan and Singleton (2008). I incorporate the VIX to investigate its additional effect on EMBI+ spreads (Table 2). The VIX index and EMBI+ spreads are highly correlated, with a coefficient of 0.4 (Table 1). Although the effect of the VIX index
is statistically significant, the VIX has limited scope for additionally explaining time variation in EMBI+ spreads. Accounting for stock market prices to predict credit spreads almost eliminates the VIX as an additional explanatory variable.

4.3.4 The distance-to-default measure

Following the approach of Moody’s KMV, it has become popular to consider the distance-to-default measure as a primary indicator of credit risk (e.g. Campbell, Hilscher, and Szilagyi, 2007, Duffie, Saita, and Wang, 2007, and Bodie et al., 2007). This measure computes the difference between the implied forward market value of the sovereign asset revenues and the default boundary scaled by the standard deviation of sovereign asset revenues. The distance-to-default measure \( DD_{i,t} \), for country \( i \) at time \( t \), thus captures the number of standard deviations the sovereign asset revenue is away from default, which is defined by

\[
DD_{i,t} = \frac{\ln\left(\frac{x^N}{x_{i,t}}\right) + \left(\mu - \frac{\sigma^2}{2}\right) T}{\sigma_i \sqrt{T}}
\]  

Bodie et al. (2007) develop a reduced-form contingent-claims model of sovereign credit risk and analyze the measure of distance-to-default for 11 developing countries in the period 2002-2004. They explain 80% of the variations in distance-to-default for 11 developing countries in the period 2002-2004. They explain 80% of the variations in sovereign spreads. In Table 4, I reproduce the analysis presented in Table 2 to investigate the explanatory power of the distance-to-default measure on EMBI+ spreads in levels (Analysis 3a, 3b, and 3c) and in differences (Analysis 4a, 4b, and 4c). Both the predicted credit spread and the distance-to-default indicator very similarly explain the time variation in EMBI+ spreads. The distance-to-default measure cannot be directly used to price sovereign debts. However, it appears to be a good indicator of country creditworthiness as it is closely related to the probability of defaulting.

Table 4 [about here]

4.4 Implications of the results

Most of the time variation in EMBI+ spreads over the period 1998-2006 can be explained by the credit spreads predicted by the structural model, with the stock market index price as the only time-varying explanatory variable. To better grasp the power of my model, Figure 6 illustrates the dynamics of the credit spreads predicted by the model compared with that of the EMBI+ spreads
for the period 1998-2006. Both series evolve very closely over time, suggesting that credit spreads are largely driven by the information on economic fundamentals incorporated in stock market prices. The structural approach for credit risk valuation offers an appealing theoretical framework in the understanding of the dynamics of sovereign credit risk spreads, which is an advantage over reduced-form models (e.g. papers based on Duffie and Singleton, 1999).

Figure 6 [about here]

4.4.1 Decomposing the components of credit spreads

The work of Black and Scholes (1973) and Merton (1974) has spawned an enormous theoretical literature on risky debt pricing. A common motivation in these studies is to provide a framework that is able to generate the high-yield spreads observed in the market. Recent research (e.g. Huang and Huang, 2003) suggests that credit risk accounts for only a fraction of the observed corporate yield spreads, even for very liquid bonds. Spreads also incorporate a risk premium, defined as the difference between expected realized return of a defaultable bond and that of a comparable Treasury bond. This premium essentially serves to compensate bondholders for credit risk, market risk, liquidity risk, and correlation risk.

A limitation of most structural models based on contingent-claims analysis is to only consider credit risk. Because other sources of risk are not accounted for, such models are generally unable to generate sufficiently high spreads. The difference $CS_{EMBI,i,t} - CS_{Model,i,t}$ is likely to capture the combination of the different risk premia and the remaining sources of risk. Results in Table 2 suggest that most of the time variation in observed spreads is explained by the time variation in the risk-neutral credit risk component. In addition, the average coefficient of elasticity $\gamma_{2,i}$ is close to unity. Hence, risk premia, in proportion to the predicted risk-neutral credit spreads, appear relatively constant over time. Studies suggest that corporate credit risk premia are highly time-varying (e.g. Berndt, Douglas, Ferguson, and Schranz, 2005, Ericsson and Elkamhi, 2007). This paper is a first step in shedding light on the dynamics of sovereign risk premia.

33The market risk is the possibility that bond prices may move against the bondholder and the liquidity risk (documented by Hund and Lesmond, 2007) is the risk that investors will not be able to liquidate their positions without depressing market prices. Finally, the risk premium also accounts for the correlation risk because of the tendency for default events to cluster, which becomes undiversifiable.
4.4.2 Final words on the low credit risk levels over the period 2003-2006

Recent years have seen a substantial and steady narrowing of sovereign spreads in emerging debt markets. Does this mean that the borrowers in these markets have become less risky? While much of the recent literature on sovereign spreads has not been very helpful in answering this question, my model’s answer is positive. Fundamentals in emerging market economies, measured by the performance of their financial markets, have significantly improved in the last 5 years (see Figure 3). The combination with low short-term interest rates in the major financial centers has created an excess liquidity since 2001, which led many institutional investors to make strategic portfolio allocations in emerging markets. As a result, returns and expectations on these stock market indices have been mostly rising. To the extant financial markets correctly forecast economic activity in emerging markets, the consistent improvement in sovereign creditworthiness over the recent years can then be explained (Figure 6).

5 Conclusion

A sovereign debt crisis is a pervasive phenomenon in emerging market economies that prevents sustainable economic growth. The basic idea of this paper is to show that defaulting is not exogenous to the sovereign’s decisions. It is an optimal outcome. When the choice of defaulting on foreign debt emanates from a value-maximizing behavior of the sovereign, the assessment of the country creditworthiness corroborates the empirics. This paper contributes to the literature in several ways. First, the paper sheds light on the role played by renegotiation between the sovereign and its lenders upon default and compares predictions on country credit risk. This model generates higher credit spreads when renegotiation is allowed because of greater incentive of defaulting. Second, the analysis of the renegotiation round upon default offers a rationale for the large debt losses observed in sovereign default crises. Last but not least, the paper investigates the power of the structural model in explaining the dynamics of sovereign credit risk. In contrast to existing studies, my model explains most of the time variation in daily observed EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998-2006.
References


**Figure 1: Credit Risk and Related Macro-Variables.** The figure illustrates the predicted relationships between economic growth, volatility, risk-free interest rates, economic costs of defaulting, the incentive of issuing debt, and sovereign credit spreads. It also shows how the potential for restructuring the terms of the debt contract at default affect credit risk. [Parameters: $\lambda=4\%$, $x=100$, $r=6\%$, $r_g=6\%$, $\mu=0.05\%$, $\sigma=30\%$, $\eta=70\%$]

**Figure 2: Short-Term versus Long-Term Credit Risk.** The figure compares the credit spreads predicted by a model with endogenous debt level (long-term effect) with those predicted by a model with constant financing policy (short-term effect). [Parameters: $\lambda=4\%$, $x=100$, $r=6\%$, $r_g=9\%$, $\mu=0.05\%$, $\sigma=30\%$, $\eta=70\%$]
Figure 3: Stock Market Indices and EMBI+ Spreads, 1998-2006. The figure plots the dynamics of observed EMBI+ spreads and IFCG stock market index levels for Brazil, Mexico, Peru, and Russia over the period 1998-2006. Both series have been normalized.

Figure 4: Model Implied Indebtedness levels, 1998-2006. The figure plots the dynamics of the debt-to-GDP ratios predicted by the model for Brazil, Mexico, Peru, and Russia over the period 1998-2006. [Parameters: $\Lambda=2\%$, $\alpha=0.4$, $\mu=0.05\%$, $\eta=70\%$, $r=4.48\%$, $\sigma_{\text{Brazil}}=29.49\%$, $\sigma_{\text{Mexico}}=24.28\%$, $\sigma_{\text{Peru}}=18.76\%$, $\sigma_{\text{Russia}}=42.04\%$]
Figure 5: Relationship between Revenues and Market Asset Values. The figure exhibits the relationship between the revenues and the value of the economy (middle panel), when assuming either an endogenous or a fixed debt policy. The total value of the economy is decomposed in two components: The debt value (upper panel) and the net sovereign value (lower panel). [Parameters: Lamda=4%, Alpha=0.4, Mu=0.05%, Eta=70%, r=4.48%, Sigma=30%]
Figure 6: Predicted versus EMBI+ Spreads, 1998-2006. The figure offers a comparison of the credit spreads generated by the structural model with observed EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998-2006. The daily predicted credit spreads are computed using IFCG stock market indices measured in U.S. Dollars. The unconditional stock market volatility and the risk-free rate are measured using the sample average of the IFCG stock market standard deviation of the log returns and the 5Y U.S. Treasuries rate respectively. [Parameters: Lamda=2%, Alpha=0.4, Mu=0.05%, Eta=70%, r=4.48%, Sigma_Brazil=29.49%, Sigma_Mexico=24.28%, Sigma_Peru=18.76%, Sigma_Russia=42.04%]
In the following tables, I perform an empirical analysis on Brazil, Mexico, Peru, and Russia over the period 1998-2006: EMBI+ CS represent the emerging market bond index (EMBI+) spreads of the four issuers. JPMorgan derives the credit spread implied in the price of each Brady bond and then computes each country's EMBI+ index, which is a weighted average index on spreads using the country's most liquid Brady bonds denominated in U.S. dollars. Stock Market corresponds to the daily level of each country’s stock market index in U.S. dollars computed by IFCG/S&P. Stock Market Volatility is a measure of conditional volatility computed with an AR(3)-GARCH(1,1) process on daily returns. US Treasuries 5Y stands for the U.S. daily Treasury rate with constant maturities of 5 years. Finally, VIX Index represents the option-implied volatility on the S&P 500 index. Regarding the sources, data for this section are taken from Datastream for IFCG stock market indices, 5Y U.S. Treasuries rate, and VIX series, and CBonds for EMBI+ spreads. All series consist of 2329 daily observations from January 1 1998 to December 31 2006.

### Table 1
Table of Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Observations</th>
<th>Mean (Standard Deviation)</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil EMBI+ CS</td>
<td>2329</td>
<td>7.64% (4.02%)</td>
<td>2.06%</td>
<td>24.36%</td>
</tr>
<tr>
<td>Mexico EMBI+ CS</td>
<td>2329</td>
<td>3.29% (1.85%)</td>
<td>.95%</td>
<td>11.60%</td>
</tr>
<tr>
<td>Peru EMBI+ CS</td>
<td>2329</td>
<td>4.63% (2.00%)</td>
<td>1.28%</td>
<td>10.61%</td>
</tr>
<tr>
<td>Russia EMBI+ CS</td>
<td>2329</td>
<td>11.35% (14.96%)</td>
<td>.92%</td>
<td>68.90%</td>
</tr>
<tr>
<td>Brazil Stock Market</td>
<td>2329</td>
<td>387.21 (210.00)</td>
<td>129.29</td>
<td>1043.96</td>
</tr>
<tr>
<td>Mexico Stock Market</td>
<td>2329</td>
<td>2422.81 (1131.06)</td>
<td>861.09</td>
<td>6243.22</td>
</tr>
<tr>
<td>Peru Stock Market</td>
<td>2329</td>
<td>288.18 (170.07)</td>
<td>132.24</td>
<td>891.32</td>
</tr>
<tr>
<td>Russia Stock Market</td>
<td>2329</td>
<td>1233.68 (1133.15)</td>
<td>137.53</td>
<td>4830.10</td>
</tr>
<tr>
<td>Brazil Stock Market Volatility</td>
<td>2329</td>
<td>32.14% (12.02%)</td>
<td>17.89%</td>
<td>121.07%</td>
</tr>
<tr>
<td>Mexico Stock Market Volatility</td>
<td>2329</td>
<td>24.73% (8.18%)</td>
<td>14.13%</td>
<td>96.08%</td>
</tr>
<tr>
<td>Peru Stock Market Volatility</td>
<td>2329</td>
<td>18.10% (5.60%)</td>
<td>11.2%</td>
<td>42.02%</td>
</tr>
<tr>
<td>Russia Stock Market Volatility</td>
<td>2329</td>
<td>35.29% (14.97%)</td>
<td>19.91%</td>
<td>121.11%</td>
</tr>
<tr>
<td>US Treasuries 5Y</td>
<td>2329</td>
<td>4.48% (1.06%)</td>
<td>2.08%</td>
<td>6.83%</td>
</tr>
<tr>
<td>VIX Index</td>
<td>2329</td>
<td>21.09% (6.96%)</td>
<td>9.90%</td>
<td>45.74%</td>
</tr>
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### Panel correlations

<table>
<thead>
<tr>
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<th>Stock Market</th>
<th>Stock Market Volatility</th>
<th>US Treasuries 5Y</th>
<th>VIX Index</th>
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<td>Stock Market</td>
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<td>Stock Market Volatility</td>
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<td>-0.096</td>
<td>1</td>
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<td>US Treasuries 5Y</td>
<td>0.185</td>
<td>-0.052</td>
<td>0.169</td>
<td>1</td>
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<tr>
<td>VIX Index</td>
<td>0.402</td>
<td>-0.392</td>
<td>0.299</td>
<td>0.053</td>
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### Table 2

**Estimation of EMBI+ Spreads with Credit Spreads Predicted by the Model**


<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Log EMBI+ spreads</th>
<th>(\Delta_t) EMBI+ spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1a</td>
<td>1b</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>0.540***</td>
<td>-0.019</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>(0.015)</td>
<td>(0.021)</td>
</tr>
<tr>
<td><strong>Log Model Credit Spread</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>0.951***</td>
<td>0.961***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.045***</td>
<td>1.016***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Peru</td>
<td>0.575***</td>
<td>0.553***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Russia</td>
<td>1.604***</td>
<td>1.521***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
</tr>
<tr>
<td><strong>US Treasuries 5Y</strong></td>
<td>7.955***</td>
<td>8.462***</td>
</tr>
<tr>
<td></td>
<td>(0.227)</td>
<td>(0.213)</td>
</tr>
<tr>
<td><strong>(\Delta_t) US Treasuries 5Y</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>0.350**</td>
<td>0.205**</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.279***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>0.915</td>
<td>0.932</td>
</tr>
<tr>
<td>Number of observations</td>
<td>9312</td>
<td>9312</td>
</tr>
</tbody>
</table>

Notes:

Standard errors reported in parentheses are robust to heteroskedasticity, cross-sectional and temporal dependence.

\(*\), **, *** relate to coefficients respectively significant at the 90, 95, 99% confidence level.
Table 3
Statistics on Predicted versus Observed Credit Spreads, 1998-2006
Table 3 provides descriptive statistics for the predicted credit spreads and observed EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998-2006. The credit spreads generated by the structural model are computed using the price of each IFCG stock market index. The unconditional stock market volatility and the risk-free rate are measured using the sample average of the IFCG stock market standard deviation of the log returns and the 5Y U.S. Treasuries rate respectively. The structural credit spreads are computed as described in Section 4.2. [Parameters: Lamda=2%, Alpha=0.4, Mu=0.05%, Eta=70%, r=4.48%, Sigma_{Brazil}=32.14%, Sigma_{Mexico}=24.73%, Sigma_{Peru}=18.10%, Sigma_{Russia}=35.29%]

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean (%)</th>
<th>Standard deviation (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
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<tr>
<td>Brazil</td>
<td></td>
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<tr>
<td>Predicted Credit Spread</td>
<td>4.27</td>
<td>2.47</td>
<td>1.78</td>
<td>8.07</td>
</tr>
<tr>
<td>EMBI+ Spread</td>
<td>7.64</td>
<td>4.02</td>
<td>1.17</td>
<td>4.75</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted Credit Spread</td>
<td>2.15</td>
<td>0.97</td>
<td>1.27</td>
<td>6.76</td>
</tr>
<tr>
<td>EMBI+ Spread</td>
<td>3.30</td>
<td>1.85</td>
<td>1.27</td>
<td>4.64</td>
</tr>
<tr>
<td>Peru</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted Credit Spread</td>
<td>2.84</td>
<td>1.90</td>
<td>0.46</td>
<td>2.15</td>
</tr>
<tr>
<td>EMBI+ Spread</td>
<td>4.63</td>
<td>2.00</td>
<td>0.02</td>
<td>2.15</td>
</tr>
<tr>
<td>Russia</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted Credit Spread</td>
<td>5.85</td>
<td>5.71</td>
<td>2.97</td>
<td>14.03</td>
</tr>
<tr>
<td>EMBI+ Spread</td>
<td>11.35</td>
<td>14.96</td>
<td>1.96</td>
<td>5.85</td>
</tr>
</tbody>
</table>
## Table 4

### Estimation of EMBI+ Spreads with Distance-to-Default Indicator


<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Log EMBI+ spreads</th>
<th>Δₜ EMBI+ spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3a</td>
<td>3b</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>-1.237***</td>
<td>-1.786***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.014)</td>
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<tr>
<td><strong>Distance-to-Default Δₜ</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>-0.723***</td>
<td>-0.714***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Mexico</td>
<td>-0.659***</td>
<td>-0.634***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Peru</td>
<td>-0.409***</td>
<td>-0.400***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Russia</td>
<td>-1.213***</td>
<td>-1.141***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td><strong>US Treasuries 5Y Δₜ</strong></td>
<td>-0.059***</td>
<td>-0.058***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td><strong>Stock Market Volatility Δₜ</strong></td>
<td>0.560***</td>
<td>0.445***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.028)</td>
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<tr>
<td><strong>VIX Index Δₜ</strong></td>
<td>0.924***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td></td>
</tr>
<tr>
<td><strong>R² Within</strong></td>
<td>0.915</td>
<td>0.933</td>
</tr>
<tr>
<td><strong>R² Between</strong></td>
<td>0.105</td>
<td>0.098</td>
</tr>
<tr>
<td><strong>R² Overall</strong></td>
<td>0.363</td>
<td>0.430</td>
</tr>
<tr>
<td><strong>Number of observations</strong></td>
<td>9312</td>
<td>9312</td>
</tr>
</tbody>
</table>

### Notes:

Standard errors reported in parentheses are robust to heteroskedasticity, cross-sectional and temporal dependence.

*, **, *** relate to coefficients respectively significant at the 90, 95, 99% confidence level.

OLS Fixed Effects | Yes | Yes | Yes
GLS Random Effects | Yes | Yes | Yes