Money Burning and Multiple Equilibria in Bargaining

Christopher Avery and Peter B. Zemsky*

Graduate School of Business, Stanford University, Stanford, California

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This paper synthesizes recent work on delay and multiple equilibria in complete information Rubinstein bargaining. We identify a general principle, "money burning," underlying prior results and develop a general model of the phenomena. Money burning is a threat by one player to destroy surplus, which can serve to enhance that player's bargaining power if it is credible. Our results show that the uniqueness and efficiency properties of the subgame perfect equilibrium of Rubinstein bargaining are vulnerable to players' ability to burn money. Journal of Economic Literature Classification Number: 026. © 1994 Academic Press, Inc.

1. INTRODUCTION

Rubinstein's model of bilateral bargaining was a breakthrough because of its unique subgame perfect equilibrium (SPE). Because the model's equilibrium always involves acceptance of the first offer, however, it does not capture the observed phenomena of delayed agreements. This has motivated the search for elaborations of Rubinstein's model which produce delays in equilibrium.

Uniqueness in Rubinstein's model is robust to many alternative specifications including asymmetries in the players' discount factors and valuations. However, a series of examples demonstrate that delay can occur in models of complete information. We now know that delayed agreements can result from outside options (Shaked, 1987), the ability to postpone

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bargaining (Kambe, 1992), and a union's option to strike during the bargaining process (Fernandez and Glazer, 1991). In all of these papers, delay results from the existence of multiple equilibria with immediate agreements. Players are kept from making acceptable offers by the threat of getting their worse possible equilibrium payoff.

We seek a unifying framework for the source of multiple equilibria in complete information Rubinstein bargaining. Consider the case of outside options. In Shaked and Sutton (1984), where a player can exercise an outside option only after rejecting an offer, there is a unique SPE. In contrast, in Shaked (1987), where a player can exercise an outside option after his offer is rejected, there can be multiplicity but only if the outside option is inefficient.

We show that all of the cited examples conform to a general principle. Multiple equilibria arise because at least one player has the ability to take some action that reduces the value of the asset after her own offer is rejected. We refer to such an action as money burning. Usually, the rationality of money burning depends on a threat to switch to a regime of play without it.

We develop a general model to demonstrate the link between multiple equilibria and money burning. Since the expositions of prior examples have focused on the enumeration of strategies for proving folk theorems, they have tended to be complex and grounded in the specifics of the individual models. Our focus on the equilibria which achieve the extreme payoffs to the players simplifies the analysis.

Section 2 reviews and generalizes Shaked and Sutton's proof of uniqueness in Rubinstein bargaining. Section 3 shows how money burning can lead to multiplicity. Section 4 shows how several prior papers on complete information delay are special cases of our model or are consistent with the money burning principle. Section 5 considers the extent to which other models with complete information delay fit into our framework.

2. Generalization of Shaked and Sutton's Approach

We begin by reviewing Shaked and Sutton's well-known analysis of the standard Rubinstein bargaining model, for it lies at the heart of our subsequent analysis. In the standard model, two risk-neutral players alternate offers to divide an object of unit value. At each turn, the responding player can accept the current offer or reject it and make a counteroffer. Each rejection incurs discounting at the rate $\delta$.

Shaked and Sutton's analysis utilizes the stationarity and symmetry of the game to derive bounds for the supremum $M$ and the infimum $m$ over the set of SPE payoffs for the offering player in period $t$. Since the re-
sponding player will get to make an offer in the next period, \( t + 1 \), her reservation value in period \( t \) is restricted to \([\delta m, \delta M]\). This lead to bounds for \( m \) and \( M \):

\[
M \leq 1 - \delta m; \quad (1)
\]

\[
m \geq 1 - \delta M. \quad (2)
\]

Coupled with the constraint \( M \geq m \), these equations have the unique solution

\[
M = m = \frac{1}{1 + \delta}. \quad (3)
\]

When played by both players, the strategy, "Always offer \( 1/(1 + \delta) \); accept an offer iff it is at least \( \delta/(1 + \delta) \)" supports this equilibrium payoff, so it is the unique SPE outcome. As Shaked and Sutton remark, this line of argument is quite general.

As a precursor to our study of money burning, we now generalize the standard alternating offer bargaining model to allow for discount factors and per period costs (or benefits) that are dependent on which player made the last offer. We define \((\alpha_{1i}, \alpha_{2i})\) to be the per period payoffs (costs or benefits) and \(\beta_{i}\) to be the discount factor when Player \( i \)'s offer is rejected by Player \( j \). The standard model is just the special case where \(\alpha_{ii} = \alpha_{ji} = 0\) and \(\beta_{i} = \beta_{j} = \delta\).

The asymmetric model uses Player \( i \)'s turn to offer as a regeneration point to produce two pairs of linear equations relating the payoffs \(M_1, m_1\) and \(M_2, m_2\), respectively. Here, \(M_i\) represents the supremum and \(m_i\) the infimum of SPE payoffs to Player \( i \) when it is his turn to make an offer in any subgame. This gives rise to paired equations for \(M_i, m_i\):

\[
M_1 \leq 1 - (\alpha_{12} + \beta_{1}m_2); \quad m_2 \geq 1 - (\alpha_{21} + \beta_{2}M_1); \quad (4)
\]

\[
M_2 \leq 1 - (\alpha_{21} + \beta_{2}m_1); \quad m_1 \geq 1 - (\alpha_{12} + \beta_{1}M_2). \quad (5)
\]

As in (1) and (2), the bounds on \(M_i\) and \(m_i\) are given by the difference between the asset value and the other player's continuation values. These equations yield the unique solutions\(^1\)

\[
M_i = m_i = \frac{1 - \alpha_{ij} - \beta_i + \alpha_{ji}\beta_i}{1 - \beta_i\beta_j}, \quad (6)
\]

\(^1\) We assume implicitly that the \(\alpha_{ij}\) are sufficiently close to zero that \(M_i \in [0, 1]\).
and these payoffs are supported by strategies analogous to those in the standard model. Thus there remains a unique SPE.

Our model incorporates three of the five generalizations of the standard model discussed by Kreps (1990). Fixed cost discounting and two versions of varied speed of response are special cases of our model and of the Eqs. (4)–(6). Our model does not include the possibility of outside options or different discount rates for each player. It would be easy to include different discount rates in the given set of equations, but this would add algebraic complexity without causing any qualitative changes in the results. Outside options do not fit neatly into the Shaked and Sutton approach and are addressed separately in Section 4.3.

3. The Effect of Money Burning

We now show that multiplicity can arise if the strategy space of one of the players is enlarged. We assume that in our general model whenever Player 2’s offer is rejected, she has the option to take some action which reduces the players’ net utility—either by increasing discounting or by reducing that period’s payoffs. We refer to such an action as money burning.

Formally, if Player 2 chooses to burn money in some period, the discount rate for that period is $\beta_2^*$ and the period’s payoffs are $\alpha_{21}^*$ and $\alpha_{22}^*$. Otherwise the standard parameters $(\beta_2, \alpha_{21}, \alpha_{22})$ hold for that period, where $\beta_2^* \leq \beta_2$, $(\alpha_{21}^*, \alpha_{22}^*) \leq (\alpha_{21}, \alpha_{22})$, and at least one inequality holds strictly. When Player 2 rejects an offer by Player 1, the game simply continues; the parameters $(\beta_1, \alpha_{11}, \alpha_{12})$ are fixed exogenously.

Note that money burning has no impact on the future structure of the game. However, a credible threat to burn money puts Player 2 at an advantage because Player 1 is more reluctant to refuse her offers. The immediate effect of Player 2’s threat is to reduce Player 1’s reservation value for the offer directly preceding the proposed act of burning money.

Note that with fixed cost discounting, for which $\beta = \beta_1 = 1$, $\alpha_{12} = -c_2$, $\alpha_{21} = -c_1$, $\alpha_{22} = 0$, Eqs. (4) and (5) have no solution for $c_i \neq c_j$. Assuming $c_i > c_j$, one must impose a boundary condition, $M_i  \leq 1$, to allow a solution. The boundary condition weakens the constraint for $m_i$ to $m_i \geq \min(1 - \beta_i M_i - \alpha_{ij}, 1)$, producing the result $M_i = m_i = c_j$; $M_j = m_j = 1$.

This condition is somewhat spurious. Any set of parameters which satisfy the rationality condition (9) and for which $M_2^* > M_1$ will lead to multiplicity. For strict inequality to hold in the $\alpha$'s, we require strict inequality in each component.

We are restricting the model to allow only one player to burn money for simplicity. All of our general results hold trivially and the rationality condition for multiple equilibria is eased considerably when both players can burn money.
in anticipation of the loss that will occur if the offer is refused. Of course the credibility of this threat is never assured. There is always an equilibrium where Player 2 never burns money. Any single deviation in such an equilibrium lowers Player 2’s payoff without increasing her bargaining power. Ironically, this standard equilibrium is necessary for the construction of an equilibrium with money burning.

Player 2 might rationally burn money if she believes that failure to do so will lead play to revert from an equilibrium involving money burning, where 2’s payoff is high, to the one without it, where 2’s payoff is low. We consider two regimes. In regime $R$ Player 2 enjoys minimal bargaining power because she never burns money. In regime $R^*$ Player 2 enjoys maximal bargaining power because she burns money whenever her offer is rejected; if Player 2 ever fails to burn money play reverts to $R$.

Regime $R$ is always an equilibrium and produces the payoffs given in (6). The analogous payoffs for regime $R^*$ are

\[ M_1^* = m_1^* = \frac{1 - \alpha_{12} - \beta_1 + \alpha_{21}^* \beta_1}{1 - \beta_1 \beta_1^*} \]

\[ M_2^* = m_2^* = \frac{1 - \alpha_{21}^* - \beta_2^* + \alpha_{12} \beta_1^*}{1 - \beta_2^* \beta_1} \]

It only remains to determine when the strategies in $R^*$ are subgame perfect. Player 2 prefers to burn money and continue following regime $R^*$ rather than to not burn money and switch to $R$ if

\[ \alpha_{22}^* + \beta_2^* (\beta_1 M_2^* + \alpha_{12}) \geq \alpha_{22} + \beta_2 (\beta_1 M_2 + \alpha_{12}). \]

This condition assures that $M_2^* > M_2$ because the per period returns and discount factor are higher for the terms on the right-hand side than for those on the left-hand side of the inequality.

Finally, note that there would be no multiplicity if Player 2 could only burn money after she rejects offers. In that case, money burning could never be rational since it would only give Player 2 more incentive to accept Player 1’s previous offer, thus enhancing Player 1’s bargaining power.

We summarize our analysis thus far in the following result.

**Theorem 3.1.** There are multiple SPE outcomes in the generalized

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5 It can be verified that these two regimes give the highest and lowest possible payoffs for Player 2 when there are multiple equilibria.
bargaining game with the possibility of money burning whenever the rationality constraint

\[ \alpha_{22}^* + \beta_2^* (\beta_1 M_2^* + \alpha_{12}) \geq \alpha_{22} + \beta_2 (\beta_1 M_2 + \alpha_{12}) \]  \hspace{1cm} (9)

holds. Player 2 achieves her highest possible payoff when she always burns money and her lowest when she never does.

We now examine two special cases through which we argue that the rationality constraint is a weak one.

**Corollary 3.2.** If \( \beta_2^* = \beta_2 = \delta \) then the rationality constraint simplifies to

\[ \delta^2 (\alpha_{21} - \alpha_{22}^*) \geq (1 - \delta^2)(\alpha_{22} - \alpha_{22}^*). \]  \hspace{1cm} (10)

This result shows the effect when discounting is fixed and money burning reduces the current period’s payoffs. Then the rationality constraint compares weighted losses from money burning to the individual players. When the players are very patient, both discount factors are close to 1, and the weight on the loss to Player 1, \( \delta^2 \), is much greater than that on the loss to Player 2, \( 1 - \delta^2 \). Therefore Player 2’s option to burn money supports multiple equilibria even in cases where Player 2 suffers most of the immediate losses from that choice.

**Corollary 3.3.** If \( \alpha_{ij} = \alpha_{ij}^* = -c \) for all \( i, j \) and \( \beta_2 = \beta_1 = \delta \), then the rationality constraint simplifies to

\[ \beta_2^* \geq (1 - c/\delta)/(1 + \delta). \]  \hspace{1cm} (11)

This corollary considers the case where per period costs are fixed (and assumed to be small) and Player 2 can burn money by reducing the discount factor from its standard value, \( \delta \). There are multiple equilibria for \( \beta_2^* \in [(1 - c/\delta)/(1 + \delta), \delta] \), a weak condition when \( \delta \) is near to 1. At the lower bound for \( \beta_2^* \), Player 2’s payoff in \( R^* \) is arbitrarily close to 1 as \( \delta \to 1 \).

We conclude this section by demonstrating that the two equilibria support a folk theorem construction to achieve all intervening payoffs.

**Theorem 3.4.** If the rationality constraint (9) holds, then the entire range of payoffs \([M_1^*, M_1], [M_2, M_2^*]\) can be supported in SPEs with immediate agreement at Player 1’s and Player 2’s turn to offer respectively.

**Proof.** This proof uses only the existence of two SPEs corresponding
to the bounding payoffs. It is otherwise independent of the possibility of burning money. Therefore, it suffices to prove the result for Player 1. For an arbitrary \( x \in (M^*_1, M^*_1) \) we construct an SPE to support a payoff of \((x, (1 - x))\) when it is Player 1’s turn to offer in period \( t \).

Our construction relates the selection of equilibrium regimes beginning in period \( t + 1 \) to Player 1’s offer in period \( t \). The players follow \( R^* \) in period \( t + 1 \) if Player 1 offers \((x, (1 - x))\), but they follow \( R^* \) if Player 1 makes any other offer. It is optimal for Player 2 to accept the offer of \((x, (1 - x))\), because \( 1 - x > 1 - M_1 = \delta M_2 \). It is optimal for Player 1 to make this offer because any other offer gives a payoff of less than \( 1 - \delta M^*_2 = M^*_1 < x \). Therefore, \((x, (1 - x))\) is the payoff in an SPE. □

The folk theorem serves as the basis for sustaining delayed agreements in which the players reach an agreement at some time \( t > 0 \). Until time \( t \), each player demands the entire value and refuses to capitulate to that demand from her opponent. If either deviates to a more equitable offer, that player is punished by reversion to the worst equilibrium for her. Then, at time \( t \) play switches to an equilibrium with intermediate valued payoffs and immediate agreement; existence of such an equilibrium is guaranteed by the folk theorem. Agreement in period \( t \) will be an equilibrium for \( t \) sufficiently small.  

4. Money Burning in Earlier Examples

We now demonstrate that money burning drives the results in several known examples of multiple equilibria. We have simplified each model to highlight the importance of money burning without, we believe, changing the essence of the results.

4.1. Timing of Offers

Kambe (1992) provides the most direct example of the money burning principle. His model is identical to the standard model with the single addition that after Player 1 rejects an offer in period \( t \), Player 2 chooses whether to continue bargaining immediately in period \( t + 1 \) or to delay until period \( t + 2 \). Calling for a one period break burns money by increasing the amount of discounting. Hence, in terms of our general model we have the special case:

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6 The maximum delay depends on the skew between the maximum and minimum payoffs to each player. See Fernandez and Glazer (1991) for details.

7 In Kambe’s more general model, either player can call for a delay of any length and all divisions of the asset can be sustained in SPE’s.
\[ \alpha_{ij}^* = \alpha_{ij} = 0 \forall i, j; \quad \beta_i^* = \beta_i^1 = \delta; \quad \beta_i^* = \delta^2. \]  

(12)

Player 2's choice to delay bargaining reduces \( \beta_i^* \) from \( \delta \) to \( \delta^2 \). Thus, the payoffs in \( R^* \) are skewed towards Player 2 as follows\(^8\):

\[ M_i^* = m_i^* = \frac{1}{1 + \delta + \delta^2}; \quad M_2^* = m_2^* = \frac{1 + \delta}{1 + \delta + \delta^2}. \]  

(13)

From Corollary 3.2 the rationality constraint for \( R^* \) to be an equilibrium is

\[ \delta^2 + \delta^3 \geq 1. \]  

(14)

Thus, there are multiple equilibria for discount factors above a critical level. Note that uniqueness still holds if it is the responding player who can call for a break after a rejected offer.

4.2. Labor Negotiations and Strikes

In the models of Fernandez and Glazer (1991) and Haller and Holden (1990), a firm and a union bargain over wages. In each period prior to an agreement, the union decides whether to strike or to work at the prevailing wage, \( w_0 < 1 \). The value of the firm is 1 if the union does not strike and 0 if it does. Since the players are bargaining over per period wages, we normalize the value of the firm to \( (1 - \delta) \) per period and the prevailing wage to \( (1 - \delta)w_0 \) per period. As long as the union does not strike, both sides achieve positive payoffs during the bargaining process.\(^9\)

In terms of our model, the firm is Player 1 and the union is Player 2. In \( R \), the union never strikes, while in \( R^* \) the union burns money by striking each time that the firm refuses its offer—that is, striking every other period while the game continues.\(^10\) The parameters defining the two regimes are

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\(^8\) Strategies which support these payoffs are: If she has called for a break at every opportunity, Player 2 offers \( 1 - M_i^* \), calls for a break whenever Player 1 rejects an offer, and accepts offers of at least \( \delta M_i^* \); Player 1 offers \( 1 - M_i^* \), and accepts offers of at least \( \delta M_i^* \). If Player 2 bypasses a chance to call for a break, then the players switch to equilibrium \( R \) and their Rubinstein strategies.

\(^9\) In fact, these authors are treating the general case where there is an existing division of the benefits from the object and one side can choose to interrupt the flow of benefits during the bargaining process.

\(^10\) Fernandez and Glazer prove that this strategy supports the maximum payoff to the union. As in Kambe's model, it hurts the union to burn money after an offer by the firm.
$$\alpha_1 = \alpha_{21} = (1 - \delta)(1 - w_0); \quad \alpha_{12} = \alpha_{22} = (1 - \delta)w_0; \quad \beta_1 = \beta_2 = \delta;$$  \hfill (15)

$$\alpha^*_1 = \alpha^*_2 = 0; \quad \beta^*_2 = \delta.$$  \hfill (16)

The resulting payoffs are then

$$M_1 = m_1 = (1 - w_0); \quad M_2 = m_2 = w_0;$$  \hfill (17)

$$M^*_1 = m^*_1 = \frac{1 - w_0}{1 + \delta}; \quad M^*_2 = m^*_2 = \frac{1 + \delta w_0}{1 + \delta},$$  \hfill (18)

and from Corollary 3.3 the rationality constraint is simply

$$\delta^2 \geq w_0.$$  \hfill (19)

Once again, for sufficiently large values of $\delta$, there are multiple equilibria. In $R$, the union is held to the prevailing wage $w_0$, but it strictly improves on that in $R^*$ because of its threat to strike.\textsuperscript{11}

In the money burning equilibrium $R^*$, the union incurs the shortrun cost of striking in order to maintain its valuable reputation for striking. The union loses this reputation if it ever fails to carry out the threat via a switch to regime $R$. This interpretation of regime switching applies to the general model as well, but not to the model of outside options or the others which follow.

4.3. Outside Options

The outside option models of Shaked and Sutton (1984) and Shaked (1987) highlight the importance of the timing of money burning. In each model, Player 2 can choose to exit at some points during the game, giving the payoff $(a, b)$, where $a < b$, and $a + b < 1$ so that exiting is inefficient. For Player 2 to gain great bargaining power, she must be able to credibly threaten to exit after Player 1 rejects her offer.\textsuperscript{12}

In Shaked and Sutton's model, Player 2 can only exit after an offer by Player 1 so that the timing of the outside option does not fit the framework of Section 3. Player 1 can defuse Player 2's threat to exit by offering her the maximum of $b$ and the Rubinstein payoff $\delta/(1 + \delta)$. There is a unique equilibrium with one of these payoffs when it is Player 1's turn to offer.

\textsuperscript{11} We omit the strategies to support these outcomes as SPEs and those for the rest of the paper, noting that they are analogous to those in Footnote 8.

\textsuperscript{12} This formulation is similar to that in Osborne and Rubinstein (1990), Section 3.12.2, except that they take $a = 0$, simplifying the conditions for multiple equilibria. Their condition for multiple equilibria, $(1/(1 + \delta)) \leq b \leq \delta^2$ is not as sharp as the condition we give below.
Player 2 cannot use the fact that $a$ is small to increase her bargaining power. Her payoff is never larger than that given by the outside option.

Shaked’s model contains the elements of the timing and inefficiency of money burning from the general model. Since the outside option ends the game, the rationality condition for multiple equilibria does not follow the simple form of (9), as Player 2 can no longer be “rewarded” in the continuation of the game for her choice to burn money. If there is a stationary SPE in which Player 2 always takes or always passes up the outside option, then this SPE is unique. In these instances, the outside option is either so attractive (corresponding to large $b$) that it dictates play at every subgame or so unattractive ($b$ is small) that it is as if the option does not exist. Any such equilibrium involves an immediate agreement and an efficient division of the asset; the players never reach a subgame where Player 2 will accept the outside option.

For intermediate values of $b$ there are two SPE’s which rely on a change of regimes. The best payoff for Player 2 occurs when money burning is credible at the next opportunity and the worst occurs when it is not. Exiting is a very powerful threat in these equilibria because Player 1’s payoff is only $a$. Indeed, Player 2’s threat to exit after a rejection by Player 1 in period $n + 2$ strengthens her position in period $n + 1$ to the point that she will not exit in period $n$. For Player 2 to accept the outside option in any period, she must intend to refuse it at the next opportunity. Once again, in these equilibria, the outside option in a given period serves to determine payoffs in previous periods, but it is never exercised on the equilibrium path.

These multiple SPE’s have the odd property that Player 2’s position is not constant. She has a strong bargaining position in any two-period cycle between decisions to exit only because her position is weak in the next cycle. Each of the two extreme equilibria relies on the existence of the other outcome in its construction. This dependence gives a two-sided rationality condition for multiple equilibria in the infinite horizon game, namely $\delta^2/(1 + \delta) \leq b \leq \delta^2(1 - a)$. Finally, we note that the outside option must be inefficient to sustain multiple equilibria for if $a + b \geq 1$, then the right-hand inequality fails. In that case, Player 2 always takes the outside option in a unique SPE.

5. Other Examples of Delayed Agreements

There are several examples of multiple equilibria in complete information bargaining which do not contain the three main elements of inefficiency, the timing of money burning, and switches in regime which were common to the earlier examples. $N$ person bargaining and retraction of
offers rely almost exclusively on switches in regime with some elements of money burning to support asymmetric outcomes. In a model with dynamic uncertainty, Avery and Zemsky (1994) (not discussed here) exhibit multiple stationary outcomes based on inefficient offers without any change in regime. Sakovics exhibits equilibria in multiple regimes in a continuous time model.\textsuperscript{13}

5.1.  \textit{N Person Bargaining}

We illustrate multiple equilibria in \textit{N} person games with a three-person bargaining game discussed by Herrero (1984). In each period of the game, one player proposes a division of a dollar among all three participants. If both others accept, the game ends. Otherwise, bargaining continues with an offer by the next player in turn after a period of discounting.

In this game, any division of the dollar can be supported as the payoff of an SPE. More specifically, there are SPE’s in which any one person receives the full dollar and the others receive nothing. Given these degenerate outcomes, the standard folk theorem construction produces SPE’s with all intermediate payoffs.

Each degenerate outcome relies on an extreme shift in regimes. Suppose that Player 1 is to receive the dollar and that it is Player 2’s turn to offer. In this SPE, any deviation by Player 2 to an offer which gives Player 1 between $\delta$ and $\frac{1}{2}$\textsuperscript{14} switches the equilibrium to one which gives the entire dollar to Player 3. Then it is impossible for Player 2 to make an alternate offer which is acceptable to both Player 1 and Player 3 except to give the full dollar to Player 1.\textsuperscript{15}

Once again, these extreme equilibria are bound together in their construction by the rationality condition. Each depends on the others and the infinite horizon for its existence. Since two players receive payoffs of zero in these equilibria, the construction of Section 3 produces further equilibria with arbitrarily long delays and payoffs arbitrarily close to zero for each player.

Money burning enters consideration when we focus on the interactions between pairs of players. In the equilibrium described above, Player 2 is powerless against Player 1, regardless of who is to offer. In the competition between Players 1 and 2, it is as if Player 1 can always make a credible

\textsuperscript{13} We do not discuss van Damme et al. (1990), where the existence of a smallest money unit renders the requirement of subgame perfection almost vacuous. Any division is an equilibrium, just as in the original Rubinstein model with the Nash solution concept.

\textsuperscript{14} Player 2 must offer at least $\delta$ to Player 1, otherwise Player 1 will prefer to refuse the offer and receive the next period continuation payoff of 1.

\textsuperscript{15} So long as the weak rationality condition $\delta > 1 - \delta$, i.e., $\delta > \frac{1}{2}$ holds, Player 3 prefers getting the dollar next period to any division this period in which Player 1 gets a payoff of at least $\delta$. 
threat to burn the entire dollar. Any deviation by Player 2 gives the dollar to Player 3 in an agreement to be reached in the next round. Then Players 1 and 2 receive an aggregate payoff of zero. Of course, Player 1 never has to make an explicit decision to burn money for it is imposed on him exogenously through the change of equilibrium and Player 3’s veto power. Therefore, the only rationality condition for multiple equilibria involves Player 3’s payoffs.

5.2. Retraction of Offers

In Muthoo (1990), each player can retract an offer after the other player accepts it, opting to continue bargaining instead. Such behavior burns money, for it is wasteful to make an offer and then retract it if the other player accepts. This model is distinct from others which allow money burning because it is only possible to reduce the value of the asset after accepted rather than rejected offers.

The Rubinstein equilibrium is an SPE of this game where no one retracts an offer. For discount factors above a critical level, there are also equilibria giving the entire dollar to one player, say Player 2. As in the earlier examples, such alternate equilibria are supported by the threat of reverting to the Rubinstein outcome in some subgames off the equilibrium path. An obvious difficulty with an equilibrium which gives the whole dollar to Player 2 is that Player 1 can make an offer of \((\delta, 1 - \delta)\) rather than conceding the dollar, and Player 2 will prefer the payoff of \(\delta\) now to the continuation payoff of 1 next period. That would eliminate such a proposed equilibrium in an ordinary bargaining game. But here we can append the Rubinstein equilibrium to the continuation game after Player 2 accepts Player 1’s preliminary offer of \((\delta, 1 - \delta)\); this shift is sufficient to cause Player 1 to retract his offer, if \(\delta^2/(1 - \delta) > 1 - \delta\) or \(\delta > \frac{1}{2}\).

Thus the possibility of Player 1 retracting his offer can sustain the division which gives the entire dollar to Player 2. To summarize the sequence of events on the equilibrium path: Player 2 is to receive the entire dollar. If Player 1 could make a credible alternate offer of \((1 - \delta, \delta)\), Player 2 would accept it and the equilibrium would unravel. However, Player 1 cannot make such an offer credibly; if he does and Player 2 accepts, there is a change of regime which induces Player 1 to then retract his offer. Consequently, Player 1 is stuck conceding the entire dollar to Player 2. In these equilibria, the option to retract an offer only works to a player’s detriment. In any equilibrium with an immediate agreement, a player receives less than the Rubinstein payoff only if he is expected to retract an offer at a subsequent subgame.

5.3. Continuous Time Models

Although the analysis becomes more complicated, the intuition from discrete time results carries over to continuous time models in which the
timing of offers is endogenous. In what they describe as "Bargaining without Procedures," Perry and Reny (1989) consider a model with exponential discounting in which the players choose when to make offers, players are committed to each of their offers for a fixed period of time $\Delta$, and bargaining ends when they make compatible offers. This produces a range of SPE payoffs with the Rubinstein payoffs to the first and second players as endpoints, but does not allow delayed agreements.

In effect, this model replicates the standard framework. An unmatched offer by Player 1 at time 0 pushes back the earliest possible agreement at any other payoff until time $\Delta$, i.e., for "one period." Any offer (other than an acceptance of the outstanding offer) by Player 2 between time 0 and $\Delta$ serves only to commit Player 2 beyond time $\Delta$. Since lengthier commitments allow Player 2 more leverage over Player 1, it is optimal for Player 2 to wait until time $\Delta^-$ for a counteroffer. If she waited any longer, Player 1 could make another offer.

If Player 1 offers at time 0 and Player 2 at time $\Delta^-$ and so on, this is merely the standard setting with periods of length $\Delta$ and discount factor $\delta = e^{-\Delta t}$. Since the timing of offers is endogenous, there is no need for a second regime to give multiple equilibria. Competition to make the first offer produces the given range of payoffs with the players making compatible offers immediately at time 0 in each SPE. That is, either player may enjoy the full first player advantage, or the players may divide it. However, this competition alone does not skew the payoffs enough to allow delayed agreements. Since the range of payoffs gives $m_i = \delta M_i$, it is never worthwhile to pass up an agreement at time 0 in hopes of a better one at time $\Delta$.

Sakovics (1990) generalizes the Perry and Reny model to incorporate the idea that it takes time to respond to an offer. He finds that this addition produces a wider range of equilibria and allows lengthy delays. In his model, a player who makes an offer at time $t$ is committed to it until time $t + \Delta$, while the other player cannot respond until time $t + \lambda$, where $\lambda < \Delta$. If Player 2 makes an offer at time $t$ and Player 1 is also eligible to offer at that time, then the players can reach an agreement at time $t$ if Player 1 makes a simultaneous compatible offer. Otherwise, they cannot agree until time $t + \lambda$. This discontinuity creates the ability to burn money and leads to the possibility of delays in this model.

Suppose it is known that Player 2 will make a given offer at time 0 and that Player 1 is considering whether to accept it by making a compatible offer at time 0 or to reject it by making a counteroffer. If Player 1 makes a counteroffer at time $t$, Player 2 cannot accept it until the later of $\Delta$ and $t + \lambda$. It is optimal for Player 1 to wait until $\Delta - \lambda$ to make a counteroffer. This allows an agreement at $\Delta$, the first instant that Player 2 can change her offer, while committing Player 1 until time $2\Delta - \lambda$ (as long as possible).
Comparing the time frame of offers shows that Player 2 is committed to her offer for a period of length \( \Delta \), while Player 1 can only commit to the counteroffer for \( \Delta - \lambda \). In other words, rejection of Player 2's offer burns more money than does rejection of Player 1's.

When Player 2 enjoys this advantage over Player 1 for the entire game, we are back in the original Rubinstein model subject to the generalization of varied speeds of responses. The equilibrium results fit directly into (4) and (5) with

\[
\alpha_1 = \alpha_2 = \alpha_{22} = 0; \quad \beta_1 = e^{-r(\Delta - \lambda)}; \quad \beta_2 = e^{-r\Delta}.
\]

(20)

This produces the payoffs

\[
m_1 = \frac{1 - e^{-r(\Delta - \lambda)}}{1 - e^{-r(2\Delta - \lambda)}}, \quad M_2 = \frac{1 - e^{-r\Delta}}{1 - e^{-r(2\Delta - \lambda)}},
\]

(21)

which work to Player 2's advantage. Here \( m_1 \) gives Player 1's payoff for an agreement at time \( \Delta \). Discounting this back to time 0 units shows that, indeed, Player 1's minimum payoff at time 0 is \( 1 - M_2 \).

To support these payoffs in an SPE, Player 2 must be able to commit for length \( \Delta \) in every cycle of \( 2\Delta - \lambda \) while Player 1 can never commit for longer than \( \Delta - \lambda \). A strategy that allows Player 2 to achieve this advantage is "Always offer \( 1 - M_2 \) at the first possible moment when both players are eligible to make an offer." In equilibrium, Player 2 expects that both sides will always make simultaneous offers to agree on \((1 - M_2, M_2)\) at the first instance when it is possible to do so, and this expectation drives the discrepancy in bargaining power between the two players.

While we assumed that Player 1 made no offer at time 0 in these calculations, we may now note there is an SPE in which Player 1 "accepts" Player 2's offer of \( 1 - M_2 \) immediately at time 0. But since neither player is entitled to the first offer, Player 1's maximum payoff is identical to Player 2's, and they can achieve all intermediate payoffs in SPE's with immediate agreement. This final set of payoffs is considerably larger than the set of payoffs in Perry and Reny's model and supports delayed agreements.

In essence, it is as if the speeds of response can be fixed in two different ways in Sakovics' model: either player may have the advantage of longer commitments to her offers, or both players may only be able to commit for the same length of time. In contrast, in Perry and Reny's model, the players always have the same effective speed of response. In terms of our general model from Section 2, Sakovics' model allows two different discounting regimes to emerge endogenously while Perry and Reny only
allow one. Then the intuition of Theorem 3.4 explains the existence of delays in Sakovics' model where they do not exist in the model of Perry and Reny.

REFERENCES


