Eliciting Honest Feedback in Electronic Markets

Nolan Miller, Paul Resnick, and Richard Zeckhauser*

February 11, 2003†

Abstract

Recommender and reputation systems seek to inform potential customers by securing current consumers’ feedback about products and sellers. This paper proposes a payment-based system to induce honest reporting of feedback. The system applies proper scoring rules to each buyer’s report, looking to how well it predicts the report of a later buyer. Honest reporting proves to be a Nash Equilibrium. To balance the budget, the incentive payment to each buyer is charged to some buyer(s) other than the one whose report that buyer is asked to predict. In addition, payment schemes can be scaled to induce appropriate effort by raters.

*Miller, Kennedy School of Government, Harvard University; Resnick, School of Information, University of Michigan; Zeckhauser, Kennedy School of Government and Visiting Professor at Harvard Business School.
†We thank Chris Avery, Bill Sandholm, John Pratt, Lones Smith, Ennio Stachetti, and Hal Varian for helpful coments.
1 Introduction

The quintessential consumer decision is what to buy and from whom. In making this decision, consumers recognize that sellers know more than they do, e.g., the seller better knows an item’s quality. Such information asymmetries make buyers reluctant to purchase; beneficial transactions are foregone.

Reputation building, a time honored business practice, reduces the inefficiencies created by asymmetric information. Using their theory scalpels, economists and decision scientists have recently laid bare the anatomy of this practice. Their models posit repeat participation in a marketplace, but usually assume few or no repeat interactions between the same partners. Some (perhaps imperfect) information about a player’s type is revealed during an interaction. We call this information feedback, generalizing from the use of that term at eBay. The feedback is then spread to other participants, who can use it to determine with whom to transact, or what strategies to employ. For example, a buyer might adjust his reservation price based on the feedback about a potential seller. The equilibria (or lack thereof) of these models suggest conditions under which reputations convey information about who is trustworthy, and the extent to which reputations provide incentives for good behavior in transactions (i.e., deter moral hazard), and the extent to which they discourage low-quality or dishonest participants from joining the marketplace (i.e., deter adverse selection).

To illustrate, Kreps and Wilson (1982) consider why it might be beneficial for a player to take costly actions to develop a reputation as being of certain type. Shapiro (1982) characterizes the conditions under which deliberate cycles of quality (building up a reputation and then cashing in on it) will be avoided. Kandori (1992) shows that moral hazard can be deterred even if information is only distributed and processed locally. Most recently, Tadelis (2002) shows that if firms’ names can be traded, then their ability to provide incentives becomes “ageless.”
Electronic markets present new challenges for reputation building. Transactions rarely involve face-to-face contact, goods can rarely be inspected before payment, repeat transactions are unusual, and other aspects of reputation that might be held hostage – e.g., standing in the community – are rarely available. When information comes by wire, severe asymmetries must be expected; market failure threatens. Moreover, because electronic merchants serve a geographically diverse clientele, individual buyers may be forced to rely on information provided by strangers, which may be of questionable relevance and dubious validity.

To create trust is critical for sellers. Brynjolfsson and Smith (2000, p. 578), observing a high degree of price dispersion for items sold over the Internet, posit heterogeneity in the trust that sellers enjoy as a possible explanation: "trust may play an enhanced role because of the spatial and temporal separation between buyer, seller, and product on the Internet."1

New mechanisms must be found to build effective reputations. Here, the Internet holds some significant advantages. Once information about sellers is collected, it can be disseminated widely at nearly zero cost. And, extraordinary information processing and storage capabilities are immediately at hand to tally scores potentially based on thousands of transactions. Thus while electronic markets are particularly vulnerable to opportunistic behavior, they are also particularly well suited to harness the power of reputations.

Most conventional models of reputations treat the elicitation and distribution of feedback as exogenous. In practice, the elicitation and distribution of feedback encourage strategic behavior. This paper proposes mechanisms that harness the capabilities of the Internet to provide monetary incentives for buyers make informed and truthful evaluations of sellers. With reliable feedback assured, reputations have the potential to make sellers act responsibly.

1They find price dispersion among different sellers fo 25% for CDs and 33% for books.
The key to our mechanism is that the experiences of buyers interacting with the same seller tend to be correlated, i.e., if one buyer has a good experience with a seller, it becomes more likely a second buyer will have a good experience. Given such correlation, reward schemes that rely on "proper scoring rules" can be constructed to induce agents to truthfully reveal their information about the seller (Johnson, Pratt, and Zeckhauser, 1990).

The abstract problem of inducing truthful revelation given that agents’ private information is correlated has been addressed in the mechanism design literature. Seminal papers by d’Aspremont and Gérard-Varet (1979; 1982) and Crémer and McLean (1985; 1988) demonstrate that it is generally possible to extract agents’ private information when types are correlated. Despite the power of these general results, they rely on a linear systems approach to demonstrate the possibility of constructing a truth-inducing mechanism. Hence, they provide little guidance on how such a scheme should be constructed in practice. This paper shows how the general results can be applied to and adapted for the particular problem of eliciting honest feedback in electronic markets.

We are concerned with the specific problem of designing a mechanism that will elicit honest feedback from agents. This problem is an instance of a more abstract problem, that of inducing agents to reveal private information. This problem has been studied extensively in the literature on mechanism design. For example, d’Aspremont and Gerard-Varet (  

The paper is organized as follows. Section 2 discusses the current use of reputation in electronic markets. Section 3 presents our main theoretical result, a mechanism that simultaneously elicits honest feedback and balances the budget. It then discusses the problem of securing appropriate effort from evaluators. Section 4 addresses alternative approaches and complications, and section 5 concludes.
2 Recommender and Reputation Systems in Electronic Markets

A recommender system collects, distributes, and aggregates product evaluations, to inform the choices of future consumers (Resnick and Varian, 1997). A reputation system performs these functions with evaluations of people’s behavior, especially that of sellers. Though few of those who provide or use the ratings know each other, these systems help people decide whom to trust, they encourage trustworthy behavior and appropriate effort, and they deter participation by those who are unskilled or dishonest (Resnick et al, 2000).

A number of websites have built formal reputation systems. The best known and most widely studied system is that of eBay. eBay is the largest person-to-person online auction site, with more than 4 million auctions open at a time. eBay does not warranty its auctions; it is only as a listing service, while the buyers and the sellers assume all the risks associated with transactions. There are fraudulent transactions to be sure. Nonetheless, the overall rate of successful transactions remains astonishingly high for a market as “ripe with the possibility of large-scale fraud and deceit” as eBay (Kollock, 1999). eBay attributes its remarkable rate of successful transactions to its reputation system, the Feedback Forum. After a transaction is completed, the buyer and seller have the opportunity to rate each other as positive, neutral, or negative (1, 0, or -1), possibly with written comments. All participants have the running total of feedback points attached visibly to their screen names, which may be pseudonyms. Yahoo! Auction, Amazon, and other auction sites feature reputation systems like eBay’s, with variations such as a rating scale from 1-5, or using several measures (friendliness, prompt response, quality of product, etc.), or averaging rather than totaling feedback scores.

Recommender and reputation systems have spread far beyond the auction sites, where their most natural application. Bizrate.com rates registered retailers by asking consumers
to complete a survey after each purchase. So-called “expert sites” (www.expertcentral.com, www.askme.com) provide Q&A forums in which experts provide answers for questions posted by clients in exchange for reputation points and comments. Product review sites (such as www.epinions.com) offer both product evaluations and rating services for product reviewers – the better the review, the more points the reviewer receives. iExchange.com tallies and displays reputations for stock market analysts based on the performance of their picks. Messages on the news and commentary site Slashdot (www.slashdot.com) can be sorted and filtered based on reader feedback, and users accumulate karma or reputation points based in part on the popularity of their posts; higher reputation scores give users successively greater privileges within the system, including the option of moderating other people’s posts.

Although recommender and reputation systems could enable the Internet to achieve its potential as a grand electronic bazaar, many challenges must be met. The principal challenge addressed in this paper is to secure sufficient and honest feedback. For, if this can be accomplished, meaningful reputations become possible.

Buyers may be reluctant to report evaluations, or to do so honestly. They may withhold positive evaluations if a seller’s capacity is limited, in which case the information is not a public good. For example, wise parents are reluctant to reveal the names of their favorite baby-sitters. “Nice” buyers may withhold negative evaluations. Sellers’ threats of retaliation for negative feedback or explicit or implicit offers of rewards for positive feedback might lead buyers to submit reports that do not accurately reflect their perceptions. The extreme paucity of negative feedback on eBay – buyers give negative or neutral feedback in fewer than 1% of all transactions – suggests that people are hesitant to leave such evaluations (Resnick and Zeckhauser, 2003).\(^2\) Dishonest feedback can also come from “phantom” transactions.

\(^2\)Dellarocas (2001) analyzes a model in which, so long as harshness in interpretation of feedback is appropriately tied with leniency in giving feedback, leniency has some advantages in deterring seller opportunism.
undertaken solely to generate feedback (that will build the reputations of friends or stain
the reputations of enemies).\textsuperscript{3}

Evaluations may be underprovided, since providing feedback about a completed transac-
tion requires some effort, and to improve the evaluation takes more, yet the information only
benefits other participants.\textsuperscript{4} There may also be insufficient testing of participants whose
reputations are not well established. That is, participants may avoid interacting with a new-
comer, or someone whose initial feedback is not favorable. Avery, Resnick, and Zeckhauser
(1999, hereafter ARZ) explore this problem in the context of product evaluations.

Their mechanism pays early evaluators, who give the most useful information, to provide
information, and charges later evaluators so as to balance the budget.\textsuperscript{5} Sellers themselves
can stimulate evaluations. In the Internet context, a new high-quality seller might offer
items at a reduced price in order to ensure that there would be transactions from which a
reputation could develop. This would represent “dues paying” by newcomers as a means to
establish a reputation, much like the real world free trial.

Providing incentives to generate feedback may create problems. If payments are not tied
to the quality of the evaluations, buyers may submit uninformative evaluations. Performance-
based payments might seem a solution, but they could induce herding or collusion by the
raters.

The problem we are concerned with here is not systematic leniency, but the failure to report negative eval-
uations, whatever threshold is in use.

\textsuperscript{3}Dellarocas (2000) models this situation and shows that phantom feedback can be limited to a small per-
centage of the total, and if true feedback follows some known distribution, phantom feedback is undermined
by dropping outlier reports from computations of reputations.

\textsuperscript{4}Despite the rational incentive to free-ride, provision of feedback at eBay is quite common, occurring in
more than 50\% of transactions for a sample from 1999 (Resnick and Zeckhauser, 2001).

\textsuperscript{5}They conclude that any two of three desirable properties for such a mechanism can be achieved, but not
all three; the three properties are voluntary participation, no price discrimination, and budget balance.
What mechanism could motivate buyers to provide evaluations, to make them honestly, and with sufficient care? The remainder of this paper addresses those questions. Our proposed mechanism recognizes that some reward is necessary; it relies on monetary payments to raters. To focus on the honest elicitation problem, we begin with the case in which acquiring and reporting information is costless to buyers. Later, we show that our basic mechanism can be adapted to situations where buyers bear costs to acquire and report information.

Our mechanism for eliciting honest feedback is based on tying the payments to buyers to the informativeness of their evaluations. Since there is no gold standard for what constitutes a “good” seller and the center has no independent information, the payment for an evaluation needs to reflect the degree to which the evaluation agrees with the evaluations of others.

3 A Mechanism for Eliciting Honest Feedback

We consider a world where a number of buyers engage with a seller and then rate her (or her product) for quality. We assume that the quality of a seller does not vary over time, and that all buyers attach the same value to a seller’s quality. Quality is observed with some error. It could be represented, for example, as a binomial process, where the quality on any trial is good or bad. After receiving the product, each buyer sends a message to a common processing facility called the center. The center has four roles: First, it continually updates the aggregate assessment of each seller’s quality. Second, it distributes these assessments to the large audience of web users. Third, it rewards or penalizes each buyer on the basis of his rating. Fourth, it plays the role of a bank, ensuring that the mechanism at least breaks even in the long run. The center has no independent information. Thus, when rewarding

---

6 For clarity, we refer to sellers as female and buyers as male.
7 Given independent verifying power, a variant of the system outlined below would be easier to implement. It could simply pay buyers on how effectively they predicted the center’s information. Utilizing information
individual raters it can only rely on the information provided by other raters.

The sellers (or products), which are already in the market, have an innate, predetermined quality level, and play no active role in our model. The central question is whether we can develop a payment system that simultaneously produces honest ratings as a Nash equilibrium and breaks even. The answer, fortunately, is positive for a quite reasonable set of assumptions. Our mechanism bases payments to agents on a proper scoring rule, a payoff structure that induces decision makers to reveal their true beliefs about the distribution of a random variable by rewarding them based on their announced distribution of the possible outcomes, and the actual realization of the random variable. Our mechanism capitalizes on the predictive power of one rater’s information for the information of other raters.

Formulation

Following tradition in the economics of information, we refer to a seller’s quality as her type. We refer to a buyer’s perception of a seller’s type as his signal.

Suppose there are a finite number of seller types, indexed by \( t = 1, ..., T \). Let \( p(t) \) be the commonly held prior probability assigned to the seller being type \( t \). Assume that \( p(t) > 0 \) for all \( t \) and \( \sum_{t=1}^{T} p(t) = 1 \).

Let \( I \) be the set of buyers, where \( |I| \geq 3 \). We allow for the possibility that \( I \) is countably infinite. Each buyer, judging from his own transaction, privately observes a signal of the seller’s type. Conditional on the seller’s type, buyers’ signals are independent and identically distributed. Let \( S^i \) denote the random signal received by buyer \( i \). Let \( S = \{ s_1, ..., s_M \} \) be the set of possible signals, and let \( f(s_m|t) = \Pr(S^i = s_m|t) \), where \( f(s_m|t) > 0 \) for all \( s_m \) and \( t \), and \( \sum_{m=1}^{M} f(s_m|t) = 1 \) for all \( t \). Further, we assume that the conditional distribution from other buyers as well as the center would increase the reliability of the mechanism, but would not affect the incentive to report honestly.

---

8 Future work should allow sellers to respond to their developing reputations.

9 We briefly address the issue of non-common priors later.
of signals is different for different values of $t$. Let $s^i \in S$ denote a generic realization of $S^i$. Frequently, we will use $s^i_m$ to denote the event $S^i = s_m$. We assume that buyers are risk neutral and seek to maximize expected wealth.

In the mechanism we propose, the center asks each buyer to announce his signal and then makes a transfer to the buyer that may depend on all buyers’ announcements. Let $a^i \in S$ denote one such announcement, and $a = (a^1, ..., a^I)$ denote a vector of announcements, one by each buyer. Let $a^i_m \in S$ denote buyer $i$’s announcement when his signal is $s_m$, and $\bar{a}^i = (a^i_1, ..., a^i_M) \in S^M$ denote buyer $i$’s announcement strategy. Let $\bar{a} = (\bar{a}^1, ..., \bar{a}^I)$ denote a vector of announcement strategies. As is customary, let the superscript “$-i$” denote a vector leaving off buyer $i$’s component.

Let $\tau_i (a)$ denote the transfer paid to buyer $i$ when the buyers make announcements $a$, and let $\tau (a) = (\tau_1 (a), ..., \tau_I (a))$ be the vector of transfers made to all agents. We are particularly interested in transfers that balance in the sense that $\sum_{i=1}^I \tau_i (a) = 0$.

An announcement strategy $\bar{a}^i$ is a best response to $\bar{a}^{-i}$ for player $i$ if for each $m$:

$$E_{S^{-i}} [\tau_i (a^i_m, \bar{a}^{-i}) | s^i_m] \geq E_{S^{-i}} [\tau_i (\hat{a}^i, \bar{a}^{-i}| s^i_m) | s^i_m] \text{ for all } \hat{a}^i \in S. \quad (1)$$

That is, a strategy is a best response if, conditional on receiving signal $s_m$, the announcement specified by the strategy maximizes that buyer’s expected transfer, where the expectation is taken with respect to the distribution of all other buyers’ signals conditional on $S^i = s_m$. Given transfer scheme $\tau (a)$, a vector of announcement strategies $\bar{a}$ is a Nash Equilibrium of the reporting game if (1) holds for $i = 1, ..., I$, and a strict Nash Equilibrium if the inequality in (1) is strict for all $i = 1, ..., I$.

Truthful revelation is a Nash Equilibrium of the reporting game if (1) holds for all $i$ when $a^i_m = s_m$ for all $i$ and all $m$, and a strict Nash Equilibrium if the inequality is strict. That is, if all the other players announce truthfully, truthful announcement is a strict best response. Since buyers do not receive any direct return from their announcement, if there
were no transfers at all then any strategy vector, including truthful revelation, would be a Nash equilibrium. However, since players are indifferent between all strategies when there are no monetary transfers, these Nash equilibria are not strict.

**The Mechanism**

Our main result shows that truthful revelation is a strict Nash equilibrium. The analysis begins by noting that although $S^i$ and $S^j$ are conditionally independent, they are not necessarily independent. Because each signal is drawn from the same distribution with unknown parameter $t$, $S^i$ and $S^j$ are generally dependent.

Our results rely on a form of dependence, which we call stochastic relevance.\(^{10}\)

**Definition:** Random variable $S^i$ is stochastically relevant for random variable $S^j$ if and only if the distribution of $S^j$ conditional on $S^i$ is different for different realizations of $S^i$.

More technically, $S^i$ is stochastically relevant for $S^j$ if for any distinct realizations of $S^i$, call them $s^i$ and $\hat{s}^i$, there exists at least one realization of $S^j$, call it $\hat{s}^j$, such that $\Pr(s^j|s^i) \neq \Pr(s^j|\hat{s}^i)$. Let $g(S^j|S^i)$ be the distribution of $S^j$ conditional on $S^i$, and let $g(s^j|s^i)$ represent $\Pr(S^j = s^j|S^i = s^i)$.

**Lemma 1:** *For generic distributions $f(s^i|t)$ and $p(t)$, $S^i$ is stochastically relevant for $S^j$ for any two distinct players $i$ and $j$.*\(^ {11}\)

**Proof of Lemma 1:** We argue by contradiction. By Bayes’ Rule:

$$
g(s^j|\hat{s}^i) = \sum_{t=1}^{T} f(s^j|t) \frac{f(s^i|t) p(t)}{\Pr(s^i)},$$

\(^{10}\)The concept of stochastic relevance is introduced in Johnson, Miller, Pratt, and Zeckhauser (2002).

\(^{11}\)That is, the closure of the set of distributions for which $S^i$ is not stochastically relevant for $S^j$ has Lebesgue measure zero. See Mas-Colell, Whinston, and Green (1995, p. 595) for a discussion of generic conditions.
where \( \Pr (s^i) = \sum_{t=1}^{T} f(s^i|t) p(t) \).

If there exists \( s^i \) and \( \hat{s}^i \) such that \( g(s^j|s^i) = g(s^j|\hat{s}^i) \) for all \( s^j \), then the following must hold for each \( s^i_m \):

\[
\sum_{t=1}^{T} f(s^i_m|t) \frac{f(s^i|t)p(t)}{\Pr (s^i)} - \sum_{t=1}^{T} f(s^i_m|t) \frac{f(\hat{s}^i|t)p(t)}{\Pr (\hat{s}^i)} = 0
\]

\[
\sum_{t=1}^{T} f(s^j_m|t) \left( \frac{f(s^i|t)p(t)}{\Pr (s^i)} - \frac{f(\hat{s}^i|t)p(t)}{\Pr (\hat{s}^i)} \right) = 0
\]

\[
\sum_{t=1}^{T} f(s^j_m|t) \Delta(t) = 0,
\]

where \( \Delta(t) = \left( \frac{f(s^i|t)p(t)}{\Pr (s^i)} - \frac{f(\hat{s}^i|t)p(t)}{\Pr (\hat{s}^i)} \right) \). Since (2) is an inner product, the set of distributions \( f(s_m|t) \) and \( p(t) \) that satisfy it is closed and has Lebesgue measure zero.\(^{12}\)

Consider two buyers, call them \( i \) and \( j \). If \( S^i \) is stochastically relevant for \( S^j \), then buyer \( i \)'s signal provides information about the distribution of buyer \( j \)'s information. Thus, if it were known that buyer \( j \) will honestly report his signal, the problem of eliciting buyer \( i \)'s information is reduced to eliciting his belief about the distribution of \( j \)'s signal.

The elicitation of beliefs about the distribution of \( S^j \) from an agent who has observed \( S^i \) is the problem for which proper scoring rules were developed.\(^{13}\) Put simply, suppose agent \( i \) privately observes the realization of \( S^i \), which is stochastically relevant for some publicly observable random variable \( S^j \), and agent \( i \) is asked to reveal his private information. A scoring rule is a function \( R(s^j|s^i) \) that, for each possible announcement of \( S^i \), assigns a score to each possible realization of \( S^j \). A convenient interpretation is that the scoring rule

---

\(^{12}\)To show this, note that (2) is satisfied only if all \( \Delta t = 0 \), or, failing this, \( f(s_m|t) \) satisfies a linear equation of the form \( \sum_{t=1}^{K} x_t \Delta t = 0 \). It is straightforward to show that these restrictions are only satisfied non-generically.

\(^{13}\)See Cooke (1991) for an introduction to the use of scoring rules.
specifies the payment made to the (risk neutral) agent following each realization $S^j$. A scoring rule is strictly proper if the expected score is uniquely maximized at the true value of the parameter (i.e., if an agent paid according to the rule uniquely maximizes his expected wealth by truthfully revealing his private information).

There are a number of strictly proper scoring rules. The three best known are:\textsuperscript{14}

1. Quadratic Scoring Rule: $R(s^j_n | s^i_m) = 2g(s^j_n | s^i_m) - \sum_{h=1}^{M} g(s^j_h | s^i_m)^2$.

2. Spherical Scoring Rule: $R(s^j_n | s^i_m) = \frac{g(s^j_n | s^i_m)}{\left(\sum_{h=1}^{M} g(s^j_h | s^i_m)^2\right)^{\frac{1}{2}}}$.

3. Logarithmic Scoring Rule: $R(s^j_n | s^i_m) = \ln g(s^j_n | s^i_m)$.

Any strictly proper scoring rule would work in our construction. We adopt the logarithmic scoring rule because of its intuitive appeal and notational simplicity. The proof that the logarithmic scoring rule is strictly proper follows immediately from Jensen’s inequality.

\textbf{Lemma 2:} Suppose that risk neutral agent $i$ knows the realization of random variable $S^i$ that is stochastically relevant for random variable $S^j$. If agent $i$ is asked to announce a realization of $S^i$ and is paid according to the logarithmic scoring rule, the expected payment is uniquely maximized by announcing the true realization of $S^i$.

\textbf{Proof of Lemma 2:} Let $s^* \in S$ be the realization of $S^i$ and let $a \in S$ be the realization agent $i$ announces.

$$\sum_{n=1}^{M} \ln g(s^j_n | a) g(s^j_n | s^*) - \sum_{n=1}^{M} \ln g(s^j_n | s^*) g(s^j_n | s^*) = \sum_{n=1}^{M} \ln \frac{g(s^j_n | a)}{g(s^j_n | s^*)} g(s^j_n | s^*)$$

$$= \ln \left(\sum_{n=1}^{M} \frac{g(s^j_n | a)}{g(s^j_n | s^*)} g(s^j_n | s^*)\right) = 0.$$

\textsuperscript{14}See Cooke (1991) p. 139.
The inequality in the second line follows from Jensen’s inequality, strict concavity of the natural logarithm, and stochastic relevance.\textsuperscript{15}

Lemma 2 proves that log-likelihood transfers can be used to induce truthful revelation by agent \( i \) as long as his private information is stochastically relevant for some other publicly available signal. However, in the case we consider, each buyer’s signal is private information, and therefore we can only check players’ announcements against other players’ announcements, not their actual signals.

We need to construct self-financing transfers that make truthful reporting a Nash equilibrium. The essence of our mechanism can be illustrated using three agents. For example, if buyer 1’s signal is stochastically relevant for buyer 2’s signal, then paying buyer 1 based on the log-likelihood of the reported realization of buyer 2’s signal induces buyer 1 to truthfully announce his type, assuming buyer 2 announces honestly. However, since player 2’s announcement affects the transfer to buyer 1, buyer 2 cannot also pay the transfers to player 1. Instead, to balance the transfers, let buyer 3 pay buyer 1 the transfer specified by the scoring rule. Such payments do not affect incentives, since the transfer buyer 3 pays to buyer 1 is independent of buyer 3’s action. Thus for each player a second player’s signal is used to determine incentive payments, and a third player makes the payments. We formalize this intuition in the following proposition.

\textbf{Proposition 1:} For generic distributions \( f ( s_m|t) \) and \( p (t) \), there exist balanced transfers under which truthful reporting is a strict Nash Equilibrium of the reporting game.

\textbf{Proof of Proposition 1:} For each buyer, choose another buyer \( r (i) \) whose announcement

\textsuperscript{15}Stochastic relevance is necessary only for uniqueness of the best response. Without stochastic relevance, the inequality in the last line of the derivation is weak, \( \leq \). Hence truth-telling is a best response, but not necessarily the only best response.
i will be asked to predict. Let

$$
\tau_{i}^{\ast} \left( a^{i}, a^{r(i)} \right) = \ln \left( a^{r(i)} | a^{i} \right).
$$

Assume buyer \( r(i) \) reports honestly: \( a^{r(i)}(s_{m}) = s_{m} \) for all \( m \). Since \( S^{i} \) is stochastically relevant for \( S^{r(i)} \), and \( r(i) \) reports honestly, \( S^{i} \) is stochastically relevant for \( r(i) \)’s report as well. Given that \( S^{i} = s^{\ast} \), player \( i \) chooses \( a^{i} \in S \) in order to maximize:

$$
\sum_{n=1}^{M} \ln g \left( s_{n}^{r(i)} | a^{i} \right) g \left( s_{n}^{r(i)} | s^{\ast} \right).
$$

By Lemma 2, (4) is uniquely maximized by announcing \( a^{i} = s^{\ast} \), i.e. truthful announcement is a strict best response. Thus, given that player \( r(i) \) announces truthfully, player \( i \)’s best response is to announce truthfully as well.

To balance transfers, choose any agent or group of agents not including \( r(i) \) and divide the payment to agent \( i \) among them.\(^{16}\) For example, suppose each agent is chosen to make transfers to one other agent. Let \( b(i) \) be the buyer to whom \( i \) pays transfers, and choose \( b(i) \) such that \( b(r(i)) \neq i \). Let the net transfer to buyer \( i \) be given by:

$$
\tau_{i} \left( a \right) = \tau_{i}^{\ast} \left( a^{i}, a^{r(i)} \right) - \tau_{b(i)}^{\ast} \left( a^{b(i)}, a^{r(b(i))} \right).
$$

These transfers induce truthful reporting and are balanced.\(^{\blacksquare}\)

Proposition 1 shows that it is generally possible to solve the problem of eliciting honest information from a number of buyers yet still meet a breakeven constraint. Each player’s expected gain from honest reporting is 0, but in particular cases it may be negative. To assure ex-post voluntary participation, a bond can be collected in advance. Let \( q = \min_{s_{m}, s_{n} \in S} \left( \ln g \left( s_{m} | s_{n} \right) \right) \). If \( -q \) is collected in advance from each player, each player will

\(^{16}\)We refer to \( \tau_{i}^{\ast} \) as the payment to buyer \( i \), even though, given the log scoring rule, the payment is always negative: \( 0 < g \left( s_{n}^{i} | s_{n}^{i} \right) < 1 \), so \( \ln g \left( s_{n}^{i} | s_{n}^{i} \right) < 0 \).
receive positive payments after the evaluations are reported.\footnote{Since $0 < g \left( s^i_n \middle| s^e_n \right) < 1$, $\ln g \left( s^i_n \middle| s^e_n \right) < 0$. However, adding $q$ to the log of the likelihood ensures that all payments to agent $i$ are positive.}

Although we illustrate using the log-likelihood scoring rule, any other strictly proper scoring rule could be employed in Proposition 1. For example, consider the quadratic scoring rule described above. Basing transfers on the quadratic scoring rule, i.e., \( \tau^*_i \left( a^i, a^{r(i)} \right) = 2g \left( a^{r(i)} \middle| a^i \right) - \sum_{h=1}^{M} g \left( s^r_{h} \middle| a^i \right)^2 \), also induces truthful reporting, and these transfers can be balanced as in (5). The logarithmic and quadratic scoring rules each has advantages. The logarithmic scoring rule is attractive for its simplicity and intuitive appeal, and also for the fact that the payoff assigned to an outcome depends on the probability of that outcome only; the probabilities of outcomes that do not occur do not factor into the payoff.\footnote{When there are more than 2 outcomes, the logarithmic scoring rule (up to a positive affine transformation) is the only strictly proper scoring rule with this property. See Cooke (1991) for a statement of the result, Winkler (1969) for a discussion of relevance, and Shuford, Albert, and Massengil (1966) for the proof that the only relevant strictly proper scoring rules are based on the logarithmic score. Criticism of the logarithmic score has focused on how it deals with low-probability events. Small changes in assessments of small probabilities can translate into very large changes in the score, and in order for the logarithmic score to be strictly proper such changes must be properly assessed by the decision maker. Further, when the distribution in question involves low-probability events, the necessary range of payments may become very large.}

Our methods easily generalize to the continuous-signal case. However, given continuous signals, the logarithmic scoring rule can be problematic. For example, if signals are normally distributed on the real line, then the signal density becomes arbitrarily small as the signal approaches positive or negative infinity. Consequently, the logarithm of the density approaches negative infinity, and the range of payments needed to induce truthful revelation may become infinite as well. In cases where the signal density is very small (or zero), the quadratic scoring rule is more appropriate.\footnote{In Johnson et al. (2002), it is shown that a lower truncation of the logarithmic score can be used to}

\[ 2g \left( a^{r(i)} \middle| a^i \right) - \sum_{h=1}^{M} g \left( s^r_{h} \middle| a^i \right)^2, \]
Sequential Interaction

In the framework above, buyers report their experiences simultaneously. The mechanism adapts readily to sequential situations. For example, if an infinite sequence of buyers interacts with the seller, then incentives for buyer $i$ can be provided by a log-likelihood payment based on the information reported by buyer $i+1$ (or any other later player for whom buyer $i$’s signal is stochastically relevant). This payment can be funded by player $i+2$ (or any combination of later players not including $i+1$). Sequential reporting is superior, since it allows later buyers to make immediate use of the information provided by their predecessors.

Suppose the seller interacts with an infinite sequence of buyers, indexed by $i = 1, 2, \ldots$. Initially, the commonly held prior distribution for the seller’s type is given by $p(t)$. Let $p_1(t|s^1)$ denote the posterior distribution when buyer 1 receives signal $s^1$. That is,

$$p_1(t|s^1) = \frac{f(s^1|t)p(t)}{\Pr(s^1)}, \quad (6)$$

where $\Pr(s^1) = \sum_{t=1}^{T} f(s^1|t)p(t)$. Buyer 1’s belief about the probability that $S^2 = s^2$ is given by

$$g(s^2|s^1) = \sum_{t=1}^{T} \frac{f(s^2|t)f(s^1|t)p_1(t|s^1)}{\Pr(s^2|s^1)}, \quad (7)$$

where $\Pr(s^2|s^1) = \sum_{t=1}^{T} f(s^2|t)p_1(t|s^1)$.

Given the distribution specified in (7), buyer 1 can be induced to truthfully reveal $s^1$ using the scoring rule specified in Proposition 1. Buyer 1 is asked the distribution of buyer 2’s announcements. Payments to buyer 1 can then be made by buyer 3 after buyer 2 announces.

---

$^{20}$Hanson (2002) applies a scoring-rule based approach in a model in which a number of experts are sequentially asked their belief about the distribution of a random event, the realization of which is revealed after all experts have reported. In our model, the seller’s type is never revealed, and therefore we must rely on other agents’ reports to provide incentives.
By recursively updating beliefs about the seller using Bayes’ rule (as in (6)), each player’s announcement feeds into the information used by subsequent players. Let
\[
p_i (t|s^1, \ldots, s^i) = \frac{f (s^i|t) p_{i-1} (t|s^1, \ldots, s^{i-1})}{\Pr (s^i)}, \quad (8)
\]
where \(\Pr (s^i) = \sum_{t=1}^{T} f (s^i|t) p_{i-1} (t|s^1, \ldots, s^{i-1})\), and let
\[
g (s^{i+1}|s^i) = \sum_{t=1}^{T} \frac{f (s^{i+1}|t) f (s^i|t) p_i (t|s^1, \ldots, s^i)}{\Pr (s^{i+1}|s^i)}, \quad (9)
\]
where \(\Pr (s^{i+1}|s^i) = \sum_{t=1}^{T} f (s^{i+1}|t) p_i (t|s^1, \ldots, s^i)\).

The sequential elicitation game has each buyer \(i\) observe \(s^i\) and then be asked the distribution of the subsequent buyer’s signal. Transfers constructed according to (3) using the conditional distribution specified in (9) elicit truthful announcement. This announcement then becomes common knowledge and is used to update beliefs about the seller according to (8). Incentives to buyer \(i+1\) are then constructed using a scoring rule that incorporates these updated beliefs. Transfers to buyer \(i\) are paid by buyer \(i + 2\).21

When a finite string of buyers purchases sequentially from the seller, given the end point, checking each buyer’s announcement against that of a later buyer has the potential to unravel. The last buyer has no incentive to lie, but also none to tell the truth, since there is no future signal upon which to base his reward. With the final announcement unreliable, the previous buyers cannot be induced to report truthfully, and so on up the line. Fortunately, by grouping some buyers together and treating the groups as if they report simultaneously, the center can create reporting “rings” that provide appropriate incentives for every buyer. For example, suppose \(N = 10\); we will consider the last three players, 8, 9, and 10. Base payments to 8 on 9’s announcement of his signal, payments to 9 on 10’s announcement, and payments to 10 on 8’s announcement. To balance the transfers, let 8 pay 9, 9 pay 10, and 10

21 Note that buyer 1 does not pay anyone.
pay 8. As long as the center can avoid revealing these three buyers’ announcements until all three have announced, effective incentives can be provided, and the chain will not unravel.\textsuperscript{22}

\textbf{Costly Reporting}

Assuming that reporting is costless allowed us to focus on the essence of the scoring-rule based mechanism. This section considers two different problems associated with costly reporting. The first, the fixed-cost problem, posits that the buyer must incur a cost to evaluate the product. That cost may include direct costs as well as the opportunity cost of being an early evaluator rather than waiting for better information from other evaluators before deciding whether to consume. If the expected benefit is less than this cost, buyers will not provide feedback. Hence the elicitation mechanism must pay buyers to allow them to recoup the fixed cost. Payments of this sort address the real-world problem that often only the most motivated buyers offer feedback, providing a biased impression of the seller.

The second problem, which we call the effort-inducement problem, supposes that the buyer, by incurring further cost, can increase the precision of his information about the seller. The effort-inducement problem implies that buyers may put too little effort into securing reliable information. The center must induce the buyer to obtain information with the proper level of precision.

As is apparent from the proof of Proposition 1, the truth-inducing incentives provided by log-likelihood payments (or any of the scoring rules mentioned above) are unaffected by either a positive rescaling of all payments or the addition of a constant to all payments. Thus, even log-likelihood rules offer significant leeway to adapt the transfers. We next argue that adding a constant to all payments can overcome the fixed-cost problem, while rescaling the payments can partially address the effort-inducement problem.

\textsuperscript{22}Player 7, the last before the ring, presents a challenge. Either he could be paid by 1, with 1’s announcement withheld until 7 chooses, or multiple rings could be created, e.g., 1 through 4, 5-7 and 8-10.
Suppose there is a fixed cost of evaluating and reporting given by \( c > 0 \). To induce truthful reporting, a truthful announcement must maximize the expected transfer (4) and offer an expected return of at least \( c \). This can be accomplished simply by adding \( c \) to \( \tau_i(a) \) from equation 5. Of course, the amount \( c \) will need to be subsidized by those who benefit from the evaluation (See Section 2 and ARZ).

A modified log-likelihood scheme can also be used when, in addition to inducing truthful reporting, the center must induce buyers to gather the appropriate amount of information. Since a positive multiple of the logarithmic scoring rule remains strictly proper, by rescaling all payments proportionately the center can affect incentives to gather information without affecting the buyer’s incentive to tell the truth.

To illustrate such a scheme, we generalize the problem to allow the buyer to obtain “more precise” information. To do so, the buyer’s experience with the seller is encoded not as a single outcome, but rather as an infinite sequence of outcomes generated by repeated, independent sampling from distribution \( f(s_m|t) \). If the buyer knew the entire sequence, by the Law of Large Numbers, the seller’s true type would be revealed. Here, however, we require the buyer to put forth effort to learn about his experience, letting \( c_i(x_i) \) be the cost of learning the first \( x \) components of his experience, where \( c_i(x_i) \) is strictly positive, strictly increasing, and strictly convex, and assumed to be known by the center. We will refer to \( x_i \) as the precision of agent \( i \)’s information.

For a buyer who already knows the first \( x \) components of his experience, learning the \( x+1^{st} \) component further partitions the outcome space. We begin by arguing that, holding fixed the precision of agents’ information, the scoring-rule based information can elicit this

\[\footnotesize\text{23Clemen (2001) undertakes a similar investigation in a principal-agent model.}\]

\[\footnotesize\text{24In single-agent context, Clemen (2001) examines the incentive problem in the case where the center does not know } c_i(x_i).\]

\[\footnotesize\text{25Savage (1954) formally studied this approach, which he calls “partition problem.”}\]
information. We then ask how the mechanism can be used to induce agents to acquire more precise information, even though such acquisition is costly.

For any fixed $x_i$, the information content of two possible $x_i$ component sequences depends only on the frequencies of the various outcomes and not on their order. Consequently, let $Y^i(x_i)$ be the $M$ dimensional random variable whose $m^{th}$ component counts the number of times outcome $s_m$ occurs in the first $x_i$ components of the agent’s information.\footnote{\textit{Y}^i(x_i) \text{ is a multinomial random variable with } x_i \text{ trials and } M \text{ possible outcomes. On any trial, the probability of the } m^{th} \text{ is } f(s_m|t), \text{ where } t \text{ is the seller’s unknown type.}}$ Let $y^i = (y^i_1, ..., y^i_M)$ denote a generic realization of $Y^i(x_i)$, where $y^i_m$ is the number of times out of $x_i$ that signal $s_m$ is received, and note that $\sum_{m=1}^{M} y^i_m = M$. Based on his observation of $Y^i(x_i)$, buyer $i$ can compute his posterior beliefs about the seller’s type, which are informative about the expected distribution of the other players’ signals. Since different realizations of $Y^i(x_i)$ yield different posterior beliefs about the seller’s type, we can extend Lemmas 1 and 2 to the multiple signal case. In the remainder of this section, we let $g(y^j(x_j)|y^i(x_i))$ denote the distribution of $Y^j(x_j)$ conditional on $Y^i(x_i)$.

**Lemma 3:** Consider distinct players $i$ and $j$, and suppose $x_i, x_j \geq 0$ are commonly known. For generic distributions $f(s_m|t)$ and $p(t)$, $Y^i(x_i)$ is stochastically relevant for $Y^j(x_j)$. If agent $i$ is asked to announce a realization of $Y^i(x_i)$ and is paid according to the logarithmic score of $Y^j(x_j)$, given that announcement (i.e., $R(y^j(x_j)|y^i(x_i)) = \ln g(y^j(x_j)|y^i(x_i)))$, then the expected payment is uniquely maximized by announcing the true realization of $Y^i(x_j)$.

**Proof:** The proof is analogous to the proofs of Lemmas 1 and 2 above.  

Proposition 2 restates Proposition 1 in the case where the precision of the buyers’ information is fixed and possibly greater than 1, i.e., $x_i \geq 1$ for $i = 1,..., I$. It follows as an
Proposition 2: Suppose buyer $i$ collects $x_i \geq 1$ signals. For generic distributions $f(s_m|t)$ and $p(t)$ there exist balanced transfers under which truthful reporting is a strict Nash Equilibrium of the reporting game.

Proof: The construction follows that in Proposition 1, using $Y^i(x_i)$ for the information received by buyer $i$ and constructing transfers as in (3) and (5). Under the equilibrium hypothesis, $j = r(i)$ announces truthfully. Let $a^i$ denote buyer $i$’s announcement of the realization of $Y^i(x_i)$, and let transfers be given by:

$$\tau^*_i(y^j|a^i) = \ln g(y^j|a^i).$$

(10)

Under these transfers, truthful announcement is a strict best response. Defining $\tau_i(a)$ as in (5) using $\tau^*_i$ as specified in (10) balances payments without affecting incentives to tell the truth.

Proposition 2 establishes that truthful reporting remains an equilibrium when buyers can choose the precision of their information. We next turn to the questions of how and whether the center can induce a buyer to choose a particular $x_i$. Let $j$ denote the buyer whose signal player $i$ is asked to predict (i.e., let $r(i) = j$), and suppose buyer $j$’s information has precision $x_j$ and that he truthfully reports the realization of $Y^j(x_j)$. For simplicity, we omit argument $x_j$ in what follows. Further, suppose that buyer $i$ is paid according to the log-likelihood scheme described in (5) and (10). Since $x_i$ affects these transfers only through buyer $i$’s announcement, it is optimal for buyer $i$ to truthfully announce $Y^i(x_i)$ regardless of $x_i$.

Since $x_i$ is chosen before observing any information, buyer $i$’s incentive to choose $x_i$ depends on his ex ante expected payoff before learning his own signal, i.e., his payoff taking
expectations over both $Y^i (x_i)$ and $Y^j$. This expectation is written as:

$$Z_i (x_i) = E_{Y^i} \left( E_{Y^j} \ln g (Y^j | Y^i (x_i)) \right).$$

Lemma 4 establishes that buyers benefit from better information, and is a restatement of the well-known result in decision theory that every decision maker benefits from a finer partition of the outcome space Savage (1954).

**Lemma 4:** For generic distributions $f (s_m | t)$ and $p (t)$, $Z_i (x_i)$ is strictly increasing in $x_i$.

**Proof:** Fix $x_i$ and let $y^i$ be a generic realization of $Y^i (x_i)$. Conditional upon observing $y^i$, buyer $i$ maximizes his expected transfer by announcing distribution $g (Y^j | y^i)$ for buyer $j$’s information. Suppose buyer $i$ observes the $x_i+1^{st}$ component of his information. By Lemma 3, $i$’s expected transfer is now strictly maximized by announcing distribution $g (Y^j | (y^i, s_m))$, and buyer $i$ increases his expected value by observing the additional information. Since this is true for every $y^i$, it is true in expectation, and $Z_i (x_i + 1) > Z_i (x_i)$.

As the proof of Lemma 4 makes clear, it applies to any strictly proper scoring rule, not merely the logarithmic scoring rule. The idea that observing additional information improves a decision maker’s expected value is well known in the decision theory literature (Savage, 1954; Lavalle, a1968).

Lemma 4 establishes that as $x_i$ increases, buyer $i$’s information becomes more informative regarding buyer $j$’s signal as $x_i$ increases. Interestingly, the direct effect of gathering more information is to provide buyer $i$ with better information about the seller, not buyer $j$. Nevertheless, as long as buyer $i$’s information is stochastically relevant for that of buyer $j$, better information about the seller translates into better information about buyer $j$.

When transfers are given by (5), the expected net benefit to buyer $i$ from collecting a sample of size $x_i$ and truthfully reporting his observation is $Z_i (x_i) - c (x_i)$. Hence, transfers
(5) induce buyer $i$ to collect a sample of size $x_i^* \in \arg \max \left( Z_i(x_i) - cx_i \right)$. Since the second term of (5), i.e., the payment agent $i$ makes to agent $b(i)$, does not affect buyer $i$'s incentives, we suppress it for the remainder of the discussion and focus on (10), the part of the payment that is relevant for buyer $i$’s incentives.

Buyer $i$’s incentives to truthfully report are unaffected by a uniform scaling of all payments in (10). Therefore, by a judicious rescaling of the payments to buyer $i$, the center may be able to induce the agent to vary the precision of his information. Expression (11) extends the transfers described in (10) to allow for multiple signals and a rescaling of all payments by multiplier $\alpha_i > 0$:

$$\tau_i^* (a^i, y^{(i)}) = \alpha_i \ln g \left( y^{(i)} | a^i \right).$$

Under transfers (11), the maximal expected benefit from a sample of size $B_i$ is $\alpha_i Z_i(x_i)$. Hence the center can induce buyer $i$ to select a particular precision, $\hat{x}_i$, if and only if there is some multiplier $\hat{\alpha} > 0$ such that $\hat{x}_i \in \arg \max \hat{\alpha} Z_i(x_i) - c(x_i)$. The simplest case has $Z_i(x_i)$ concave, i.e., where $Z_i(x_i + 1) - Z_i(x_i)$ decreases in $x_i$.

**Proposition 3:** If $Z_i(x_i + 1) - Z_i(x_i)$ decreases in $x_i$, then for any precision $x_i \geq 0$ there exists a scalar $\hat{\alpha}_i \geq 0$ such that when paid according to (11), buyer $i$ chooses sample size $\hat{x}_i$.

**Proof:** Since $Z_i(x)$ is concave, sample size $\hat{x}_i$ is optimal if there exists $\hat{\alpha}_i$ satisfying

$$\hat{\alpha}_i Z_i(\hat{x}_i) - c_i(\hat{x}_i) \geq \hat{\alpha}_i Z_i(\hat{x}_i + 1) - c_i(\hat{x}_i + 1),$$

and

$$\hat{\alpha}_i Z_i(\hat{x}_i) - c_i(\hat{x}_i) \geq \hat{\alpha}_i Z_i(\hat{x}_i - 1) - c_i(\hat{x}_i - 1).$$

Solving each condition for $\hat{\alpha}_i$,

$$\hat{\alpha}_i \leq \frac{c_i(\hat{x}_i + 1) - c_i(\hat{x}_i)}{Z_i(\hat{x}_i + 1) - Z_i(\hat{x}_i)},$$

and

$$\hat{\alpha}_i \geq \frac{c_i(\hat{x}_i) - c_i(\hat{x}_i - 1)}{Z_i(\hat{x}_i) - Z_i(\hat{x}_i - 1)}.$$
Such an $\hat{\alpha}_i$ exists if and only if $\frac{Z_i(\hat{x}_i) - Z_i(\hat{x}_i - 1)}{Z_i(\hat{x}_i + 1) - Z_i(\hat{x}_i)} \geq \frac{c_i(\hat{x}_i) - c_i(\hat{x}_i - 1)}{c_i(\hat{x}_i + 1) - c_i(\hat{x}_i)}$; by our assumptions, this expression is always true.\[\]

If $Z_i (x_i + 1) - Z_i (x_i)$ does not decrease in $x_i$, then there may be some levels of precision that are never optimal.\[27\] Nevertheless, increasing the scaling factor never decreases optimal precision, and so while the center may not be able to perfectly control the buyers’ effort choices, it can always induce them to put forth greater effort if it wishes.

In practice, the center will not know each individual’s cost structure for extracting more precise signals. However, the center may be able to estimate costs, and then pick a scaling factor that, in expectation, induces each buyer to give signals of the optimal precision.\[28\]

**Non-common prior information**

We have addressed the case in which the center and buyers share common prior beliefs about the seller. Fortunately, the proper-scoring-rule mechanism easily adapts to the case of non-common priors. In particular, buyers’ prior beliefs need not be commonly held. However, in order for the center to properly interpret the buyers’ announcements, it must have some knowledge of the buyers’ prior beliefs.

To briefly illustrate, suppose that at time 0 all parties share “original” prior beliefs about the seller $p(t)$. At time 1, each buyer privately observes a finite-valued random variable

---

\[27\]While we are not aware of any general results pertaining to the shape of the $Z ()$ function, Clemen (2001) provides a number of examples of cases in which $Z_i (x_i + 1) - Z_i (x_i)$ does decrease in $x_i$.

The problem of finding the set of sample sizes that maximize $\alpha Z_i (x_i) - c_i (x_i)$ for some $\alpha$ when $Z_i (x_i)$ is not concave is isomorphic to the problem in production theory of finding the set of outputs that maximize profit for some output price when the production function is not concave. The solution involves finding the set of outputs that remain on the convex closure of the technology set. See, for example, Mas-Colell, Whinston, and Green (1985, Section 5.D).

\[28\]The center chooses the scale that induces the optimal ex ante precision. Ex post, if buyers knows their costs, they will tend to choose lower precision if they are high cost and vice versa.
Let $H^i$ be a typical realization. Information $h^i$, which we call buyer $i$’s history, summarizes buyer $i$’s information before his interaction with the current seller. The history could take into account past interactions with other sellers, other information about the current seller, or anything else that influences his beliefs about the seller’s type before interacting with her.

Let $\eta(h^i|t) > 0$ be the probability that $H^i = h^i$ when the seller is type $t$, where the distribution of $H^i$ differs depending on $t$. Based on the original priors $p(t)$ and his history $h^i$, buyer $i$ can use Bayes’ rule to compute new beliefs about the seller’s type. Denote these beliefs as $p(t, h^i)$, and note that generically, different histories lead to different interim beliefs. Thus, buyers with non-common priors are modeled as having observed different histories.

At time 2, buyer $i$ observes signal $S^i$, which characterizes his current interaction with the seller. Thus the buyer’s information in this model consists of both his history and signal, $(H^i, S^i)$. This problem has identical structure to our original one, and therefore analogs to Lemmas 1 and 2 follow. Generically, buyer $i$’s information, $(H^i, S^i)$, is stochastically relevant for buyer $j$’s, $(H^j, S^j)$. Hence, the center can elicit buyer $i$’s information by asking buyer $i$ to predict buyer $j$’s announced information and paying him according to a strictly proper scoring rule. The analog to Proposition 1 then shows that there are balanced payments that elicit truthful information from all buyers in a strict Nash Equilibrium.

The assumption of a common, original prior can be relaxed slightly. The arguments go through essentially unmodified if the original priors of the various buyers are not common but are known to the center. The key is that buyer $i$’s belief about the distribution of buyer $j$’s signal depends on buyer $i$’s prior beliefs, but not on buyer $j$’s.

29 Since $(H^i, S^i)$ is also stochastically relevant for $S^j$ alone, buyer $i$ could be asked to predict only buyer $j$’s experience and not his history.
If the center does not know a buyer’s original prior, then it cannot differentiate between information contained in the buyer’s current experience and information deriving from his history. That is because proper scoring rules can elicit only beliefs about the posterior distribution of the other buyer’s signal. In the models considered in this paper, a one-to-one mapping between information and posterior distributions allows the information to be elicited. If the center does not know the buyer’s original prior, then it cannot separate the influence of the buyer’s prior from his experience with the seller. For example, a buyer with a good history who has a bad experience with the seller could have beliefs about the seller’s quality similar to a buyer with a bad history but a good experience with the seller. Thus, while common knowledge is not crucial to our mechanism, knowledge of the buyers’ priors is.

In certain circumstances, it may be possible for the center to elicit a buyer’s prior information by asking him to reveal his beliefs about another agent’s announcement before observing his own signal. For example, consider the case where each buyer’s prior beliefs about the seller’s type are private information, but the distribution of signals given types, \( f(s_n|t) \), is commonly known. Suppose buyer 2’s announcement is being used to provide incentives to buyer 1. Let \( p' = (p_1, ..., p_T) \) be buyer 1’s prior beliefs about the seller’s type, where \( p_t \geq 0 \) and \( \sum_{t=1}^{T} p_T = 1 \), and suppose buyer 1 is asked, before observing his signal, to announce his beliefs about the distribution of buyer 2’s announced signal. If buyer 1 is paid according to a strictly proper scoring rule based on his announced distribution, he will truthfully announce his beliefs about the distribution of buyer 2’s signal.\(^{30}\)

Let \( q' = (q_1, ..., q_N) \) be buyer 1’s initial belief about the distribution of buyer 2’s announcement. Since \( q \) is a distribution, \( q_n \geq 0 \) and \( \sum_{n=1}^{N} q_n = 1 \). Under a strictly proper scoring rule, buyer 1 reveals \( q \). Let \( p' = (p_1, ..., p_T) \) be buyer 1’s prior belief about the seller’s quality similar to a buyer with a bad history but a good experience with the seller. Thus, while common knowledge is not crucial to our mechanism, knowledge of the buyers’ priors is.

\(^{30}\)These payments could be made, for example, by buyer 3.
type, where \( p_t \geq 0 \) and \( \sum_{t=1}^{T} p_t = 1 \). Distributions \( p \) and \( q \) are related according to:

\[
q = Fp,
\]

where \( F \) is the \( N \) by \( T \) matrix whose elements are \( f(s_n|t) \).

The usefulness of this procedure depends on whether buyer 1’s initial announcement effectively reveals his prior distribution. If \( N = T \) and \( F \) is invertible, then \( F^{-1}q = p \), and \( q \) reveals \( p \). However, if \( N \neq T \) or \( F \) is not invertible, then there may be multiple prior distributions \( p \) consistent with a particular announcement \( q \).\(^{31}\) If the center is unable to form beliefs about which of the possible priors the buyer holds, then it may not be possible to extract meaningful information about the buyer’s prior beliefs. Nevertheless, for many classes of prior beliefs such extraction is possible.

When priors can be extracted, buyer 1’s signal can be elicited using a second round of transfers. These transfers are based on the scoring rule employed above, using the prior distribution revealed by the first round of elicitation.

4 Alternative Scoring Approaches

Two logical alternative scoring approaches will not work. The first would have the center pay on the basis of how close a rater’s report was to that of a future rater. The obvious problem is that buyers would not have an incentive to report their private signals in this case but rather their posterior beliefs about the next signal. The posterior beliefs would be informed not only by the private signal but also by the prior beliefs as well.\(^{32}\)

\(^{31}\)Since buyer 1 wants to truthfully reveal his information, there will always be some \( p \) consistent with his announced \( q \).

\(^{32}\)The proper scoring rule approach avoids this problem because the payment scheme automatically takes into account the buyer’s priors; hence reporting the private signal yields the optimal payoffs.
The second approach would pay on the basis of the divergence between the mean of a rater’s posterior and the posterior mean for some subsequent rater. Since the mean of the prior equals the expected value of the posterior mean after further information is secured, and given the mean’s well known least-squared-error properties, it might seem that charging the rater on a squared distance measure would induce him to report his true posterior mean.

Scoring based on the squared distance of the posterior means would work if the first report can do nothing to shift the variance of the future mean. However, for many important situations a player’s report will affect that variance, which introduces an incentive to distort the report. The Appendix presents an example.

5 Conclusion

The grand bazaar of electronic markets produces many temptations: for sellers they are sloth and misrepresentation, for buyers the lure is to shirk from the collective endeavor of producing accurate seller reputations. Both these temptations could be overcome through the effective use of reputation systems.

We show that a mechanism that capitalizes on stochastic relevance between the reports of different buyers, and that employs proper scoring rules and monetary payments, can give buyers appropriate incentives to report honestly. Beyond this, it can balance the budget and induce the right amount of effort when effort is costly.

Any such mechanism would require the widespread collection and dissemination of information about reputation, and computationally intensive calibration and distribution of payments. The vast reach of the Internet and its ready connection to superfast computers suggest that electronic markets could implement such mechanisms. The reputation systems

\[ \text{33 Thus the variance could be known and constant, or shrinking at a known rate with further evaluations.} \]
of the future compared to those of today, such as eBay’s, will be as a modern jumbo jet is to a Model T Ford. They will accomplish much the same task, but will operate at many times the scale, with vastly greater sophistication, using a radically different technology.
References


A Example Where Charging on Divergence in Posterior Fails

Consider the following example, where there are two seller types, G and B (for good and bad), and two possible signals, a and b. Initially, types G and B are equally likely. Type G sellers generate the two signals with equal probability. Type B sellers always generate signal a.

There will be two buyers in order. The first buyer gets a signal a. Employing Bayes’ rule, the posterior placed on types G and B become 1/3 and 2/3, respectively. Assuming the second buyer reports honestly, if the second buyer receives signal a, the posterior probability of type G is $\frac{1}{5}$. If the second buyer receives signal b, the posterior probability of type G is $\frac{1}{3}$.

It is readily computed that if the first buyer honestly reports the signal a, the expected charge is $K\left[\frac{5}{6}(\frac{1}{3} - \frac{1}{5})^2 + \frac{1}{6}(\frac{1}{3} - 1)^2\right] = K\frac{4}{45}$, where $K$ is the scaling constant and $\frac{4}{45}$ is the expected squared distance of the posterior probability of type G from the $\frac{1}{3}$ estimate.

However, buyer 1 has a superior strategy. If he simply lies and says he saw b, the posterior goes to Type G = 1, Type B = 0. Moreover, whatever the next buyer observes, the posterior will remain at Type G = 1. Thus, his expected charge will go to 0.

The explanation for this apparent paradox is that the variance of the posterior mean after buyer 2’s report depends on what buyer 1 says. Given that buyer 1 can reduce the variance with one report but not the other, a quadratic loss function will no longer give him an incentive to report honestly.