Rules and Standards When Compliance Costs Are Private Information

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A regulator, seeking to maximize net benefits, must choose between rules and standards and then set a level of care. The regulated agents have private information about their compliance costs. Rules are set ex ante, so agents know the required level of care. Standards are established after agents have taken initial actions, in anticipation of the regulating bureau’s directive. Those actions allow the bureau to make inferences about agents’ costs and thus set more appropriate requirements. Standards, however, expose agents to adjustment costs when they misjudge the required level of care before it is set. Nuanced trade-offs emerge. Standards are relatively more attractive when adjustment costs are low and compliance costs are more uncertain. If some agents are large relative to the market, those agents will choose their actions strategically to influence the ultimate standard. Rules, in contrast, are immune to strategic posturing. We discuss applications to financial regulation.

1. INTRODUCTION

Consider a regulation designed to control an externality-causing activity. A regulator, for example, might set the level of required bank reserves out of a concern about systemic effects of a bank failure or require the use of pollution abatement technology to limit external harms from emissions. To be efficient, such a regulation should set the marginal externality equal to the marginal compliance costs for each agent.

Compliance costs, however, are often private information, not known...
to the regulator. For example, the cost to implement a bolstered reserve requirement will vary from bank to bank, and only individual banks will know their idiosyncratic costs. Similarly, the use of a specific pollution abatement technology will be more costly for some firms than for others; firms will know these costs even if regulators do not. Accurate benefit-cost analysis depends on the regulator’s ability to estimate the magnitude of these costs.

A tax or subsidy equal to the expected marginal externality addresses the problem of private compliance costs. Individual actors, knowing their private costs and facing the tax, will act in a socially optimal fashion (Pigou 1932; Kaplow and Shavell 2002). Similarly, a tort system that imposes a charge equal to actual harm allows agents to choose appropriate actions on the basis of their private costs.

In many settings, however, externalities are addressed through command-and-control regulations in which the regulator specifies the level of care or the use of a particular technology. This is particularly applicable to the regulation of the financial system, in which Pigouvian taxes are not commonly used and ex post tort liability may not be adequate to address externalities.1 Regulators in this setting must estimate private costs to determine the appropriate level of care.

We consider how the choice between rules and standards affects the ability of a regulator to infer private costs. A regulator using a standard gives detailed content to the law only after the regulated agents have taken costly actions. By observing these actions, the regulator may be able to make inferences about private costs. A regulator using rules must promulgate them prior to when agents take action and, therefore, must estimate private costs from public information. Standards, therefore, may have an advantage over rules because they allow regulators to infer private costs by observing actions taken in anticipation of the standard.

Weighing against this advantage, standards suffer two disadvantages relative to rules. First, because agents take actions before they know where the final requirement will be set, they have to estimate what the required level of care will be. If they overshoot the requirement, they will have wasted resources. If they undershoot it, they will have to retrofit (at additional cost) to meet it. Rules, on the other hand, offer agents

1. An agent’s risky actions could raise the risk for the entire industry or even the economy. The tort system would not work since the agent would be judgment proof should a massive harm occur.
clarity up front regarding the final law and, therefore, avoid the problems of under- and overshooting.

Second, if a regulator uses a standard, an agent knows that a regulatory bureau will try to learn about his compliance costs. If the agent is large relative to the market, he will have an incentive to take a less-than-optimal action today to secure a more favorable regulation tomorrow. That is, standards create the possibility of strategic action that rules avoid.

Prior work, particularly Kaplow (1992), analyzes the choice between a rule and a standard using a similar framework but under the assumption that costs are public information. Kaplow compares the costs of acquiring the information ex ante (for rules) or ex post (for standards). If the regulator uses a rule, it incurs the costs of determining the appropriate level of care and incorporates this information into the law. If the regulator uses a standard, each regulated agent must incur the costs of determining the appropriate level of care. The choice between rules and standards picks the option with the lesser costs. Our analysis employs the same basic timing of actions as in Kaplow (1992). However, we emphasize that compliance costs are private information, which implies a more nuanced trade-off. Standards have a built-in flexibility that may allow them to incorporate private information in arriving at the appropriate regulation. Rules, being set ex ante, cannot take advantage of agents’ regulation-relevant private information.

We highlight three variables that affect how the equilibrium standard performs relative to a rule. These factors are distinct from the information-acquisition costs that have been traditionally emphasized in prior literature.

Strategic Influence. A standard allows the regulatory bureau to observe agents’ behavior before settling on a final law. However, if agents know that their actions today will affect the law tomorrow, they may modify their behavior accordingly. Standards, therefore, may be susceptible to strategic manipulation by regulated parties. Whether this is the case, however, depends on the agents’ strategic power. If a few agents, or a single agent, are large relative to the market, they will likely have considerable sway over the enforced standard. Conversely, if there are many agents, none large, no single agent will be able to exert appreciable influence. At times, the risk of strategic manipulation can bias a standard sufficiently that the bureau would prefer to employ a rule instead, as rules are immune to an agent’s strategic posturing. In other cases, the additional information revealed in a standards regime is sufficiently valuable that the
bureau is willing to accept the trade-off. Despite strategic manipulation, valuable information can often be secured.

Adjustment and Retrofitting Costs. Whenever agents act not knowing what the law requires, there is a chance they will err in the action they take. Thus, when a standard is employed, agents may overshoot (over-comply) or undershoot (take an insufficient action). Both errors beget a welfare loss, excess initial expenditure (overshooting), and costly ex post adjustment (undershooting and possibly overshooting). Other factors equal, high adjustment costs make standards relatively less attractive than rules. However, as we argue below, other factors may not be equal. The threat of a costly adjustment may enable the bureau to draw clearer inferences from agents’ actions, a factor that tilts in favor of standards.

Magnitude of Uncertainty. The choice between a rule and a standard often depends on the regulator’s uncertainty regarding the right policy. When uncertainties are small, rules can be set appropriately. When uncertainties are large, gathering information from agents’ actions can often be beneficial, despite the costs (noted above) that the process incurs.

We stress the importance of these variables through a formal model, which we develop in Section 3. Section 2 presents a simple example that highlights most of the model’s key intuitions. Section 4 discusses possible extensions and conclusions.

2. A MOTIVATING SITUATION

Consider the problem of setting the required level of (equity) capital that a bank must keep on its balance sheet. Assume that the reason for a capital-adequacy requirement is that there is a positive externality from high(er) levels of capital such as an increase in the safety of the banking sector overall.2 Because banks have no incentive to internalize this benefit, they will set their capital levels too low. In the language of the literature on tort liability, banks take a suboptimal level of care. The problem facing the regulating bureau is to set a capital-adequacy threshold that balances the external benefit of additional capital and banks’ costs of meeting this requirement.

To simplify the problem, we consider a two-by-two model. Capital levels (or more generally care) can be set to one of two levels, high or low, and the safety benefits are achieved only if banks have high capital

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2. This argument has been recently advanced by Admati and Hellwig (2013), among others.
levels. In addition, banks can have one of two levels of compliance costs, high or low. Compliance costs are such that if they are high, the bureau prefers a low threshold level of capital; if costs are low, the bureau prefers a high capital threshold. While we assume that the bureau knows the benefits of additional capital, compliance costs are private information known only to each bank, and, at the outset, the bureau and the banks have only an estimate of the distribution of these costs across the banks.

Although compliance costs differ among banks, we assume that the threshold level of capital cannot be differentiated so as to be agent specific. The bureau must set the same capital requirement for all banks. Political and legal restrictions, or the costs of additional complexity, often prevent the implementation of agent-specific regulations or laws. The bureau can regulate using a rule or a standard. When employing a rule, the bureau will decide on the threshold capital requirement ex ante on the basis of its prior beliefs about the agents’ costs. We assume perfect enforcement, verifiability of agents’ actions, and a sufficiently high level of penalties for noncompliance, so that banks always comply with the rule. If the bureau sets a high threshold, all banks set high levels of capital.

With standards, the bureau imposes a broad-brush requirement, such as “banks must take due care” in their operations. Indeed, the regulatory bureau might even merely state that it will be paying particular attention to capital levels when assessing banking practices. Banks, anticipating the enforcement of a standard and mindful of the bureau’s objective, then choose an initial level of capital. After some time, the bureau examines the banks’ behaviors and then determines if their initial actions were adequate. Through this assessment it establishes a requirement, sometimes explicit but often de facto, that all banks must meet. Those not in compliance are required to take a corrective action. As compared to meeting the requirement initially, such adjustment increases costs further.

Our analysis seeks to determine when establishing a rule will be preferable to a standard. The rule has the advantage that agents know from the outset what is required. There will be no overshooting or undershooting, no wasted expenditures or pare-down costs from the
former or retrofitting costs from the latter. The standard has the advantage that the bureau secures additional information before setting its requirement.

While we frame the example in terms of bank capital levels, the model is completely general. It applies to any regulation that imposes a threshold level of care because of an externality. It applies to positive externalities from care, as we consider here, or negative externalities from ordinary activities, such as those that generate pollution. It encompasses numerous financial regulations as well as environmental, safety, and many other types of regulations. With further elaboration, it could also relate to regulations that prohibit certain activities, as does the Volcker Rule. At the outset, banks, anticipating such a rule, will steer clear of some activities. As a result, those activities are more likely to be placed on the bureau’s prohibition list.

**Key Results and Intuition**

Our analysis addresses two distinct strategic environments. In the first case, there is a large number of banks, none large relative to the market, and no bank can influence the preferred regulation through its particular actions. In the second case, there is a single bank, and its strategic behavior must be expected. In most applications, reality likely resides somewhere in the middle. This distinction reflects the first critical factor identified in the Introduction.

Whether there is a single agent or many, setting a rule is conceptually straightforward. On the basis of its prior beliefs regarding the distribution of agents’ costs, the bureau can mandate either a high capital requirement or a low capital requirement. The bureau therefore simply maximizes expected utility on the basis of its prior beliefs. In our setting, where the bureau must pick either high or low level of capital, it assesses the probability that the bank has (banks have) low costs. If that probability exceeds a critical value, the bureau requires a high level of capital; otherwise, it sets a low capital requirement.

Whether there is a single agent or many, regulating via a standard is only seemingly straightforward. The standards approach relies on identifying an equilibrium between the behavior of the bank(s) and that of the bureau. The bank chooses (banks choose) an initial level of capital

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4. The Volcker Rule is a product of the Dodd-Frank Wall Street Reform and Consumer Protection Act. The provision seeks to limit the domain of banks’ proprietary trading activities.
and an adjustment strategy. The bureau observes initial capital level(s) and then sets a compliance threshold that all banks ought to exceed. In the interim, both parties update their beliefs and information on the basis of all observed behaviors. The bank chooses (banks choose) a strategy to maximize its (their) expected profits, anticipating the bureau’s response. The bureau sets its policy to maximize expected welfare, accounting for bank(s)’ compliance costs, (possible) adjustment behavior and costs, and the beneficial externality.

The resulting bureau-agent interactions are rich with nuanced implications. Consider the case of a single bank. A compelling intuition is that if there is a high likelihood that the bank’s costs are low, the bureau should tilt its policy toward a high-capital requirement. If it does so via a standard, however, it may become hostage to the bank’s strategic behavior. If the bank moves first and adjustment costs are high, the bank will opt for a low capital level. The bank recognizes that the bureau will not enforce a high standard because the adjustment costs would render that policy suboptimal ex post. In the presence of high adjustment costs, stringent standards may suffer a credibility crisis. In such a circumstance, the bureau would be better off retaking the first-mover advantage by employing a rule.\footnote{We assume that if the bureau announces a rule requiring high capital levels, it will enforce the rule even if a high-cost bank were to opt for a low capital level because there may be external costs on bureaus that do not enforce announced rules.}

Consider now the alternative scenario in which there are many banks, none large. With many banks, the issue of interest becomes how to draw appropriate inferences about the distribution of banks’ costs, as this fact determines the most preferred policy. When a bank is small, its particular behavior negligibly affects the inferred distribution of costs; hence, it cannot influence the standard via strategic manipulation. Moreover, since the standard will depend on the actions of other banks, each bank faces further uncertainty concerning the final law, as it can make only probabilistic predictions about the law’s stringency. Despite these complications, the bureau can often extract useful information from banks’ actions and can then tailor its regulation to the inferred distribution of banks’ compliance costs. Even though some banks incur adjustment costs ex post, these costs are often insufficient to negate the gain from the better-tuned policy overall. In short, there is much richness in the ensuing interaction, with costs and benefits trading off along multiple dimensions.
Because we simplify our model to capture the interactions between the bureau and agents, we abstract from many aspects of the choice between rules and standards. In particular, the information acquisition costs discussed in Kaplow (1992) are absent from our model; we assume that such costs have already been incurred. The bureau knows the size of the externality and has developed probabilistic beliefs about the distribution of agents’ costs on the basis of experience, hearings, consultant studies, and so forth. Second, rules are perfectly enforced, thus obviating the type of rule avoidance found in Weisbach (1999) and the uncertainty-related compliance effects found in Craswell and Calfee (1986) and Ben-Shahar (1998). For similar reasons, the model finesses the enforcement-cost considerations discussed in Shavell (2013). To be sure, such considerations will affect the ultimate choice between rules and standards. Our focus here is on the potential advantage that standards offer in eliciting information and their disadvantage because they will sometimes require retrofitting or lead to overshooting.

3. THE MODEL

This section presents a formal model of the problem. Although we often restrict parameters, we do so mainly to highlight the model’s interesting cases as well as to simplify exposition. Our conclusions apply more generally, as we note throughout. The model is phrased generally, but it closely tracks the situation explained in Section 2.

Suppose there is a set of profit-maximizing agents. Each agent can take an observable and verifiable level of care that we generically call an action, \( x \in \{0, 1\} \). The action \( x = 0 \) costs the agent nothing but generates no positive externalities (a normalization). The action \( x = 1 \) generates a positive social benefit of \( \sigma \) relative to \( x = 0 \). This benefit does not accrue to the agent and instead flows to other actors.\(^6\) Although \( x = 1 \) generates a social benefit, it burdens the agent with a private cost, \( x_p \), which depends on his type, \( \theta \in \{L, H\} \). An agent’s type is private information to him and is unknown to the bureau and to other agents. It costs an agent \( c_v \) to choose \( x = 1 \).\(^7\) We refer to a type L agent as a low-

\(^6\) We focus on the case of a positive externality, but the negative-externality case is symmetric. In this case, \( x = 0 \) would be associated with a negative externality, while \( x = 1 \) would be linked to an abatement of that negative effect.

\(^7\) We have assumed that the costly action does not generate any private benefits for the agent. We can accommodate a private benefit by interpreting \( c_v \) as the agent’s net cost of taking the action \( x = 1 \).
cost agent and to a type H agent as a high-cost agent. To focus attention on an economically interesting case, we henceforth assume that

\[ 0 < c_L < \sigma < c_H. \]  \hfill (1)

Each agent takes the action that maximizes the expectation of his private payoff. The bureau’s objective is to maximize the likelihood that an agent takes the action that maximizes (expected) aggregate welfare. Thus, weighing costs and benefits, it seeks to get agents’ choice of action up to the socially optimal level. A choice beyond that level is not desirable, since the agent’s costs will exceed the external benefits.

Given the available actions and the restrictions noted in equation (1), the following interpretation of the model might be helpful. We consider the action \( x = 0 \) to be the status quo level of care that an agent has been taking thus far. For example, it may reflect current banking practices or a polluting production process. A regulator becomes aware of an alternative but costly action \( (x = 1) \) that an agent can take. This action is believed to be socially beneficial relative to the default or status quo. For example, \( x = 1 \) may involve adopting a more prudent financial position or installing a new pollution abatement technology. Because of equation (1), no agent will voluntarily choose the costly action since \( x = 0 \) is privately optimal for all agents.

The situation is further complicated by the private costs associated with the socially beneficial action. A welfare-maximizing bureau would like a low-cost agent to take the action \( x = 1 \) and a high-cost agent to continue with the status quo action \( x = 0 \). Thus, our setting posits an agent-bureau conflict of interest and agent-level heterogeneity. Both elements are common in policy situations, and they present the bureau with a complex problem.

Many mechanisms have been proposed to solve the bureau’s regulatory problem when an externality is present. As mentioned above, a carefully calibrated tax (or tort liability) scheme is one option. To see how this idea works in our model, suppose that the bureau charges any agent taking the action \( x = 0 \) a tax of \( t \in (c_L, c_H) \) or, equivalently, an injured party was able to impose a tort liability of \( t \in (c_L, c_H) \). This tax or tort liability would be sufficient to incentivize a low-cost agent to take the costly action since \( -c_L > -t \). However, a high-cost agent prefers to pay \( t \) and take the action \( x = 0 \) instead.

Although a tax or tort liability can solve the regulatory problem in our setting, we assume that the bureau may not employ these policy tools because of political or other administrative constraints and instead
must choose a threshold action that all agents must meet or exceed. That is, it adopts a command-and-control approach, which is common in many policy domains. To set this threshold, the bureau may implement a rule or set a standard. Below we formalize both procedures; however, we first clarify some implicit assumptions.

First, we assume a public interest motive to regulation, in which broadly inclusive benefit-cost calculations are the guideline. In practice a regulator may have more narrowly defined interests, motives, or goals. Political pressure, for example, may lead regulators to weigh some parties’ costs and benefits more heavily than others. Regulatory capture may tilt the nature of the regulatory policies that are implemented (see, for example, Stigler 1971; Peltzman 1976; Laffont and Tirole 1991; a recent survey is Dal Bó [2006]). If these cases apply, \( \sigma \) would encode the more narrowly defined benefits motivating the bureau’s regulation. Except for the qualified interpretation, the analysis to follow would be unchanged.

Second, to avoid additional complications, we assume that both rules and standards are error-free regulatory instruments. Hence, we abstract from judicial errors or bias. Enforcement is certain, it occurs at negligible cost, and the penalties are high enough to ensure compliance.

Finally, we abstract from the situation’s temporal dimension in the sense that regulations implemented via rules or standards take effect over a comparable time period. Standard formal qualifications notwithstanding, our framework can accommodate the above extensions without substantively altering our conclusions.

**Rules.** If the bureau uses a rule, the law is specified in detail before any agent takes his action. Figure 1 summarizes the bureau-agent interaction in this case. Utilizing only information available to it at the outset, the bureau commits to a specified rule. That rule defines a minimal action \( y \) that the bureau will accept as adequate in meeting its objective. Taking this baseline as given, each agent then selects his preferred action \( x \) from those that comply with the new regulation. Thereafter, the interaction ends and payoffs are realized.

**Standards.** Standards constitute an alternative strategy. They specify the details of the regulation after agents have taken a costly action, usually in anticipation of the regulation’s eventual content. The overall sequence

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8. Recent models incorporating such features include Schwartzstein and Shleifer (2013) and Gennaioli and Shleifer (2008).
proceeds along the time line in Figure 2. First, each agent learns his type \( \theta \), which is private information. The bureau’s objectives and interests are assumed to be common knowledge, and with this in mind each agent takes an initial action \( x_0 \). Thereafter, the bureau observes the agents’ actions and utilizes that information to establish a minimum threshold \( y \), which gives content to the standard. The bureau is committed to setting some minimal threshold. Hence, an adjudication of agents’ actions in relation to a common standard will occur. Finally, agents can adjust their initial action to any action meeting or exceeding the set threshold. The term \( z_\theta \) is the final action an agent takes after adjustment (if any) has occurred. Adjustment is mandatory for any agent who falls short of that threshold.\(^{10}\) Adjustment is voluntary if an agent overshoots the threshold. When an agent adjusts his action ex post, he incurs an adjustment cost of \( k > 0.\(^{11}\) Table 1 outlines an agent’s total cost as a function of his initial action \( (x_0) \) and final action \( (z_\theta) \), which is the action taken after the opportunity to make adjustments. For example, if an agent falls short of a high threshold and must adjust his action \( (x_0 = 0 \implies z_\theta = 1) \), he incurs the adjustment cost \( k \) plus the cost of the higher action. Conversely, an agent initially overshooting the threshold who decides to economize if the threshold is low \( (x_0 = 1 \implies z_\theta = 0) \) incurs only the adjustment cost \( k \). In this regard, \( c_\psi \) can be interpreted as a long-term cost associated with the

\(^{10}\) In practice, bureaus often go beyond forcing only costly adjustment following non-compliance. Fines, penalties, or even imprisonment may follow. These supplementary policy tools come with additional degrees of freedom. To avoid confounds and to keep our analysis focused, we assume that the bureau cannot employ such measures.

\(^{11}\) We assume the same adjustment cost for both upward and downward adjustments. This assumption is easily relaxed. In a more general model, in which agents can choose from multiple actions, adjustment costs would vary with the magnitude of adjustment. We assume binary actions, and a fixed adjustment cost is therefore sufficient.
high-threshold action. An agent overshot the threshold initially may therefore be willing to adjust his action downward to save on these costs.

The bureau’s objective when it sets a standard follows the same benefit-cost principle as when it sets a rule. Importantly, when setting a standard the bureau is mindful of adjustment costs. This concern does not apply to rules-based regulations, as agents comply from the start.

The effectiveness of a rules- or standards-based regulation depends on the details of the environment. The distribution of agents’ compliance costs, the magnitude of the adjustment costs, and the severity of the social externality all matter. This is to be expected. A more subtle issue concerns the capacity of an agent to influence the bureau’s preferred policy through his initial action. If there is one agent, the bureau is likely to weigh this agent’s initial action heavily when enforcing a standard. The same phenomenon applies, though less strongly, if there are a few agents, each of whom can exert some modest influence on the standard. If, however, there are many small agents, each agent will take an action without trying to exert influence, since no individual agent can be influential given others’ behavior. Thus, with many agents, the bureau focuses on the aggregate outcome. We organize our study around the two extreme scenarios, beginning with the latter.

3.1. Many (Nonstrategic) Agents

When there are many small agents, each agent’s action and cost only negligibly affect aggregate welfare. Instead, the aggregate distribution of agents’ costs and actions is the key determinant of welfare. However, the bureau is uncertain regarding this distribution ex ante. For example, a regulator may be unsure what fraction of banks have high compliance
costs. As noted above, it may wish to implement a different requirement if many agents’ costs are high than if they are low.

To formalize such variability with a large number of agents, we assume two kinds of uncertainty in our model. First, let \( p \) be the probability that an agent’s type is \( \theta = L \). Conditional on \( p \), agents’ types are independent. Thus, in a very large population of agents (for example, in a continuum), a fraction \( p \) of agents will have low costs. However, suppose that the true value of \( p \) is not known ex ante to either the bureau or the agents. It is common knowledge that \( p \) is the realization of the random variable \( P \), which is distributed according to the cumulative distribution function \( F(\cdot) \). To avoid technicalities, suppose that \( F(\cdot) \) admits a strictly positive probability density function, \( f(\cdot) \).

We first consider rule setting, which is surprisingly simple: a bureau formulating the optimal rule plays the role of leader in a Stackelberg leader-follower problem. The bureau anticipates that each agent will take the least costly compliant action and sets its threshold accordingly. Formally, the bureau will choose \( y \in \{0, 1\} \) to maximize \( \{\sigma - \hat{p}c_L - (1 - \hat{p})c_H\}y \), where \( \hat{p} \equiv \mathbb{E}[P] \) is the expected value of \( P \). This is the bureau’s best ex ante estimate of the fraction of agents who have low costs. Hence, if we define

\[
p^* = \frac{c_H - \sigma}{c_H - c_L},
\]

the threshold, \( y^* \), that maximizes expected aggregate welfare is

\[
y^* = \begin{cases} 0 & \text{if } \hat{p} < p^* \\ 1 & \text{if } \hat{p} \geq p^*. \end{cases}
\]

The analysis is more complex when the bureau adopts a standards-based approach. Notably, both agents and the bureau will (attempt to) learn new information. For example, if there is sufficient heterogeneity in agents’ actions, the bureau will be able to infer \( p \). Similarly, employing Bayes’s rule, each agent will estimate \( p \) before he takes an action. Consequently, the resulting policy will be an equilibrium outcome depending
on beliefs and agents’ actions. Since each agent is small, we assume in definition 1 that the bureau does not respond to his particular action. Rather, the bureau bases its decision only on observed market aggregates.

**Definition 1.** A (type-symmetric) multiple-agent equilibrium in a standards regime consists of the following four elements:

1) For each \(v\), there is an adjustment strategy \(z^*_v(x, y)\) that assigns an adjustment action given the initial action \(x\) and the threshold \(y\). The function \(z^*_v\) maximizes a type \(v\) agent’s expected payoff given \(x\) and \(y\).

2) There is a threshold strategy \(y^*(q)\) for the bureau that specifies the threshold given the observed fraction of agents \(q\) selecting action \(x\).

3) For each \(v\), there is an initial action \(x^*_v\) that maximizes a type \(v\) agent’s expected payoff given \(y^*, z^*_v\), and the agent’s beliefs regarding the value of \(P\) and other agents’ initial actions.

4) The bureau and the agents have probabilistic beliefs that are updated using Bayes’s rule, whenever possible, conditional on the agents’ strategies and all available information.

Definition 1 restricts attention to type-symmetric outcomes. Thus, it considers equilibria in which all agents of the same type adopt the same strategy. Implicit in the definition is that our focus is on pure strategies.

The many-agents setting features two main classes of equilibria. The first class is a separating equilibrium in which each type of agent takes a distinct initial action. Such a strategy reveals the realized distribution of agents’ costs. The second class is a pooling equilibrium in which all agents take the same initial action. In this case no information is revealed, and at best the bureau draws only on its prior beliefs to set policy.

In studying separating equilibria, we focus on the natural case in which type L agents take the costly action while type H agents take the no-cost, status quo action. Thus, agents take initial actions that correspond to the first-best outcome. However, as shown below, a standard will not implement the first-best action, as a uniform regulation must apply to all agents. Generally, if few agents take the costly action, the

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12. We require that \(z^*_v(x, y) \geq y\) to ensure ex post compliance.

13. A third case, a semiseparating equilibrium, is possible too. However, this would require agents to adopt mixed strategies. We restrict attention to pure strategies.
bureau will set a low standard. If many take the costly action, a high standard is likely to follow. If the bureau insists on a high standard, high-cost agents who undershot initially need to make a costly upward adjustment. On the other hand, if the bureau sets a low standard and adjustment costs are low, low-cost agents who overshot initially will adjust their action downward, thereby undoing the external benefits and incurring private adjustment costs. Together, both cases imply that the first-best outcome is not achieved. A key driver in the undershooting by high-cost agents and the overshooting by low-cost agents is that their beliefs differ about which standard is likely to prevail. High-cost agents are inclined to believe that many other agents also have high costs.\(^{14}\) Thus, they anticipate that the bureau will set a low standard. A low-cost agent places his bets conversely.

**Proposition 1.** In the case of many agents, there exists a separating equilibrium such that \(x_L^* = 1\) and \(x_H^* = 0\) if and only if

1) \(\kappa \leq c_L\) and

2) 
\[
\int_0^{p^*} \frac{(1 - q)f(q)}{1 - p} dq \geq \frac{1}{2} \geq \int_0^{p^*} \frac{qf(q)}{p} dq,
\]

where \(p^* = \frac{c_H - \sigma + \kappa}{c_H - c_L + 2\kappa}\).

In such an equilibrium, the following may occur:

a) The bureau sets a low threshold if fraction \(q \leq p^*\) of agents take the costly initial action. Otherwise, it sets a high standard.

b) Whenever a low threshold is set, type L agents adjust their initial action downward voluntarily.

c) Whenever a high threshold is set, type H agents adjust their initial action upward as mandated.

The proof of proposition 1, and of other formal results, appears in the Appendix. The two sufficient and necessary conditions identified in proposition 1 have natural interpretations and implications. Condition 1 asserts that adjustment costs have to be relatively small. This condition is important, since all agents adjust their action (with positive probability) in equilibrium. If such costs are too high, they will not be willing to accept such adjustment risk. At its core, condition 2 is a statement about the prior distribution of \(P, \ F(\cdot)\). Roughly, it says that the distribution of \(P\) cannot be heavily skewed toward very low or very high

\(^{14}\) This follows from Bayes’s rule as agents attempt to infer the realized value of \(P\).
values. It ensures that agents who are of different types hold interim beliefs justifying the different actions that they take. The following numerical example reinforces the proposition’s conclusions.

**Example 1.** Suppose that \( c_L = 1/10, c_H = 1, \sigma = 1/2, \) and \( \kappa = 1/20 \). Assume that \( P \sim U[0, 1] \).\(^{15}\) A direct calculation shows that \( p^* = .55 \). Thus, 55 percent of the time, low-cost agents take an initial action that overshoots the standard’s threshold. Subsequently, they adjust their action downward. High-cost agents fall short 45 percent of the time and must adjust their action upward.

The second case is a pooling equilibrium in which all agents take the same initial action. In this case, the bureau learns nothing about the realized distribution of agents’ types and must base its policy only on prior beliefs. Pooling equilibria in which the initial action is \( x^*_p = 0 \) or \( x^*_p = 1 \) are possible. Proposition 2 distinguishes between the cases. Roughly, if the bureau believes many agents have high costs, it will opt to set a low standard. Thus, agents’ actions will coalesce around the \( x^*_p = 0 \) equilibrium. The alternative equilibrium, in which \( x^*_p = 1 \), obtains when adjustment costs are high or the bureau believes sufficiently many agents are type L.

**Proposition 2.** In the case of many agents, there exist pooling equilibria in which all agents take the default action and all agents take the costly action. In particular,

1) there exists an equilibrium in which all agents take the initial action \( x^*_p = 0 \) if and only if

\[
p \leq \frac{c_H - c - \sigma + \kappa}{c_H - c_L}.
\]

In such an equilibrium, the bureau sets a low threshold.

2) There exists an equilibrium in which all agents take the initial action \( x^*_p = 1 \) if and only if

a) \( \kappa \geq c_H - \sigma \)

b) \( \kappa \leq c_L \) and \( (c_H - \sigma - \kappa)/(c_H - c_L) \leq \hat{p} \).

In such an equilibrium, the bureau sets a high threshold.

A curious feature of proposition 2 concerns the gap between the instances when \( x^*_p = 1 \) is the agents’ initial action. When \( c_L < \kappa < c_H - \sigma \), no such equilibrium exists. For such intermediate values of \( \kappa \),

\(^{15}\) The conditions of proposition 1 and equation (1) are satisfied by these parameters.
only type H agents wish to adjust their action downward conditional on having taken the costly initial action. Given this fact, the bureau will optimally set a low standard if \( x^* = 1 \) for all \( \theta \). But if a low standard is anticipated, an agent has an incentive to deviate to the low-cost action so as to save on adjustment costs. (The bureau does not react to a solitary agent’s deviation and continues to set a low standard.) Hence, this (non)equilibrium unravels.

Comparing Rules and Standards. We seek to delineate the benefits and costs of the different enforcement mechanisms. While we have identified many practical differences between rules and standards, we focus our comparison on the aggregate welfare that these approaches secure. We say that regulatory mechanism A is superior to mechanism B if A generates strictly more expected aggregate welfare.

First, consider the case in which adjustment costs are moderate or high; that is, \( \kappa > c_t \). From propositions 1 and 2 we know that a standards regime will feature only a pooling equilibrium. Given this, the bureau learns nothing about the distribution of agents’ costs that it did not know at the outset, before there were any actions taken by the agents. Thus, the potential benefits associated with a standards regime do not materialize. Indeed, a simple calculation (omitted) shows that for all \( \hat{p} \) and \( \kappa > c_t \), the optimal rule generates at least as much expected welfare as the welfare-maximizing standards equilibrium. Moreover, when \( \hat{p} > p^* \) and \( c_t < \kappa < c_{tt} - \alpha \), the rule is strictly superior.

When \( \kappa < c_t \), neither a rule nor a standard is superior if we restrict attention to pooling-standards equilibria. Often, however, coordinating on the separating-standards equilibrium can be beneficial. For instance, consider the case of example 1. With those parameters, the optimal rule delivers an expected aggregate welfare of zero, but a separating-standards equilibrium delivers .076. The beneficial flexibility under the standard compensates for the associated adjustment costs.

Whether a separating-standards equilibrium is superior to a rule ultimately depends on the distribution of \( P \) and on the magnitude of adjustment costs. Thus, unconditional comparisons between the two mechanisms are not straightforward. Nevertheless, we offer two propositions that reinforce compelling intuitions.

First, as suggested in the Introduction, if adjustment costs are sufficiently small we expect a separating-standards equilibrium to dominate the rule. A standard allows the final policy to be tailored to the distribution of agents’ costs. If adjustment costs are small, a standard should
come out ahead since the bureau need not place great emphasis on avoiding costly ex post adjustment.

**Proposition 3.** Suppose that

\[
\rho^* \int_0^\infty \frac{(1 - q)f(q)}{1 - \hat{p}} dq > \frac{1}{2} > \int_0^\infty \frac{qf(q)}{\hat{p}} dq. \tag{5}
\]

If adjustment costs are sufficiently low, the aggregate expected welfare generated by the optimal standard exceeds the aggregate expected welfare generated by the optimal rule.

Condition 4 in proposition 2 follows from condition 2 in proposition 1 as \( \kappa \to 0 \). It ensures that for all \( \kappa \) sufficiently small, there exists a separating-standards equilibrium. It is essentially technical.

A complementary result considers the distribution of agents’ types. Intuitively, as uncertainty regarding the realized value of \( P \) increases, a standard performs relatively better. Again, this is because of the standard’s ex post flexibility.

To illustrate the idea, suppose that \( c_l = 1/10, c_{H} = 1, \sigma = 1/2, \) and \( \kappa = 1/20 \). Let \( F(p) = 3p^2 - 2p^3 \). This distribution has a mean of 1/2, and its density, \( f(p) = 2 - 6p + 6p^2 \), is mound shaped as illustrated in Figure 3. Under these parameters, the optimal rule delivers an expected payoff of zero. The optimal standard, however, achieves a payoff of .046 in a separating equilibrium. Therefore, the standard is preferable. Now suppose that uncertainty increases in the sense that more extreme values of \( P \) become more likely. Suppose that the cumulative distribution becomes
$G(p) = p$, the uniform distribution on the unit interval. Relative to $F(\cdot)$, $G(\cdot)$ spreads out probability mass to more extreme values but maintains the same mean. From example 1, we know that the separating-standards equilibrium delivers .076 in expected welfare and bests the optimal rule. Therefore, as the environment’s ambient uncertainty increases, a standard’s effectiveness increases. Proposition 4 generalizes the preceding intuition.

Proposition 4. Fix $0 < \kappa < c_1$. Let $F(\cdot)$ and $G(\cdot)$ be two distributions for $P$ with the same mean satisfying the conditions of proposition 1. Suppose that $F(\cdot)$ second-order stochastically dominates $G(\cdot)$. The separating-standards equilibrium when the distribution of $P$ is $G(\cdot)$ generates greater expected aggregate welfare than the separating-standards equilibrium when the distribution of $P$ is $F(\cdot)$.

Building on proposition 4, we can make two points. First, an immediate corollary is that if a standards regime is superior to the optimal rule when the distribution of $P$ is $F(\cdot)$, a standard continues to be superior as the distribution of $P$ spreads out. Second, parallel reasoning applies to the reverse comparison. If uncertainty regarding $P$ decreases, the relative benefits of a rules-based approach become more pronounced.

3.2. A Single (Strategic) Agent

Suppose now that there is only a single agent. Because he alone draws the bureau’s interest, it is reasonable to posit that the agent will try to influence the final regulation through his initial action.

No such influence is possible with a rule, since a rule is set before the agent acts. The bureau simply anticipates that the agent will take the least costly compliant action, namely, the one that just meets the threshold, and it sets its threshold accordingly. The bureau will choose $y \in [0, 1]$ to maximize $[\sigma - \hat{p} c_L - (1 - \hat{p}) c_H] y$. Hence, if $\hat{p}^* = (c_H - \sigma)/(c_H - c_L)$ as in equation (2), then the optimal threshold $y^*$ is

$$y^* = \begin{cases} 0 & \text{if } \hat{p} < \hat{p}^* \\ 1 & \text{if } \hat{p} \geq \hat{p}^*. \end{cases}$$

Expression (6) says that if the likelihood that an agent has low costs is sufficiently large, the bureau will set a high threshold. Otherwise, a low threshold is preferable. This is the same result as in the many-agents case.

16. Alternatively, we can say that $G(\cdot)$ is a mean-preserving spread of $F(\cdot)$. See Rothschild and Stiglitz (1970).
The analysis is considerably more complex when the bureau uses a standard. Since the agent makes the first move, his action may reveal information about his type. Moreover, since there is but one agent, the bureau will care about this agent’s type and action. Therefore, the strategic implications of the agent’s action are more subtle than in the many-agents case.

Formally, the case of a single agent is a signaling game (Spence 1973). For example, if an action reveals that the agent has low costs, the bureau may wish to implement a different policy than it would had the action signaled the alternative case. Conversely, an action might not reveal any information, especially if the bureau believes that all types of agents will choose the same initial action. The final result, therefore, will be an equilibrium outcome depending on the bureau’s beliefs and the behavior of different agent types.

We first define what we mean by an equilibrium in a standards regime. Our definition adapts the notion of a perfect Bayesian equilibrium to our specific application (see, for example, Fudenberg and Tirole 1991; Osborne and Rubinstein 1994). Roughly, we require that both the agent and the bureau behave optimally given the strategy and beliefs of the other. It contrasts with the definition of an equilibrium with many agents in that the bureau now responds to the action of the single agent and draws inferences based on that action.

Definition 2. A single-agent equilibrium in a standards regime consists of the following four elements:

1) For each $\theta$, there is an adjustment strategy $z^*_\theta(x, y)$ that assigns an adjustment action given the initial action $x$ and the threshold $y$. The function $z^*_\theta$ maximizes a type $\theta$ agent’s expected payoff given $x$ and $y$.

2) There is a threshold strategy $y^*(x)$ for the bureau that specifies the threshold given the agent’s initial action $x$. The function $y^*$ maximizes the bureau’s expected payoff given the agent’s initial action, the agent’s adjustment strategy, and the bureau’s beliefs regarding the agent’s type.

3) For each $\theta$, there is an initial action $x^*_\theta$ that maximizes a type $\theta$ agent’s expected payoff given $y^*$ and $z^*_\theta$.

4) The bureau’s beliefs regarding the agent’s type are probabilistic beliefs...
updated using Bayes’s rule, whenever possible, conditional on the agent’s strategy.18

Given the interaction of beliefs and actions, a standards regime may lead to multiple equilibria. That is, for a given set of parameters, the conditions of definition 2 hold for different combinations of initial actions or threshold strategies. Whenever this is the case, we focus on the aggregate welfare-maximizing equilibrium. As in the many-agents case, implicit in definition 2 is a restriction to pure strategies in which neither the agent nor the bureau chooses an action at random.

Signaling games traditionally feature two main classes of equilibria.19 In a separating equilibrium, each type of agent takes a distinct initial action. Hence, the bureau is able to perfectly infer the agent’s type from the observed action. In a pooling equilibrium, both types of agents take the same initial action. No information is revealed, and the bureau relies on its prior beliefs to set policy.

We first focus on separating equilibria, which feature in many policy-oriented applications of signaling games. For example, Spence (1973) famously argued that educational attainment can signal innate ability. Intrinsically more productive workers may choose to get more schooling solely to convince employers of their higher ability. Surprisingly, such separation is not possible in our setting.

Proposition 5. In the case of a single agent, there does not exist a separating equilibrium.

The logic of proposition 5 is straightforward. If there were a separating equilibrium, then any agent taking the least costly initial action must be required to adjust his action upward. Were this not the case, the agent taking a higher-cost action could profitably deviate to the lower-cost option. But if the lower-cost action is met with a mandated costly adjustment, the agent taking the lower-cost action would prefer to take the higher-cost action initially to avoid the adjustment costs. Thus, separation cannot be supported.20

18. When beliefs cannot be derived using Bayes’s rule, we are free to assign beliefs to the bureau as desired. We do not employ any equilibrium refinements constraining off-equilibrium path beliefs.
19. A third class, the semiseparating equilibrium, is also possible. Such an equilibrium involves mixed strategies. We restrict attention to pure strategies.
20. A separating equilibrium can be supported when the agent’s action space is richer. For example, suppose that 0 ≤ x ≤ 1 and assume that an agent’s costs, c(x), satisfy the Spence-Mirrlees single-crossing condition. Such a generalization can admit separating equilibria.
It is worth considering why a separating-standards equilibrium exists in the many-agents case but is not attainable in the single-agent setting. The key difference is the residual uncertainty regarding the ultimate threshold in the many-agents setting. With many agents, the realized standard depends on the distribution of agents’ actions. Thus, when a particular agent decides on his action, he is unsure what the bureau will do. Given the residual uncertainty and agents’ beliefs, different types may well coalesce around different initial actions in an equilibrium.

What we call residual uncertainty is frequently identified in studies of ex ante and ex post regulation, especially in the literature on negligence and liability. For example, Kolstad, Ulen, and Johnson (1990) study how uncertainty about the legal standard affects the level of care taken by an agent. However, the uncertainty we focus on is distinct from their approach. Like Craswell and Calfee (1986), Kolstad, Ulen, and Johnson (1990) view the variation in the legal standard as exogenously driven. For example, it may stem from a court’s idiosyncratic judgment or interpretation of evidence.21 Residual uncertainty in our setting is an equilibrium outcome that depends on agents’ behavior.22

The nonexistence of a separating equilibrium turns our focus to the second class of equilibria. Broadly, there are three types of pooling equilibria that can occur. We illustrate their occurrence as a function of the underlying parameters in Figure 4. The figure distills the key conclusions from proposition 6, which we present below. On the figure’s horizontal axis, we plot the ex ante probability that an agent has low costs \( \hat{p} \). The adjustment cost \( \kappa \) is shown on the vertical axis. For each pair \( (\hat{p}, \kappa) \), we identify the (pooling) equilibrium that yields the greatest expected aggregate welfare. We summarize this equilibrium with the pair \( x^* \mid y^* \), where \( x^* \) is the agent’s initial action and \( y^* \) is the bureau’s equilibrium standard. Figure 4 also imposes the parameter restrictions

\[
c_{L} + c_{H} < 2 \sigma \quad \text{and} \quad \sigma + c_{L} < c_{H}
\]

so as to illustrate the full range of equilibria that occur.23

There are three types of equilibria. In a 0|0 equilibrium, aggregate

21. In the model of Kolstad, Ulen, and Johnson (1990), the legal standard is also a threshold that an agent must exceed, \( \xi(e) \). However, \( e \) is a random variable with a pre-specified distribution.

22. Diamond (1974) identifies an alternative channel whereby an agent taking an action is unsure whether his action is sufficient. Diamond (1974) considers a situation in which the agent’s action is mapped stochastically onto the outcome-relevant domain.

23. The conditions in equations (1) and (7) are satisfied by \( c_{L} = 1/10, c_{H} = 1 \), and \( \sigma = 3/4 \).
welfare is zero. This pooling equilibrium occurs primarily when \( \hat{p} \) is low. Surprisingly, it also features when \( \hat{p} \) is not low but adjustment costs are high. This counterintuitive conclusion rests on the bureau’s inability to credibly implement a higher standard when adjustment costs are high. When adjustment costs are sufficiently high to exclude ex post adjustment, the strategic agent takes the least costly action knowing that the bureau will not move to enforce an alternative outcome. In a 1|1 equilibrium, the expected welfare is \( \sigma - \hat{p}c_l - (1 - \hat{p})c_{11}, \) and only the high-cost action is taken. Finally, in a 1|0 equilibrium, the expected welfare is \( \hat{p}(\sigma - c_i) + (1 - \hat{p})(-\kappa). \) In this case, corresponding to the hatched region of Figure 4, the initial action is costly, but a type H agent adjusts his action downward ex post. The bureau sets a lax threshold since it knows that only a type H agent will find it worthwhile to adjust. Although costly, this adjustment is welfare improving as \( -\kappa > \sigma - c_{11}, \) and agents’ final actions align with the first-best outcome.

More formally, proposition 6 characterizes the pooling equilibria in our model. As suggested above, the precise form of the pooling equilibrium depends on the prevailing parameter values.

**Proposition 6.** In the case of a single agent there exist pooling equilibria...
in which all agents take the default action or all agents take the costly action. In particular,

1) there exists an equilibrium in which \( x^*_p = 0 \) if and only if

\[
\hat{p} \leq \frac{c_{hl} - \sigma + \kappa}{c_{hl} - c_L}. \tag{8}
\]

The bureau’s threshold strategy is such that \( y^*(0) = 0 \).

2) There exists an equilibrium in which \( x^*_p = 1 \) if and only if \( \kappa \leq \sigma - c_L \). Moreover,

a) if \( \kappa \leq c_L \), then the bureau’s threshold strategy is

\[
y^*(1) = \begin{cases} 
0 & \text{if } \hat{p} < \frac{c_{hl} - \sigma - \kappa}{c_{hl} - c_L} \\
1 & \text{if } \hat{p} \geq \frac{c_{hl} - \sigma - \kappa}{c_{hl} - c_L}.
\end{cases} \tag{9}
\]

Whenever \( y^*(1) = 0 \), the agent adjusts his action downward.

b) If \( c_L < \kappa \leq c_{hl} \), then the bureau’s threshold strategy is

\[
y^*(1) = \begin{cases} 
0 & \text{if } \kappa \leq c_{hl} - \sigma \\
1 & \text{if } \kappa > c_{hl} - \sigma.
\end{cases} \tag{10}
\]

Whenever \( y^*(1) = 0 \), only a high-cost agent adjusts his action downward.

c) If \( c_{hl} < \kappa \), then the bureau can set any threshold. Neither type of agent adjusts his action ex post.

**Comparing Rules and Standards.** The above analysis confirms that rules and standards lead to different outcomes. But which method is better? We can refer to Figure 4 to aid in the comparison. The shaded and hatched regions identify the superior regime as a function of \( (\hat{p}, \kappa) \). A rules-based approach is superior when adjustment costs are high and there is a high likelihood that the agent has low costs. The rule insulates the bureau from the credibility concerns that plague the standards regime when adjustment costs are high. Standards-based approaches are superior when adjustment costs are intermediate. In that case, corresponding to the hatched region in Figure 4, a standards regime allows for a flexible and responsive regulation. The low standard allows a high-cost agent to adjust his action downward while a low-cost agent finds it optimal to maintain his initial action. In the remaining cases, the optimal rule and the welfare-maximizing standards equilibrium yield the same expected aggregate welfare.

An interesting result emerges when we compare this conclusion with the multiagent case. We see that there exists a welfare trade-off that depends on the agents’ strategic capability. As shown in Figure 4, when
adjustment costs are intermediate \((c_L < \kappa < c_{H} - \sigma)\), a standard is often preferable to a rule when there is a single agent. The opposite conclusion applies in the case of many agents. A rule is beneficial if adjustment costs are relatively large \((\kappa > \sigma - c_{1})\) when there is but a single agent. This strict advantage disappears in the case of many agents.

4. CONCLUSIONS

Most regulations employ a command-and-control format. They are implemented as either rules or standards. Rules provide certainty from the outset but do not allow the regulator to observe agents’ actions prior to being promulgated. Standards enjoy a built-in flexibility that may prove valuable to the regulator but that creates costly uncertainty for regulated agents. When one or more of the regulated agents is large relative to the market, it may behave strategically under a standards regime to camouflage costs. This creates a multidimensional trade-off when choosing the optimal regulatory mechanism.

Our analysis points to qualities a policy maker may wish to keep in mind when choosing between an ex ante rules-based or an ex post standards-based regulatory regime. We note, however, that our conclusions rest on several assumptions that have been noted throughout our analysis. In particular, our analysis does not incorporate many of the considerations relevant to the choice between rules and standards that have been discussed in the literature. For example, we do not include the differential costs of gathering publicly available information for rules and standards, nor do we consider the possibility that rules and standards may optimally have different levels of complexity. We also assume that the regulator is proceeding on a benefit-cost basis, in which the sum of costs and benefits to those regulated and to external parties guides his choices. In many cases, political desires or legislative requirements may tilt the regulator’s interest away from this approach.

We emphasize two additional caveats. First, our discussion presents a stark choice between rules and standards. In practice, the choice between the approaches is more nuanced. Often a hybrid approach may be preferable.\(^{24}\) Such a hybrid approach has considerable appeal in an extended version of our model. For example, suppose that agents could take three actions—low, medium, high—instead of just two actions, as

\(^{24}\) Arguments supporting the joint use of ex ante and ex post regulations are advanced by Shavell (1984) and by Kolstad, Ulen, and Johnson (1990). See also Schmitz (2000).
in our original analysis. In such an instance, a hybrid regime may first specify a rule restricting the set of available options ex ante, say, to medium or high. Only later would it invoke a standard, once agents’ actions have revealed additional information.

Second, we have assumed that the bureau knows the magnitude of external benefits generated by the regulated agents’ actions. This is the $\sigma$ parameter in our model. The bureau’s preferred action and regulatory regime depend on $\sigma$. Thus, accurate estimates of $\sigma$ are critical for setting an optimal policy. In practice, knowing $\sigma$ may be a formidable requirement. Consider again our motivating example of bolstered capital requirements from Section 2. Quantifying the benefits of heightened capital requirements is likely to be a difficult task. It necessarily requires many judgment calls and estimates to arrive at a number that can guide decision making.25 One possibility, of course, is that the bureau could adjust either a rule or a standard as its knowledge about $\sigma$ improves.

The real-world choice between a rule and a standard will require an array of wrangles in the regulatory arena. A spare model, such as ours, can hardly dictate the outcome. Nevertheless, we believe that the variables we have identified—the broader strategic setting, the adjustment regime, and the extent of uncertainty—should be key inputs in any regulator’s policy choice.

APPENDIX

Proof of Proposition 1

Suppose that $x_1^L = 1$ and $x_{1H} = 0$. Conditional on this type-symmetric strategy profile, the bureau knows that all agents taking action 1 (0) are of type L (type H). Let $q$ be the faction of agents inferred to be type L. If the bureau sets $y^*(q)$ equal to one, its expected payoff is $q(\sigma - c_1) + (1 - q)(\sigma - c_{1H} - \kappa)$. If the bureau sets a threshold of zero, its payoff is $q(\sigma - c_1)$ if $\kappa > c_1$ and $-q \kappa$ if $c_1 \geq \kappa$.

Suppose that $\kappa > c_1$. In this case, no type L agent who took the initial action $x_1^L = 1$ will adjust his action in response to a low threshold. But since $\sigma - c_{1H} - \kappa < 0$, it is optimal for the bureau to set a low threshold. Therefore, with a probability of 1 the standard will be set at zero given the agents’ collective strategy. If an agent anticipates the bureau’s stan-

25. Coates (2014) presents a case study outlining many of the associated challenges and debates.
standard to be zero, that agent’s optimal initial action is \( x = 0 \). But this implies a profitable deviation for all type L agents. Therefore, a separating equilibrium as described cannot exist.

Suppose that \( c_L \geq \kappa \). The bureau will set a low threshold if and only if

\[
-q\kappa \geq q(\sigma - c_L) + (1 - q)(\sigma - c_H - \kappa) \Rightarrow q \leq p^*_L \equiv \frac{c_H - \sigma + \kappa}{c_H - c_L + 2\kappa}.
\]

Otherwise, a high threshold is optimal.

Having identified the bureau’s optimal strategy, we must verify that neither type of agent wishes to deviate from the proposed strategy. First, the agent must hold beliefs regarding the fraction of agents who are type L. Conditional on \( p \), this fraction will equal \( p \) in a large population. Thus, each type of agent must form beliefs regarding the distribution of \( P \) conditional on his type.

1) Consider a type L agent. Given the agent’s type, his posterior beliefs concerning the distribution of \( P \) are

\[
f_L(p) = f(p|\theta = L) = \frac{pf(p)}{\hat{p}},
\]

where

\[
\hat{p} = \int pf(p)dp = \mathbb{E}[P].
\]

Therefore, given the bureau’s strategy and the other agents’ strategies, such an agent believes that with probability

\[
F_L(p_L^*) = \int f_L(q)dq
\]

the bureau will set a threshold of zero. Whenever the threshold is zero, a type L agent will adjust his action to zero. Hence, taking the action \( x_L^* = 1 \), as prescribed, is optimal if and only if

\[
F_L(p_L^*)(-\kappa) + [1 - F_L(p_L^*)](-c_L) \geq F_L(p_L^*)(0) + [1 - F_L(p_L^*)](-c_L - \kappa)
\]

\[
\Rightarrow \frac{1}{2} \geq F_L(p_L^*).
\]

2) Consider a type H agent. Given the agent’s type, his posterior beliefs concerning the distribution of \( P \) are
Therefore, given the bureau’s strategy and the other agents’ strategies, such an agent believes that with probability

\[ F_i(p^*) = \int_{0}^{1} f_i(q) dq \]

the bureau will set a threshold of zero. Whenever the threshold is zero, a type H agent will adjust his action to zero (if he took an initial action of one). Hence, taking the action \( x_i^* = 0 \), as prescribed, is optimal if and only if

\[ F_i(p^*)(0) \geq \frac{1}{2} \]

Q.E.D.

**Proof of Proposition 2**

It can be verified that for all values of \( \hat{p} = E[P] \) and \( \kappa \), at least one of the sufficient conditions identified in the proposition are satisfied. Therefore, we focus on showing that an equilibrium exists in each case.

1) Suppose that \( x_i^* = 0 \) for all \( \theta \). For this initial action to be consistent with equilibrium, the bureau must set a low threshold. Otherwise, if the anticipated standard is high, an agent will prefer to take the alternative initial action to avoid the adjustment costs. Setting \( y^* \) equal to zero is optimal if and only if

\[ F_i(p^*)(0) + [1 - F_i(p^*)](-c_H - \kappa) \geq F_i(p^*)(-\kappa) + [1 - F_i(p^*)](-c_H) \]

\[ \Rightarrow F_i(p^*) \geq \frac{1}{2} \]

2) Suppose that \( x_i^* = 1 \) for all \( \theta \). There are three cases:

a) Suppose that \( c_H < \kappa \). In this case, neither type of agent wishes to adjust his initial action. The adjustment cost is too high. Therefore, both threshold levels are optimal for the bureau. Thus, the bureau can set \( y^* = 1 \), and the expected aggregate welfare is \( \sigma - \hat{p}c_L - (1 - \hat{p})c_H \).

b) Suppose that \( c_L < \kappa \leq c_H \). In this case, a type H agent will adjust his action downward if a low standard is realized. A type L agent will not adjust his action. If \( x_i^* = 1 \) for all agents, the bureau will set \( y^* \) equal to zero if and only if
\[
\hat{p}(\sigma - c_L) + (1 - \hat{p})(-\kappa) \geq \hat{p}(\sigma - c_L) + (1 - \hat{p})(\sigma - c_{H})
\]
\[
\Rightarrow c_{H} - \sigma \geq \kappa.
\]

Otherwise, the bureau will set \( y^* = 1 \) as the standard.

If \( c_{H} - \sigma > \kappa \), the bureau will set a low standard, and an individual agent will wish to deviate to the less costly action in anticipation of the low standard. This, of course, precludes the existence of an equilibrium in which all agents take the more costly initial action. Hence, there exists an equilibrium if and only if \( c_{H} - \sigma \leq \kappa \) and \( y^* = 1 \) is the enforced standard. The expected aggregate welfare is \( \sigma - \hat{p}c_L - (1 - \hat{p})c_{H} \).

\( c \) Suppose that \( \kappa \leq c_L \). In this case, both types will adjust their action downward if the low standard is set. Setting \( y^* \) equal to zero is optimal for the bureau if and only if

\[
-\kappa \geq \sigma - \hat{p}c_L - (1 - \hat{p})c_{H} \Rightarrow \hat{p} \leq \frac{c_{H} - \sigma - \kappa}{c_{H} - c_L}.
\]

Otherwise, the bureau will set \( y^* \) equal to one. As in the preceding case, if \( y^* = 0 \) is the anticipated standard, then an agent will have a profitable deviation. Therefore, it must be true that \( \hat{p} \geq (c_{H} - \sigma - \kappa)/(c_{H} - c_L) \) for an equilibrium to exist. The expected aggregate welfare is again \( \sigma - \hat{p}c_L - (1 - \hat{p})c_{H} \). Q.E.D.

**Proof of Proposition 3**

We argue that for \( \kappa \) sufficiently small, the separating-standards equilibrium (see proposition 1) will generate greater expected aggregate welfare than will the optimal rule. By equation (5), the conditions of proposition 1 are satisfied for all sufficiently small \( \kappa \). Thus, as \( \kappa \to 0 \), there exists a separating-standards equilibrium in the standards regime.

If \( \kappa > 0 \), the expected aggregate welfare from the separating-standards equilibrium is

\[
\int_{0}^{1} \max\{-q\kappa, q(\sigma - c_L) + (1 - q)(\sigma - c_{H} - \kappa)\}f(q)dq.
\]  
(A1)

Since there is a large number of agents, ex ante the fraction of agents taking the high-cost action (all of whom are type L) is \( p \). Thus, expectations are taken with respect to the distribution \( F(\cdot) \). As \( \kappa \to 0 \), equation (A1) converges to

\[
E[\max\{0, P(\sigma - c_L) + (1 - P)(\sigma - c_{H})\}].
\]
The function $\max\{0, q(\sigma - c_L) + (1 - q)(\sigma - c_H)\}$ is convex (but not affine) in $q$ and $f(q) > 0$ for all $q$. Hence, by Jensen’s inequality,

$$E[\max\{0, P(\sigma - c_L) + (1 - P)(\sigma - c_H)\}]$$

$$> \max\{0, E[P](\sigma - c_L) + (1 - E[P])(\sigma - c_H)\}.$$ 

The strict inequality follows from the nonlinearity of the integrand and the full support of the distribution $F(\cdot)$. The right-hand side of the preceding expression is an upper bound for the expected aggregate welfare generated by the optimal rule. Thus, there exists a sufficiently small such that $0 < \kappa < \bar{k}$ implies that

$$\int_0^1 \max\{-q\kappa, q(\sigma - c_L) + (1 - q)(\sigma - c_H - \kappa)\}f(q)\,dq$$

$$> \max\{0, E[P](\sigma - c_L) + (1 - E[P])(\sigma - c_H)\}.$$ 

Q.E.D.

**Proof of Proposition 4**

From Rothschild and Stiglitz (1970), we know that if $F(\cdot)$ second-order stochastically dominates $G(\cdot)$, then for any bounded convex function $\varphi(\cdot)$,

$$\int_0^1 \varphi(q)f(q)\,dq \leq \int_0^1 \varphi(q)g(q)\,dq,$$

where $f(\cdot)$ and $g(\cdot)$ are the distributions’ density functions. As a function of $q$, the aggregate welfare is

$$\max\{-q\kappa, q(\sigma - c_L) + (1 - q)(\sigma - c_H - \kappa)\},$$

and this function is convex in $q$ and bounded for $q \in [0, 1]$. Thus, the conclusion follows since the ex ante equilibrium distribution of $q$ is $f(\cdot)$ or $g(\cdot)$, per the prevailing case. Q.E.D.

**Proof of Proposition 5**

The proof is by contradiction. Consider an equilibrium in which $x_H^x = 0$ and $x_L^x = 1$. Thus, if an agent takes action $x = 0$, the bureau infers that the agent is of type H. Therefore, it would set a low threshold.

Given this behavior by the bureau, a type L agent has an incentive to deviate and to take the less costly initial action.

Suppose there exists an equilibrium in which $x_H^x = 1$ and $x_L^x = 0$. 
Mirroring the preceding case, if it is optimal for the bureau to set a low standard given the initial action $x = 0$, the type H agent will deviate. Suppose instead that the bureau sets a high threshold following the initial action $x = 0$. Thus, the payoff of a type L agent is $-c_L - \kappa$. In this case a type L agent would strictly benefit from taking the initial action $x = 1$ to avoid the adjustment costs. Q.E.D.

**Proof of Proposition 6**

The existence of a pooling equilibrium follows from cases 1 and 2. In particular, case 2 shows that if $\sigma \geq c_L + \kappa$, then there exists a pooling equilibrium in which $x^*_0$ equals one. On the other hand, if $\sigma < c_L + \kappa$, then $(c_H - \sigma + \kappa)/(c_H - c_L) > 1$. Hence, condition (8) holds for all $\hat{p}$. Thus, there exists a pooling equilibrium in which $x^*_0$ equals zero. Therefore, there exists at least one pooling equilibrium for all parameter values.

1) Suppose there exists a pooling equilibrium in which $x^*_0$ equals zero for all $\theta$. In any such equilibrium, the bureau’s threshold must be zero. Otherwise, in anticipation of the high threshold, an agent would prefer to take the higher-cost action to avoid the adjustment cost. Since $x = 0$ is the agent’s most preferred action, to confirm that there exists such an equilibrium it is sufficient to identify conditions so that the bureau does not prefer to set a high threshold. This condition is

$$0 \geq \sigma - \hat{p}c_L - (1 - \hat{p})c_H - \kappa \Leftrightarrow \hat{p} \leq \frac{c_H - \sigma + \kappa}{c_H - c_L}.$$  

2) For there to exist a pooling equilibrium in which both types of agents take the higher-cost initial action, the bureau must adopt a strategy that ensures that neither type of agent wishes to take the lower-cost initial action. Hence, the bureau must hold (off-equilibrium path) beliefs such that it mandates an upward adjustment following the initial action $x = 0$. Otherwise, an agent would not be deterred from this action. If the bureau believes that an agent taking the action $x = 0$ is of type 0, it is optimal for the bureau to mandate an upward adjustment if and only if $\sigma \geq c_L + \kappa$. Similarly, if it believes that an agent taking the action $x = 0$ is of type 1, it is optimal for the bureau to mandate upward adjustment if and only if $\sigma \geq c_H + \kappa$. This condition implies that $\sigma \geq c_L + \kappa$. Therefore, for the bureau to set a high threshold, the restriction $\sigma \geq c_L + \kappa$ must hold.

Finally, we verify that $\sigma \geq c_L + \kappa$ is also a sufficient condition. In each case below, suppose that following an initial action of $x = 0$, the bureau believes that the agent’s type is $\theta = 0$ with a probability of 1, and it sets
a high threshold. Given this strategy for the bureau, neither type of agent wishes to take the initial action \( x = 0 \). Thus, all that remains is to identify the bureau’s optimal threshold.

a) Suppose that \( x \leq c_1 \). If the bureau sets a threshold of zero following the initial action \( x = 1 \), both types of agents adjust downward to \( z^*_0 = z^*_1 = 0 \). Setting \( y^* \) equal to zero is optimal if and only if \( -\kappa \geq \hat{p}(c_1 - a) - (1 - \hat{p})c_{\text{II}} \) or \( \hat{p} \leq (c_{\text{II}} - a - \kappa)/(c_{\text{II}} - c_1) \). Otherwise, \( y^* = 1 \) is optimal.

b) Suppose that \( c_{\text{II}} < k \leq c_1 \). If the bureau sets a threshold of zero following the initial action \( x = 1 \), only a type \( \text{II} \) agent adjusts his action downward to \( z^*_0 \). Setting \( y^* \) equal to zero is optimal if and only if \( \hat{p}(\sigma - c_1) + (1 - \hat{p})(-\kappa) \geq \sigma - \hat{p}c_1 - (1 - \hat{p})c_{\text{II}} \) or \( k \leq c_{\text{II}} - \sigma \). Otherwise, \( y^* = 1 \) is optimal.

c) Suppose that \( c_{\text{II}} < \kappa \). In this case, neither type of agent adjusts his action downward since the adjustment costs are too high. Q.E.D.

REFERENCES


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