Loss Hedging Financial Transmission Rights

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January 15, 2002
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LOSS HEDGING FINANCIAL TRANSMISSION RIGHTS

Scott M. Harvey and William W. Hogan

I. OVERVIEW

This note outlines a number of designs for defining Financial Transmission Rights (FTR) that hedge losses as well as congestion and the associated FTR auction. The motivation for the discussion of multiple approaches is the potential for some formulations of loss hedging FTRs to expose the holder to energy price risks and an uncertainty as to the ability and willingness of market participants to bid for FTRs that expose them to such energy price risks. Conversely, some alternative formulations of loss hedging FTRs would impose unhedged energy price risks on the ISO. In the case of congestion-only Transmission Congestion Contracts (TCC), there is no exposure to energy imbalances through the holding of TCCs, and the issue does not arise. However, under this usual TCC formulation, market participants are fully exposed to variations in the cost of losses. The purpose of the FTR extension is to include the effect of losses and provide a perfect hedge for both congestion and loss costs for future use of the transmission system.

II. LOSS HEDGES

By definition, in accounting for losses the aggregate of the FTR awards cannot be perfectly balanced. The net difference is the energy that would be required to meet the loss requirements of the FTR dispatch. In principle, an individual FTR could be defined for inputs and outputs at any bus, in any combination or mix. In the aggregate, the simultaneous feasibility requirement is only that the net inputs must equal the net outputs plus losses. There is no need for individual FTRs to balance with losses, only for the aggregate to balance to preserve feasibility in the system.

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Although a great deal of flexibility could be allowed, it is convenient to separate the key issues by defining two generic types of FTRs. Any pattern of simultaneously feasible FTRs could be decomposed into balanced FTRs between locations, with the inputs and outputs equal, and unbalanced FTRs at a location, where the requirement is to inject or withdraw a given amount of energy. Consistent with current implementations, we define these as FTR obligations. Treatment of FTRs as options would be more complicated and would raise some additional issues not discussed in this paper.\(^2\)

Hence, a general FTR for 208 MW in at A and 200 MW out at B could be decomposed into a balanced FTR for 200 MW between A and B, and an unbalanced FTR of 8 MW in at A. Each FTR would operate in the same way, being a financial contract to pay the difference in value for the withdrawals minus the injections. In the case of the balanced FTR, this is \(p_B200 - p_A200 = (p_B - p_A)200\). In the case of the FTR at A this would be \(-p_A8\). In both cases, the FTR payment would exactly offset the payments in the spot market if the holder’s schedules matched its FTR holdings. The transmission charge to the participant for the movement of 200 MW in the spot market would be \((p_B - p_A)200\), and the payment to the participant for 8 MW of energy would be \(p_A8\). If the participant always injected 208 MW and withdrew 200 MW, there would be no net payments by the participant.

Furthermore, to the extent that the participant actually had the ability to inject 208 MW and withdraw 200 MW and submitted bids reflecting the incremental costs of these injections and withdrawals, any deviation from schedules matching the FTR holdings, with the accompanying net payments for FTRs and spot transactions, would be profitable compared to following a schedule matching the FTR holdings. Whether the hedge would reduce or increase the cost of the transaction, however, would depend on the price of the hedge, the amount of the unbalanced obligation, and the actual levels of energy prices and marginal losses, both in the hours in which the transactions flows and those in which it does not. Once purchased, however, either a balanced or unbalanced FTR would lock in the cost of losses associated with the hedged transaction, as well as the congestion costs.

The ISO could award both balanced and unbalanced FTRs in a single auction. Market participants could bid for any mix of balanced and unbalanced FTRs they chose. The auction problem would reduce to a form of economic dispatch with the side constraint that some bids for inputs and outputs must be individually balanced. There would be a market clearing price for each type of FTR, and the awards would be made at the market clearing price, not the amount bid. The mix of FTRs awarded would be determined in the auction to maximize the aggregate value as measured by the bids, subject to the constraint that the awards would be simultaneously feasible.

In the case of an unbalanced FTR at a location, the participant would be selling energy forward at the price determined in the FTR auction (or buying energy forward if unbalanced purchases

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2 While the concept of loss hedging FTR options would fit within the general simultaneous feasibility approach generally applied to financial rights, the extension to loss hedging options would raise additional complexities that we have not resolved. The concept of loss hedging flowgate rights would also raise a variety of implementation issues that we have not resolved.
were permitted in the auction). If the winner has a matching capability to produce the energy, or an equivalent contract for energy input, then there would be a perfect hedge. Of course, the holder of the unbalanced FTR forgoes the ability to sell the same energy in the future when spot prices might be higher, but that is the implication of the forward contract. The holder is exposed to opportunity cost risk, but not to cash flow risk as prices change.

In the case of the balanced FTR between locations, there appears to be less exposure to energy price changes, because the FTR obligation does not entail net injections. However, this is less true than it appears. If the holder of a balanced FTR also matches this with the ability to produce and use the energy, or with supply and demand contracts to the same effect, then there is no cash flow risk. However, there is the same opportunity cost risk on both ends of the transaction. Moreover, the price paid for the balanced FTR will reflect the cost in the auction of purchasing losses on a forward basis.

Moreover, regardless of the forward loss hedges, each market participant is exposed to market prices at the margin, reflecting the cost of marginal losses as well as congestion. The loss hedges therefore do not interfere with marginal incentives. The degree of risk and the willingness of market participants to accept the risk exposure associated with loss hedging FTRs is an open question. It may be that introduction of a losses component into congestion hedges creates no serious problems, or it may be that market participants would be unwilling to accept anything other than balanced FTRs. Hence, here we address some of the choices that would be available in defining different approaches to FTR definition and the associated auction.

Certain common implications of all of these auction models are discussed in Section F below.

A. Design A: The Pure Market Case

In this design, auction participants can bid for and offer balanced FTRs between locations, or unbalanced FTRs at a location. Unbalanced FTRs are individual injections of a specified quantity of MW or withdrawals of a specified quantity of MW at a particular bus. Such unbalanced FTRs to inject MW are the equivalent of selling energy forward, while those to withdraw MW would be the equivalent of buying energy forward. The balanced FTR would identify a quantity of energy that would be input at one location and withdrawn at another location.

The overall award of balanced and unbalanced FTRs would be constrained to satisfy a simultaneous feasibility criteria, that would include losses. In aggregate, the FTRs would therefore reflect injection quantities that are greater than the aggregate withdrawal quantities. Thus, the FTRs in total would reflect the actual losses for an equivalent level of loads and pattern of injections.

The choice of bids would be left to the market participants, and because the simultaneous feasibility criteria would account for both transmission congestion and losses, the awards would involve no financial commitments or obligations for the ISO under market equilibrium with the anticipated grid available.
Example

The operation of loss hedges can be illustrated for a simple system in which energy is injected at A and delivered at B. For this system:

\[
q = \text{Total deliveries at B}
\]

Total losses = \(.0005 q^2\)

Average losses per MW delivered = \(.0005 q\)

Marginal losses per MW delivered = \(.001 q\)

Thus, if the quantity of FTR hedges for deliveries at B that cleared in the auction were 100MW, then there would be corresponding obligations to deliver 105MW in the auction. If the actual loads at B hedged by these FTRs were 100MW at B and the price of energy at A were $30/MWh, then the ISO would be revenue neutral in the settlements process, collecting a net losses residual of $150 in the energy market, and making net payments of $150 to FTR holders.

If the actual loads at B hedged by the FTRs were more than 100MW, then the losses residual collected by the ISO would exceed the required payments to FTR holders and the ISO would have residual energy market revenues to credit against charges for sunk costs. It is noteworthy that if the actual loads at B hedged by the FTRs were less than 100MW, then the losses residual collected by the ISO would also exceed the required payments to FTR holders, because the obligation to deliver energy for losses implicit in the unbalanced FTRs would exceed the energy required to supply losses in the actual dispatch (see the cases for loads of 50 and 75MW in Table 2).

Figure 1

If the actual loads at B hedged by the FTRs were more than 100MW, then the losses residual collected by the ISO would exceed the required payments to FTR holders and the ISO would have residual energy market revenues to credit against charges for sunk costs. It is noteworthy that if the actual loads at B hedged by the FTRs were less than 100MW, then the losses residual collected by the ISO would also exceed the required payments to FTR holders, because the obligation to deliver energy for losses implicit in the unbalanced FTRs would exceed the energy required to supply losses in the actual dispatch (see the cases for loads of 50 and 75MW in Table 2).

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3 The net payment by energy market participants would be 100 * $33 – 105 * $30.
4 The net payment to FTR holders would be 100 * $33 – 105 * $30.
## Table 2
### Settlements

<table>
<thead>
<tr>
<th>Loss Example</th>
<th>FTR Awards</th>
<th>Actual Dispatch</th>
</tr>
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<tbody>
<tr>
<td>q</td>
<td>100</td>
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<tr>
<td>Total Loss</td>
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<td>Average Loss</td>
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<td>105</td>
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<tr>
<td>Withdrawals</td>
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<td>100</td>
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<td>30</td>
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<td>ISO Net Energy Market Revenues</td>
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<tr>
<td>ISO Net FTR Payments</td>
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</tr>
<tr>
<td>Net ISO</td>
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<td>84.375</td>
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</table>

<table>
<thead>
<tr>
<th>q</th>
<th>100</th>
<th>100</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>200</th>
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<tbody>
<tr>
<td>Total Loss</td>
<td>5</td>
<td>5</td>
<td>0.3125</td>
<td>1.25</td>
<td>2.8125</td>
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<tr>
<td>ISO Net Energy Market Revenues</td>
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<td>9.375</td>
<td>37.5</td>
<td>84.375</td>
<td>600</td>
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<tr>
<td>ISO Net FTR Payments</td>
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<tr>
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<td>37.5</td>
<td>9.375</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

It can also be seen in this example that if the actual loads are sufficiently low relative to the loads hedged by FTRs, the settlement of the loss hedging FTRs will entail payments to the ISO by the aggregate of the FTR holders.

It is important to understand that although the simultaneous feasibility criteria ensures that the aggregate injections exceed aggregate withdrawals by the amount of losses, absent other restrictions there is no necessary relationship between the injections and withdrawals cleared by an individual market participant in the auction. Thus, FTRs providing for injections of 50 at A and withdrawals of 75 at B, injections of 35 at A and withdrawals of 25 at B, and injections of 20 at A would satisfy the overall simultaneous feasibility criteria, as would many other combinations of injection and withdrawal awards.

### Advantages

1. Design A would be revenue adequate for the ISO if the awarded FTRs were simultaneously feasible on the full grid representation.

2. Market participants submitting bids to buy balanced FTRs would be able to specify the maximum price they would be willing to pay for the balanced hedge through their bids.
3. Market participants submitting offers to supply energy for losses through unbalanced FTRs could determine the minimum price they would be willing to accept for the energy through their offer prices.

4. Implementing this design may require only minimal changes to existing software for the feasibility test.

**Disadvantages**

1. Auction participants who seek to self-provide losses by submitting bids for both balanced and unbalanced FTRs may end up being awarded only unbalanced FTRs or only balanced FTRs.

In this circumstance, the market participants awarded the unbalanced FTRs would find themselves locked into a long-term contract to sell energy. There is a potential concern that market participants seeking hedges for transactions may see the risks associated with being awarded such an unbalanced energy sale as too great and offers to supply energy for unbalanced FTRs, therefore, might be sparse.

2. If interest in unbalanced FTRs is sparse, this will limit the amount of balanced FTRs that can be awarded in the auction while satisfying the simultaneous feasibility test.

It is therefore possible that the bundling of loss and congestion hedges could raise the cost of acquiring balanced hedges by more than the value to buyers of the associated loss hedge, leading to a reduction in the number of congestion hedges sold in the FTR auction, relative to the number of TCCs that would have been awarded in a TCC auction.

3. The ISO would be required to monitor the performance risk of entities selling unbalanced FTRs in the auction. To the extent that the energy sellers lack generating assets, unfavorable changes in energy prices could dramatically raise the cost of covering such forward sales, requiring the ISO to actively track performance risk.

Design A is a pure price driven approach to FTR definition and auction. Everything would be voluntary and we would let the bid and offer prices determine which FTRs were awarded and the degree of imbalance, as well as the prices of the hedges.

Given the concern with design A that the unbalanced FTR product might be rejected by the market participants, we can consider what participants might find more commercially appropriate within the limits of what is possible. As the argument goes, market participants really want balanced FTRs. However, they recognize that someone must take responsibility for the losses. This might be acceptable if this onerous responsibility were spread equitably among the various participants. In other words, by this argument, most everyone who obtained a balanced FTR would willing to accept a proportionate or fair amount of the necessary unbalanced FTRs.

If equitable sharing is important, the pure market auction would be complicated for auction participants who could not predict the final losses ratio before they bid. They could not
determine beforehand what the injection quantity would be for a given withdrawal quantity in an unbalanced FTR. The loss ratio, however, would affect the amount of their bid. Such risk in unbalanced FTRs may lead auction participants to prefer bidding only on balanced FTRs.

The search for an appropriate definition of and equitable allocation, therefore, is the motivation for the designs below. This includes mechanisms that allow market participants to express the willingness to accept unbalanced FTRs in terms of the award of balanced FTRs, and designs that impose some of this responsibility on the ISO.

**Design A-1: Conditional Bids**

A modified version of Design A, called Design A-1, “Conditional Bids,” would also allow customers to bid for balanced FTRs and unbalanced FTRs. This design, however, would allow the buyers to impose a constraint on the ratio of unbalanced to balanced FTRs awarded to them. In this way, customers are protected, if they so desire, from receiving only a long-term contract to sell energy without a corresponding balanced FTR, as is possible under Design A.

To effectuate such an outcome, the ISO would allow side constraints that set a range on the unbalanced FTR, perhaps expressed as a percentage of the withdrawal quantity of the balanced FTR. For example, the range may be set such that between 100 and 110 percent of the withdrawal quantity of the balanced FTR would be sold forward. If a customer bid for a balanced FTR with 100 MW of withdrawals, it could set 100 to 110 MW as the range of its agreeable injection quantity. If the customer received only a 50 MW FTR in the auction with this side constraint, its injection quantity would range from 50 to 55 MW.

This would be equivalent to a balanced FTR for up to 100 MW, and an unbalanced FTR for up to 10 percent of the award of the balanced FTR.

The additional restriction on the awards would reduce the set of feasible FTRs compared to design A, given the same bids. However, the ability to impose the constraint may expand the range of bids, allowing both lower FTR costs and higher total awards of FTRs in terms of both number and value.

This restriction would not change the aggregate properties of the FTRs as illustrated in the example for Design A.

**Advantages**

1. This design gives auction participants some certainty as to the composition of their FTRs before bidding into the auction.

The market participants would know that any FTRs received could not fall outside the limits set in their side constraint. They would not be awarded an unbalanced FTR to sell energy forward while being denied a corresponding balanced FTR, unless they had indicated a willingness to accept this outcome by bidding for unbalanced FTRs without the constraint.
2. Market participants submitting bids to buy balanced FTRs would be able to determine the
maximum price they would be willing to pay for their hedge through their bids.

3. This design helps to avoid the potential problem of unduly expensive transmission FTRs. By
making receipt of unbalanced FTRs conditional on the receipt of balanced FTRs, it negates
some of the potential risks inherent in offering to supply energy to support the losses
associated with a FTRs.

4. The side constraints for limits on the awards could be included as simple linear constraints
which would be a modest addition to the auction software.

5. By awarding unbalanced FTRs in conjunction with balanced FTRs and applying the
simultaneous feasibility criteria to both transmission constraints and losses, the ISO would
not have financial responsibility for injecting losses into the system.

Disadvantages

1. The participants could still face some uncertainty in the range of unbalanced awards.

2. The price of unbalanced offers might still be high, reflecting aversion to energy price risk,
but by submitting coupled bids for balanced and unbalanced FTRs, the net price of FTRs
hedging deliveries would not be too high.

3. There would still be performance risk associated with forward sales of unbalanced FTRs in
the forward auction. Because the forward FTR auction would still include such unbalanced
FTRs the ISO would still need to carefully track the financial position of entities holding
such forward positions. This might require more active tracking of forward positions than
most ISO currently undertake.

B. Design B: Predetermined Loss Ratio

Design B defines the unbalanced FTR requirement in advance. Through some process, the ISO
selects a pre-determined loss ratio and, in effect, imposes this ratio as a side constraint on the
award of balanced FTRs. The constraint would now be an equality specified by the ISO, instead
of a range or ratio specified by the bidder. The loss ratio, moreover, would be calculated before
the auction and it would not necessarily satisfy the simultaneous feasibility criteria for losses.
The simultaneous feasibility criteria for transmission constraints would still be enforced in the
auction.

The operation of this design can be illustrated with the same example used to illustrate design A.
If the ISO established a pre-determined loss ratio of 1.05 and 100 FTRs from A to B were
acquired in the auction, then the settlements and revenue adequacy would be exactly as shown in
Table 2.

Suppose, however, that 200 FTRs were acquired in the auction at the pre-determined 1.05 loss
ratio, as portrayed in Table 3. In this circumstance there would be a potential for ISO revenue
inadequacy, even if the actual load at B never exceeded 100MW. The revenue adequacy
problem would arise because the loss payout associated with the 200 FTRs exceeds the loss rentals collected on 100MW of load.

<table>
<thead>
<tr>
<th>Loss Example</th>
<th>FTR Awards</th>
<th>Actual Dispatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Total Loss</td>
<td>10</td>
<td>5</td>
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<tr>
<td>Average Loss</td>
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<td>Marginal Loss</td>
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<tr>
<td>Injections</td>
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<tr>
<td>Withdrawals</td>
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<tr>
<td>Price at B</td>
<td>33</td>
<td>30.75</td>
</tr>
<tr>
<td>Price at A</td>
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<tr>
<td>ISO Net Energy Market Revenues</td>
<td>150</td>
<td>9.375</td>
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<tr>
<td>ISO Net FTR Payments</td>
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<td>-150</td>
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</tr>
<tr>
<td>ISO Net FTR Payments</td>
</tr>
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<td>Net ISO</td>
</tr>
</tbody>
</table>

Competition in the FTR auction should in principle drive auction clearing prices to a level at which the ISO is revenue adequate on an expected value basis, but the absence of ISO reservation prices in the auction combined with risk aversion by market participants, could result in auction prices that leave that ISO revenue inadequate on an expected value basis. Moreover, even if the ISO were revenue adequate on an expected value basis, the ISO would not be hedged against its FTR obligations and unfavorable outcomes could result in substantial revenue shortfalls by the ISO.

**Advantages**

1. An advantage of this approach is that the ISO can determine and make known to customers the loss ratio before the auction.

   This approach provides bidders with certainty about the required injection quantity for a given output quantity. The administration of the auction itself would also be fairly straightforward.

**Disadvantages**

1. The preset loss ratio would never be exactly correct in the aggregate with respect to the loss contributions required for ISO revenue adequacy. Hence, there must be some method for handling the residual or making-up the difference. Under this design, the ISO would incur
the financial obligation to buy power to cover the actual losses, and would in effect take on the risk of forward energy contracts. Moreover, if market participants were risk averse, the ISO could be revenue inadequate on an expected value basis.

2. This design requires a method for pre-specifying the loss ratio associated with every possible combination of injection and withdrawal points. If a uniform loss ratio were used, the loss ratio might be relatively high between some points and low between others. This would be particularly problematic for counterflow transactions, which would reduce losses but would be required to inject more energy than withdrawn to cover the counterflow FTRs. This could magnify rather than reduce the energy price risk associated with the holding of FTRs and discourage market participants from offering counter-flow FTRs.

3. Alternatively, the pre-determined loss ratio could vary by location, perhaps based on a scaling down of the marginal losses in a high load power flow, with all loss ratios defined relative to the reference bus. This approach could potentially reduce energy price risk by reducing the likely imbalance between the losses reflected in the FTRs and those in the actual dispatch, particularly for counter-flow transactions. On the other hand, the pre-determined loss ratio could potentially expand the potential for ISO revenue inadequacy as a result of more opportunity for market participants to submit bids for large quantities of FTRs between individual points with inappropriate pre-determined loss ratios.

**Design B-1: Predetermined Losses and Market Balancing**

A close relative of this approach would be to combine the intent of Design B with the idea in Design A-1. Here the ISO would announce some predetermined loss ratio between locations, and allow those who choose to do so to bid with the equality constraint implied. However, the auction would also allow for market bids for unbalanced FTRs at locations, and the auction simultaneous feasibility criteria would be enforced for losses as well as transmission constraints, requiring purchases or sales of enough unbalanced FTRs that the ISO would be fully hedged. In this way, the ISO would avoid either taking on energy price risk or being exposed to revenue inadequacy on an expected value basis if market participants were risk averse. This design is effectively equivalent to design A-1 with a pre-specified ratio.

Note that this idea could be extended by allowing a variant of Design A-1 where the limits on the ratio of unbalanced to balanced FTRs would be accepted as an equality or an inequality. The equality would be equivalent to setting the FTRs of the form $1.x$ in at A, 1 out at B, at any price up to z\$. The market participants could decide for their own bids. The ISO’s announcement of the predetermined loss ratio, therefore, would be for information only.

**C. Design C: The ISO Provides for Losses**

Design C would go in the opposite direction of Design B-1. Here, the ISO would allow customers to bid for balanced FTRs only, where the injection quantity equals the withdrawal quantity. The ISO would be financially responsible to pay for the injection of all losses.
The operation of this design can be illustrated with the same example used to illustrate the preceding designs. If the ISO auctioned balanced FTRs and 100 FTRs from A to B were awarded in the auction, then the ISO settlements and energy market revenue adequacy would be as shown in Table 4. Because the ISO would be offering balanced FTRs that hedge losses, the ISO would necessarily be revenue inadequate in the energy market regardless of the actual level of load in relation to the awarded FTRs. In principle, the value of the balanced FTRs as loss hedges would be reflected in the equilibrium prices of those FTRs in the auction.

The ISO’s overall financial position under Design C would therefore depend on the relationship between the auction prices of the FTRs and cost of FTR settlements. Even more so than under design B, the absence of energy loss reservation prices in the FTR auction would make it more likely that the ISO would be exposed to a revenue risk.

<table>
<thead>
<tr>
<th>Loss Example</th>
<th>FTR Awards</th>
<th>Actual Dispatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Total Loss</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Average Loss</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Marginal Loss</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Injections</td>
<td>100</td>
<td>105</td>
</tr>
<tr>
<td>Withdrawals</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Price at B</td>
<td>33</td>
<td>30.75</td>
</tr>
<tr>
<td>Price at A</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>ISO Net Energy Market Revenues</td>
<td>150</td>
<td>9.375</td>
</tr>
<tr>
<td>ISO Net FTR Payments</td>
<td>-300</td>
<td>-75</td>
</tr>
<tr>
<td>Net ISO</td>
<td>-150</td>
<td>-65.625</td>
</tr>
</tbody>
</table>

Table 4
Settlements

It should be noted that the energy market revenue inadequacy would be exacerbated under this approach as the number of FTRs sold in the auction increases, even if the number of FTRs sold greatly exceeds actual loads. Thus, suppose that 200 FTRs balanced FTRs were acquired by auction participants as portrayed in Table 5. In this circumstance the ISO’s revenue inadequacy in the energy market could be greatly expanded, because the ISO would be selling more energy forward.
<table>
<thead>
<tr>
<th>Loss Example</th>
<th>FTR Awards</th>
<th>Actual Dispatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Total Loss</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Average Loss</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Marginal Loss</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Injections</td>
<td>200</td>
<td>105</td>
</tr>
<tr>
<td>Withdrawals</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Price at B</td>
<td>33</td>
<td>30.75</td>
</tr>
<tr>
<td>Price at A</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>ISO Net Energy Market Revenues</td>
<td>150</td>
<td>9.375</td>
</tr>
<tr>
<td>ISO Net FTR Payments</td>
<td>-600</td>
<td>-150</td>
</tr>
<tr>
<td>Net ISO</td>
<td>-450</td>
<td>-140.625</td>
</tr>
</tbody>
</table>

As before, competition in the FTR auction should in principle drive auction clearing prices to a level at which the ISO is revenue adequate on an expected value basis, but the absence of ISO loss energy reservations prices in the auction combined with risk aversion by market participants, could result in auction prices that leave the ISO revenue inadequate on an expected value basis. Moreover, even if the ISO were revenue adequate on an expected value basis, the ISO would not be hedged against its FTR obligations and higher than expected energy prices could result in substantial revenue shortfalls by the ISO in settling FTRs.

**Advantages**

1. Design C would provide a full hedge to customers.

2. There would be no need to determine a loss ratio in advance.

**Disadvantages**

1. This design would be risky for the ISO. In effect, the ISO would be selling a forward energy contract at the market clearing price in the auction, which might turn out to be either higher or lower than the price of energy in the day-ahead market.

2. Depending on participation in the FTR auction and the level of risk aversion with respect to forward energy prices, the ISO might not be revenue adequate even on an expected value basis.
3. It would be necessary to prohibit unbalanced FTRs as withdrawals. Otherwise the ISO would be selling energy forward for free.

**Design C-1: The ISO Provides and is Paid for Losses**

A slight variant of Design C would have the ISO providing the losses at a predetermined price. In effect, the ISO would step in as the bidder for unbalanced FTRs. In other words, rather than making up the losses at any positive market clearing price, the ISO would set a price for losses that would be its reservation price for selling losses forward. The effects of this design would be similar to Design C, except the risks for the ISO would be reduced as the price of losses rises, but higher losses prices could artificially discourage the purchase of congestion hedges.5

**Advantages**

1. Design C-1 would provide a full hedge to customers.

2. There would be no need to determine a loss ratio in advance.

3. The payment for losses in the auction would reduce the ISO’s exposure to low clearing prices in the auction.

**Disadvantages**

1. Although the ISO could be paid for this exposure in the auction, at a price that the ISO would have to specify in advance, the actual spot price would differ and the ISO would face the risk of the changing spot price of energy.

**D. Design D: Proportional Allocation Design**

Under this design, customers would submit balanced bids for withdrawal quantities and injection locations. In the feasibility test, all losses would be assumed to be made up at the reference bus. The total losses would then be allocated to the holders of balanced FTRs in proportion to each FTR’s marginal contribution to losses.6 The FTR holders would be responsible for the injection of their allocated losses at the reference bus.

**Example**

From a settlement perspective, this approach would operate like the example in Table 1. In effect, each market participant would bid for the amount of withdrawals at the sink it wanted to

---

5 The ISO’s risks would not be symmetric because if the price of losses were too high, market participants would offer counter-flow transactions in the auction that would eliminate losses. Thus, the ISO would have a losing position on an expected value basis.

6 The loss ratio would therefore vary by FTR and would be negative (injections less than withdrawals) for counter-flow FTRs.
hedge and would find out at the conclusion of the auction both how many withdrawals were hedged and how many MW it would be obligated to inject. Because the simultaneous feasibility criteria would be enforced for both transmission constraints and losses, the unbalanced FTRs would always generate sufficient energy market revenues to cover payments to FTR holders and the ISO would be revenue adequate.

Advantages

1. One advantage of this approach is that it would solve the problem that the price of unbalanced FTRs might be quite high, reflecting energy price risk, and thereby potentially raise the cost of congestion hedges. In this design, unbalanced FTRs, in and of themselves, are allocated to each FTR holder who is required to take them.

2. The ISO would not incur any energy market risk by offering loss hedges.

Disadvantages

1. One disadvantage is that, as in Design A, the loss ratio would not be known to auction participants before the auction. The amount of losses expected to be allocated to an FTR holder might materially impact the amount that the auction participant would be willing to bid for that FTR. The bidder, however, would not have the benefit of that knowledge when submitting its bids.

2. The FTR holder would retain energy price risk at the reference bus. The price of energy at the reference bus might be significantly different than the price of energy at the FTR holder’s injection point(s) due to congestion.

E. Design E: A Mix of Everything

Although it is not quite true that there is a dominant solution, these designs are not all mutually exclusive. Hence, we could think of a combination that might look attractive for the market participants and the ISO.

One approach would combine the elements of A-1, B-1, and C-1. It would not include the proportional sharing of losses, nor would it require the ISO to sell energy forward for free. However, it would allow just about everything that the market could want: balanced FTRs, unbalanced FTRs, side constraints that set ranges for unbalanced FTRs as a function of balanced FTR awards, side constraints that set equalities for unbalanced FTRs as a percentage of balanced FTR awards, and the ISO being the provider of last resort for losses (at a price). Market participants would be able to make their own choices in deciding what they want to include in their bids.
F. Further Comments

In addition to the considerations discussed above with respect to the specific formulation of loss hedges, there are a few considerations that are common to these approaches.

Options A, B and D in effect sell transmission service in the auction based on the uniform hourly average loss factor determined in the auction. If the auction awards hedge peak load, the average loss factor in the auction may exceed the actual marginal loss factor in some or many hours during the period covered by these hedges. To the extent that average losses are less than marginal losses in high load hours but exceed marginal losses in low load hours, the ownership of FTRs would be an economic benefit in some hours and an economic burden in other hours.

Thus, under options A, B and D, losses can in effect be hedged by awards in the FTR auction on an average basis, but this average basis is calculated for the solution to the auction which may be a peak hour in terms of losses. If the loss factor determined in the auction were higher than the average of the marginal losses, market participants would in effect be taxed with high losses charges for purchasing forward congestion hedges.

If, on the other hand, the loss factor determined in the auction were lower than the average of the marginal losses, market participants could in effect earn a discount on losses charges by buying losses forward. If the forward auction market were fully efficient, these values would be reflected in forward auction prices. As noted above, either market inefficiency or risk aversion may result in forward auction prices that may not reflect the expected value of the FTR payments.

In all cases, if the participant's deliveries exactly match the FTR, then there is a complete hedge for that participant. In effect, the FTR is a combined forward sale and purchase of energy at different locations. Any allocation of the forward sale between losses and balancing energy is an accounting convention that has meaning in the aggregate but not at the individual level.

The valuation of both balanced and unbalanced FTRs may be complex requiring that auction participants not only value future congestion but also assess forward energy prices, marginal losses rates in the day-ahead market over the term of the auction and the relationship between the two. This complexity is exacerbated by the fact that FTRs sell transmission based on average losses, versus marginal losses in real-time and the fact that FTRs are an economic burden if the loss ratio embodied in the FTR exceeds the real-time marginal loss ratio.

7 These costs would, of course, be reflected in the bid prices for the FTRs.

8 Unlike a TCC, an FTR will result in positive or negative cash flows in every hour, even if there is no congestion and no transactions are scheduled. In any hour in which the marginal losses between the point of injection and withdrawal of the FTR exceed the ratio of the unbalanced FTR, the FTR would provide a positive cash flow if no transaction is scheduled, while in any hour in which the marginal losses between the point of injection and withdrawal of the FTR are less than the ratio of the unbalanced FTR, the FTR would require net payments. The overall cash flows implications would depend on the losses in the auction (which is in effect solved for a single load level) and the variation in the level of losses from hour to hour in the day-ahead market (assuming that FTRs settle in the day-ahead market).
Given some of the possible limitations of loss hedging FTRs, it would be attractive to offer a design in which the auction offered both FTRs with losses and TCCs for congestion only with the mix awarded determined by the bids of market participants.

There are, however, fundamental elements of the simultaneous feasibility test and the modeling of losses that make such an approach problematic. Congestion modeling and the simultaneous feasibility test for transmission constraints must be applied to the flows associated with all injections and withdrawals simultaneously. While the marginal losses associated with each transaction are well defined, there is no unique way of assigning average losses to specific transactions. This is especially true if the transactions are not balanced injections and withdrawals. In applying the simultaneous feasibility test one can either apply the simultaneous feasibility test to all injections and withdrawals including losses, or assume that losses are made up at the reference bus or some other location. It is not readily apparent how to apply the simultaneous feasibility test assuming that losses are free for some transactions but not for others.

A combined auction for loss hedging FTRs and congestion-only TCCs might be implemented by accepting revenue adequacy risk. If losses were not modeled at all for the flows associated with congestion-only rights, it might be possible to award both FTRs and TCCs, but there would be a risk that the awards would not be feasible with losses included and thus a risk of ISO revenue inadequacy. In addition, the loss factors calculated based on loss hedging FTRs alone would be low relative to the losses attributable to these schedules in combination with the flows attributable to transactions with congestion-only hedges. This would not lead to revenue inadequacy and would be reflected in FTR auction prices.

Finally, the close connection of the form of the FTR auction and the real-time dispatch raises a question about the incentive effects for any other design than the pure market case in Design A. The principal incentives for efficient operation to include the effect of marginal losses would not be altered because the restrictions on the form of the FTRs do not apply to the real dispatch. And long-term investment incentives would not be changed for risk neutral investors. To the extent that the restrictions affect the marginal awards, therefore, the pathway would be through changes in the bids to something other than the expected value of the FTRs resulting from risk-averse bidders. This could have some impact on the value of hedges and therefore the value of transmission investments. We have not attempted to estimate these effects, but think that they should be small.

### III. PRICING AND FTR AUCTIONS WITH LOSS BIDDING CONSTRAINTS

The various compromise proposals to share the risks of providing loss energy forward could complicate the interpretation of the FTR auction prices. Here we consider an example that combines features as described above and work out the details in terms of the FTR with losses auction prices.

Due to the risk exposure for selling losses forward, relying completely on energy bids to provide the losses associated with balanced FTRs might be unacceptable to market participants. Here we consider a design to assign losses to otherwise balanced FTRs with different presumed loss factors, and then to scale these loss factors to find an optimal balance in the system. For sake of
discussion, suppose that the loss factors were determined from some pre-selected dispatch and set equal to the marginal loss at each location. Then the scaling would be equivalent to attributing the obligation to supply losses in (relative) proportion to the contribution to marginal losses. In other words, every otherwise balanced FTR from A to B would have 1 MW withdrawn at B and \(1 + xL_A\) injected at A. The factor \(L_A\) would be determined from the pre-selected dispatch. In general, the factor \(L_A\) would be the difference in the marginal loss factors at A and B as found in the pre-selected dispatch. The scaling factor \(x\), common for all FTRs, would be determined as part of the solution of the FTR auction. Further, we limit the maximum scaling factor \(x\) to ensure that there is some upper bound on the exposure to losses that come with balanced FTRs. The remainder of any losses sold forward in the FTR auction would come from unbalanced energy bids which would also be allowed as part of the auction. However, the ISO takes no role in providing losses or selling energy forward. Hence, a feasible sale of FTRs would preserve revenue adequacy.

In order to focus on the issues here, we use a simplified model to work with differentiable functions so that we can avoid the complications of dealing with endpoints in bid steps. Hence, consider the following representation:

- \(t\) a vector of balanced FTRs,
- \(g\) a vector of unbalanced energy inputs,
- \(B(t)\) a differentiable benefit function for the FTR bids,
- \(C(g)\) a differentiable offer function for the energy inputs,
- \(D\) a matrix with the "to" load location (\(+1\)) and the "from" generation location (\(-1\)),
- \(L\) a matrix of ex ante loss factors for FTRs expressed at the "from" locations,
- \(x\) the allocated loss scaling factor,
- \(A\) the line-node incidence matrix,
- \(\Omega\) the diagonal matrix of line transfer factors,
- \(y\) the net loads at each location resulting from the FTR allocation including losses,
- \(z\) the vector of line flows in the DC-load approximation,
- \(R(z)\) the vector of resistive losses at each location at either end of the lines,
- \(\theta\) the vector of angles at each location,
- \(f(\theta)\) a restriction on one of the angles (e.g., swing bus equal to zero).
The FTR bid-benefit function and energy bid function correspond to the usual sum of the bids times the awards. The gradients of these functions are the marginal benefits or costs that define the market clearing prices for the awards.

With these definitions, the net load at each location consists of the effect of the balanced part of the FTR (Dt) plus the required unbalanced contribution at the from location (Lt) plus the true unbalanced sales of energy forward (g), or

\[ y = [D - xL]t - g. \]

Suppose for this example we limit the proportional scaling of marginal losses to 100%, or

\[ x \leq 1. \]

With these definitions, the FTR auction model employs the usual DC-load formulation assumptions to calculate average line flows but includes one-half the associated line loss as additional load at each end of the line. Hence, we have the resulting FTR auction model as:

\[
\begin{align*}
\text{Max} & \quad B(t) - C(g) \\
\text{s.t.} & \quad y - [D - xL]t + g = 0, \\
& \quad y = A'z - R(z), \\
& \quad z = \Omega A\theta, \\
& \quad z \leq z_{\text{max}}, \\
& \quad x \leq 1, \\
& \quad f(\theta) = 0.
\end{align*}
\]

The corresponding Lagrangian could be defined as:

\[
B(t) - C(g) + \lambda' \left[ y - [D - xL]t + g \right] - \omega' \left[ y - A'z + R(z) \right] \\
- \rho' \left[ z - \Omega A\theta \right] - \phi' \left[ z - z_{\text{max}} \right] - \mu [x - 1] - \eta \left[ f(\theta) \right].
\]

With this we have the first order conditions:
(1) \( \nabla L_x = \nabla B - [D - xL]' \lambda = 0, \)
(2) \( \nabla L_y = \lambda - \omega = 0, \)
(3) \( \nabla L_z = -\nabla C + \lambda = 0, \)
(4) \( \nabla L_t = [A - \nabla R] \omega - \rho - \phi = 0, \)
(5) \( \nabla L_s = [Lt]' \lambda - \mu = 0, \)
(6) \( \nabla L_b = A\Omega \rho - \nabla f' \eta = 0. \)

From these we can interpret the prices. The FTR auction with losses produces a price for net loads at every location. This has an interpretation as the implied forward value of congestion and loss energy needed to support the FTRs. If the solution includes unbalanced forward sales for loss energy other than that required of all balanced FTRs, this locational price has an interpretation as the forward price of energy. The usual locational prices are by definition the marginal costs of the energy bids, which by (3) appear as \( p = \nabla C = \lambda \). The balanced FTR prices are equal by definition to \( \nabla B \), the marginal benefits, and according to (1) these are in turn equal to the difference in the locational prices adjusted for the contribution to allocated losses.

If the scaling factor constraint is not binding, then \( \mu = 0 \), and we see from (5) that the sum of the locational prices times the assigned losses is zero. In the case of all positive loss factors (where all balanced FTRs have positive losses assigned at the "from" location) and non-negative locational prices, this would imply that the locational prices at the "from" locations are all are zero. Therefore, all the FTRs bid are awarded at the clearing price as determined solely by the effects of congestion. In essence, when the scaling constraint does not bind, losses are free.

When the scaling constraint is binding, then \( \mu > 0 \), and there is an opportunity cost for losses. In this case the locational prices in \( \lambda \) will reflect the marginal cost of losses. Furthermore, the effect will include the second and third order impacts, because flows induced by losses create more losses, and so on. This impact can be complicated, but a simple illustration below shows the calculation in a special case.

Fixing the value of the scaling factor at the optimal level, and then solving the reduced problem with that value for the scaling factor clearly would not affect the optimal solution. Furthermore, we can see from (1) and (3) that fixing the scaling factor at the optimal value would not affect the locational prices or the FTR prices at the loss adjusted difference in the locational prices.

The congestion effects enter in the same way as they would for a model with no losses. There is an interaction with the losses, but the price impacts on the FTR awards follow the same intuition.

A series of simple one-line examples illustrates these conclusions. Suppose we have a line from A to B. There are two FTR bids with implied loads at B of \( d_1 \) and \( d_2 \) and associated generation at A of \( g_1 \) and \( g_2 \). The scaling factor is \( x \) and the ex ante loss factor at A is \( l \). The required connection for the assignment of losses to the otherwise balanced FTR is \( g_i = d_i (1 + x l) \). There is a third generator that can supply energy at A, denoted by \( g_3 \).
To make things simple, the bid-benefit functions are represented as $b_i \sqrt{t_i}$ and the energy cost function is just $c g_3$. There is a maximum quantity for each bid, $t_{max_i}$. There is also a maximum line flow of $z_{max}$. The loss factor is $r$ and losses $r z^2$ appear as loads split equally between A and B.

To begin, choose $b_1=10$, $b_2=50$, $c=15$, $t_{max_1}=300$, $t_{max_2}=300$, $z_{max}=350$, $l=0.2$, $r=0.0005$.

Hence, the ISO solves for the best allocation of FTRs and purchases of unbalanced energy subject to the constraint that the scaling factor is never greater than 1.0 and, therefore, that the maximum assigned loss contribution is 20% at A for the balanced FTRs. With these data, examine five cases with results:
In Case 1, there is congestion on the line but the scaling constraint is not binding. Hence losses are essentially free, and the locational price at A is zero. The line shadow price is 0.94, and the locational price at B is 1.14. Were it not for losses, the price at B would be 0.94, reflecting the marginal cost of line flow congestion. But since flow induces losses, each unit of delivered energy induces more than a unit of flow on the line, which raises the congestion cost even though the losses are free. The price of the FTR award is 1.14, equal to $p_B - p_A(1+xl) = 1.14 - 0$. The bid awards are $t_1=19.38$ and $t_2=300$ with load (injection) obligations of $[-23.09, 19.38]$ and $[-357.53, 300]$ for bidders one and two, respectively.

In Case 2 the line capacity is increased to 360, which raises the flow and losses and almost makes the scaling constraint binding, but not quite. Hence the prices are similar to those in Case 1 with the same effect of free losses at the margin, congestion, and loss induced congestion. The marginal congestion cost is lower, which results in a lower market-clearing price for the FTRs. The price of the FTR awards is 0.95, equal to $p_B - p_A(1+xl) = 0.95 - 0$.

The third example increases the line capacity to 370, and this just removes the flow limit. It also allows for increased FTR awards to the point where the scaling constraint is binding. Hence,
losses are no longer free at the margin. The total value of losses is now 244.56, which can be obtained from the shadow price on the scaling constraint or the price at A times the total losses, $3.70 \times 66.02 = 244.56$. There is no congestion, so the locational prices must represent only the impact on marginal losses. The value of marginal energy at A is 3.70, which is less than the bid price for unbalanced energy at 15. Hence, there is no purchase of the bid-in unbalanced energy; we are on a vertical portion of the unbalanced energy supply curve.

The effect on losses at each location reflects the direct and indirect impact on losses, losses on losses, losses on these losses, ad infinitum. This is easy to verify in this case at location A because we don't have to untangle the effect on counterflow. Consider the use of an increment of one unit of energy generated at location A. The FTR award for bidder two is at a maximum, but the increment of generation at A would allow for an increased award to bidder one. In this case, an increase in FTR award to bidder one creates a net added loss of $ML - l + ML \left[ \frac{0.5ML}{1 - 0.5ML} \right]$, where ML is the marginal loss factor on the line. The first term is the direct impact on losses, the second term is the contribution providing allocated losses, and the third term is the value for the infinite series of losses on losses recognizing that half of these must flow through the transmission line. The marginal value of an increment on the award for the first bidder is $0.5b_l / \sqrt{t_l} = 0.5 \times 10 / \sqrt{30.58} = 0.90$. It must be that the locational price is the benefit of injecting one unit of energy to allow for more losses that can be used to increase the bid award. Hence, the marginal value of an increment on the award for the first bidder is $0.90 = \frac{1}{0.36 - 0.2 + 0.36} \left[ \frac{0.5 \times 0.36}{1 - 0.5 \times 0.36} \right] = 3.70$.

The story is more complicated for location B, due to the effects of counterflow if an increment of energy is injected at location B. But the price at the locational tells us the net value of that injection would be 5.34. Further, we see that the price of the FTR awards is the same marginal benefit of 0.90, equal to $p_B - p_A(l + x_l) = 5.34 - 3.70 \times (1 + 0.2) = 0.90$. Since the price of energy at location A is below the bid price from $g_3$, there is no energy bid input and all the losses are assigned to the balanced FTRs.

The fourth case is similar to the third case, except here the limits on the line flows and on both FTR bids are not constraining. Hence, there will be a tradeoff between the awards to the two FTR bidders. Only the scaling constraint is binding, and prices are not high enough to purchase unbalanced energy from the $g_3$ bidder.

The marginal value of an increment on the award for the first bidder is $0.5b_l / \sqrt{t_l} = 0.5 \times 10 / \sqrt{12.72} = 1.40$. Since we also have an interior solution for the second bidder, the same marginal value applies with $0.5b_2 / \sqrt{t_2} = 0.5 \times 50 / \sqrt{317.86} = 1.40$. It must be that the locational price is the benefit of injecting one unit of energy to allow for more losses that can be used to increase either the bid award at the margin. Hence, the marginal value of an increment on the award for the first bidder is $1.40 = \frac{1}{0.36 - 0.2 + 0.36} \left[ \frac{0.5 \times 0.36}{1 - 0.5 \times 0.36} \right] = 5.74$.  


However, because the curvature of the bid functions is not the same and both awards are unconstrained by the bid maximum, increasing the availability of loss energy at A would change the mix of the FTR awards.

Again, the story is more complicated for location B, due to the effects of counterflow if an increment of energy is injected at location B. The locational prices at A and B of 5.74 and 8.29, respectively, capture the total impact on losses. The price of the FTR awards is the marginal benefit of 1.40, equal to \( p_B - p_A(1+x_l) = 8.29 - 5.74(1+0.2) = 1.40 \). The locational price at A still does not support any award to \( g_3 \).

Finally, the last case reduces the loss allocation parameter \( l \) to reduce the imputed supply of free losses. This is enough to make it attractive to accept some of the unbalanced energy bid. Given the assumed constant bid price, this necessarily means that the price at location A must be 15. The input from \( g_3 \) is 1.08. Since the balanced FTR bids are no longer providing free losses at the margin, the size of the award goes down and the price goes up. However, the same pricing interpretation applies. The price of the FTR awards is the marginal benefit of 1.85, equal to \( p_B - p_A(1+x_l) = 18.35 - 15.00(1+0.1) = 1.85 \).

In the last case, we have ignored how bidders would change their bids if we assigned a different allocation factor \( l \) for the assignment of losses. A lower factor should reduce the risk and increase the value of the FTR, thereby raising the bid and mitigating or eliminating the reduction in the size of the award.

In summary, the prices seem to have natural and sensible interpretations. The change in locational prices could be abrupt as the scaling constraint binds, which reflects the movement from implicitly free losses to expensive losses at the margin. But the value of the FTRs, which is the difference in the locational prices, should not be as dramatically affected.

IV. EXTENSIONS OF FTRs TO A NETWORK

The analysis of FTRs with losses becomes less transparent in the case of a network with loop flows. Here we approach the problem by illustrating the auction idea and the application of the rights in the simpler case of a loss-less DC-Load framework and transmission congestion contracts (TCCs). With this as background, we take the same examples to introduce losses and FTRs that include loss payments.

A. TCCs in a Network

Consider first the case of TCCs for congestion only in a pure DC-Load model where we ignore losses. A three bus example network illustrates the elements of a concurrent auction of TCCs. Here the three buses are connected by three identical lines. There is only one constraint which limits the flow of power on the line between buses 1 and 3 to a maximum of 600 MW.

The various actors in the market have identified two types of TCCs that would have value, from bus 1 to bus 3 and from bus 2 to bus 3. The assumption is that there are many bidders with different maximum evaluations of the amount they would pay for the respective TCCs. These evaluations become bids in the concurrent auction. The collection of all the bids appears as a bid
curve for each type of TCC. For simplicity, the bids are assumed to be the same for both types of TCCs, but any bids would be allowed.

The three bus example is the simplest case that includes the effects of loop flow and network interactions. However, there is no necessary connection between the definition of the TCCs and the ownership of the lines between buses. The example could be expanded by adding other lines and buses. The TCCs would still be defined from one bus and to another bus, without any requirement that there be a direct link between the two buses.

Here the highest bid is at $60 per MW, and the bid prices decline to zero at the level of 1200 MW. The objective is to find the combination of awards that maximizes the area under the bid curves, which is the sum of the value of the successful bids. In principle, all the transmission capacity could be awarded to TCCs from either source. If all the TCCs came from bus 1, then the line limit would constrain and the maximum award possible would be 900 MW with a price of $15. The value would be the area under the bid curve, $15(900) + 45(900)/2 = $33,750. If all 1200 MW of bids for TCCs from bus 2 were accepted, the price for these would be zero and there would be excess capacity. The value for these awards would be the area under the bid curve, $0(1200) + 60(1200)/2 = $36,000. Neither extreme would provide the highest valued use of the transmission grid. However, the concurrent auction formulation takes into consideration
all the bids and the interactions in the network to find the maximum value award and the associated market clearing prices for the TCCs.

The result of the concurrent auction in the figure awards 480 MW for TCCs from bus 1 to bus 3 and 840 MW of TCCs from bus 2 to bus 3. The market clearing prices for the respective TCCs are $36 and $18. The value for these awards would be the area under the bid curves, \(36(480) + 24(480)/2 + 18(840) + 42(840)/2 = 55,800\). In this simple case, the ratio of the prices is just the inverse of the tradeoff between the two types of TCCs. In order to maintain feasibility, given the constraint on the line from 1 to 3, each MW from bus 1 to bus 3 displaces 2 MW from bus 2 to bus 3.

The existence of the TCCs tells us nothing about the total price of power that might be arranged under contract or that would be determined in the spot market. Apparently the winning TCC bidders believe the average differences in the prices between buses will be at least as large as the concurrent auction award prices. However, with these TCCs in place, the holders would have a perfect hedge for the spot price of transmission congestion. If the spot price of transmission is high, then the TCC congestion payment would compensate the holder for the spot price. However, the spot price of transmission could be higher or lower than the cost of the TCC.

For example, suppose that the actual dispatch conditions conform to those in where economic dispatch leads to much of the load at bus 3 being supplied by generation at bus 3 with a cost of
$26. Here the generation at bus 2, where the opportunity cost is $23, is too expensive to run, and the remaining generation at bus 1 is supplied at a price of $20.

Everyone using the transmission grid is paying at these short-term prices. Those buying and selling through the ISO employ the appropriate locational prices. Those transmitting power from one bus to another pay the spot price of transmission equal to the difference in the locational prices. The total net usage charge collected by the ISO is $3000(26) - 2100(26) - 900(20) = $5,400. The difference in congestion charges between bus 1 and bus 3 is $6, requiring a payment of 480(6) to the holders of TCCs from bus 1 to bus 3. The difference in congestion charge from bus 2 to bus 3 is $3, requiring a payment of 840(3) to the holders of TCCs from bus 2 to bus 3. The total payment to TCC holders is 480(6) + 840(3) = $5,400. Hence, as shown in the accompanying table, the total congestion payments for use of the grid are large enough to pay the TCC obligations, even though the dispatch and economic conditions have changed.

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Price</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>900</td>
<td>20</td>
<td>($18,000)</td>
</tr>
<tr>
<td>Bus 2</td>
<td>0</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Bus 3</td>
<td>2100</td>
<td>26</td>
<td>($54,600)</td>
</tr>
<tr>
<td>Bus 3</td>
<td>-3000</td>
<td>26</td>
<td>$78,000</td>
</tr>
<tr>
<td>TCC 1-3</td>
<td>480</td>
<td>6</td>
<td>($2,880)</td>
</tr>
<tr>
<td>TCC 2-3</td>
<td>840</td>
<td>3</td>
<td>($2,520)</td>
</tr>
<tr>
<td>Net Total</td>
<td></td>
<td></td>
<td>$0</td>
</tr>
</tbody>
</table>

For the holders of TCCs from bus 1 to bus 3, the effective cost of power delivered at bus 3 is $20, the same as the price at bus 1; for the holders of TCCs from bus 2 to bus 3, the effective cost of power delivered to bus 3 is $23 cents, the same as the price at bus 2. The holders of the TCCs have a perfect hedge for the spot price of transmission congestion. Of course, in this case the holders paid more for the TCCs than they were worth for this particular dispatch. With economic conditions changing, presumably there would be other periods when congestion could be greater than the price paid for the TCC. Whether the average congestion costs would justify the price of the long-term protection is uncertain, and would be a business risk in the competitive market. However, the TCC holders would be assured of getting what they paid for: long-term protection from the uncertain congestion costs of transmission, no matter what the changing pattern of the loads.
B. FTRs in a Network

We can now compare this example with the changes that would apply to incorporate losses and provide Financial Transmission Rights (FTRs) that would include a hedge for the price of losses. In this case, we would modify the original auction and the award of FTRs to incorporate the effect of losses. Using the same line parameters and the same limit on the average flow through the line connecting buses 1 and 3, we apply a version of a DC-Load model with losses included at the source and destination bus for each line.\(^9\)

We need to change the FTRs that are awarded to reflect the effect of losses. The FTRs are treated as obligations. For sake of illustration, we assume that there are two types of FTRs. The FTRs between buses are balanced with the same inputs and outputs. The remaining FTRs provide for energy to meet losses, and are obligations to provide a certain amount of power at a bus.

For simplicity, we suppress the details of the auction and suggest a set of feasible FTRs that would be similar to the TCCs above. As shown in the figure, with the same line capacity, recognition of transmission losses reduces the capacity for FTRs. Here we assume that the FTRs for 840 MW between buses 2 and 3 can be allocated. However, with these FTRs, the maximum that remains available between buses 1 and 3 is 370 MW.

\(^9\) The per unit reactance is 0.002 and the resistance is 0.0002. Line losses are a quadratic function of the flows (with flow \(z\), line loss \(= 0.000202z^2\)) with half the losses on the line included as apparent loads in at the source and sink buses, respectively, in the DC-Load formulation.
In addition, the balanced FTRs with equal inputs and outputs must be combined with FTRs at bus 1 to provide the 172 MW of losses that would accompany these flows. The combined set of FTRs, between and at buses, is feasible and yields the line flows shown in the figure.

As before, the normal operation of the system need not match the pattern of the FTRs. Suppose, for example, that the actual dispatch follows the pattern in the accompanying figure. Here there is a total load of 3000 MW, as before with the TCCs. However, now the generation input of 3109 MW includes 109 MW of losses.
Constraints with Out-of-Merit Costs

(WITH LOSSES, IDENTICAL LINKS, CONSTRAINT ON LINE 1-3)

Bus 2 generation cost goes above $23
Load and flows change; constraint binds
Price includes losses and congestion charge

\[ P = 20 + 2.67 + 0.23 = \$22.9 \]

The pricing assumptions are designed to match closely the aggregate prices in the example without losses. In this case, the marginal losses account for most of the difference in prices across locations. The figure shows the decomposition of prices into marginal generation at the reference bus price, marginal losses and marginal congestion. Hence, the marginal losses at bus 2 yield a loss price component of $2.67. The balance of $0.23 is the congestion component. The corresponding decomposition at bus 3 has a generation reference price of $20, plus a marginal loss price of $5.5 and a congestion price of $0.5 for a total of $26.

In this case, the payments under the FTRs do not exhaust the transmission rents, and there is a net residual of $1,513. This reflects the combined effect of the difference between the marginal and average cost of losses and the higher relative losses in the case of the actual dispatch compared to the FTRs system flows. The detailed summary of the net payments appears in the accompanying table.
<table>
<thead>
<tr>
<th>System Operator Revenues</th>
<th>Quantity</th>
<th>Price</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>955</td>
<td>20</td>
<td>($19,100)</td>
</tr>
<tr>
<td>Bus 2</td>
<td>0</td>
<td>22.9</td>
<td>$0</td>
</tr>
<tr>
<td>Bus 3</td>
<td>2154</td>
<td>26</td>
<td>($56,004)</td>
</tr>
<tr>
<td>Bus 3</td>
<td>-3000</td>
<td>26</td>
<td>$78,000</td>
</tr>
<tr>
<td>FTR 1-3</td>
<td>370</td>
<td>6</td>
<td>($2,220)</td>
</tr>
<tr>
<td>FTR 2-3</td>
<td>840</td>
<td>3.1</td>
<td>($2,604)</td>
</tr>
<tr>
<td>FTR at 1</td>
<td>172</td>
<td>20</td>
<td>$3,443</td>
</tr>
<tr>
<td>Net Total</td>
<td></td>
<td></td>
<td>$1,513</td>
</tr>
</tbody>
</table>

For purposes of further comparison, we could take the reverse case with a set of feasible FTRs that closely match a dispatch with no generation at bus 2, and then have a pattern of flows that include a heavy generation at bus 2. Using this as the allocated set of FTRs, we can illustrate the impact on revenues when the actual dispatch losses turn out to be greater than the losses accounted for in the FTRs.

The corresponding set of feasible FTRs appear in the following figure. The FTRs between buses 1 and 3 amount to 846 MW. There are no FTRs between buses 2 and 3. The final requirement is for the FTRs at bus 1 in the amount of 109 MW to meet the requirements for losses.
Suppose that the actual dispatch now follows the pattern in the next figure. Here we have the case where the cost of generation at bus 3 has risen, and 840 MW of available generation at bus 2 is now competitive at the opportunity cost price of $29. The remaining transported power comes from bus 1, up to the limits of transmission capacity. The generation at bus 1 is 542 MW. Total generation in this case is 3172 MW, including losses of 172 MW.

The decomposition of prices reveals the increased importance of congestion costs. At bus 3, for example, the price consists of the reference bus generation price of $20, plus $5.5 for the cost of marginal losses, and $24.5 in congestion rents for a total of $50. At bus 2, the marginal losses contribution is negative, -$0.7, reflecting the fact power is flowing from bus 2 to bus 1 along the line and increased load at bus 2 would reduce total losses. The congestion cost component of $9.7 raises the total opportunity cost price at bus 2 to $29.
The FTRs guarantee the same net effect for those who hold them. Hence, for the holders of the 846 MW of FTRs between buses 1 and 3, the average price of the delivered energy for the 846 MW is $20, the same price as at bus 1.

Even though the losses in this dispatch are greater than those for the set of FTRs, it is still true that the net payments residual to the ISO is positive. The corresponding table summarizing the payments follows.
System Operator Revenues

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>542</td>
<td>20</td>
</tr>
<tr>
<td>Bus 2</td>
<td>840</td>
<td>29</td>
</tr>
<tr>
<td>Bus 3</td>
<td>1790</td>
<td>50</td>
</tr>
<tr>
<td>Bus 3</td>
<td>-3000</td>
<td>50</td>
</tr>
<tr>
<td>FTR 1-3</td>
<td>846</td>
<td>30</td>
</tr>
<tr>
<td>FTR 2-3</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>FTR at 1</td>
<td>109</td>
<td>20</td>
</tr>
<tr>
<td>Net Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The net residual is small, but positive. In general, in the model including losses, with the dispatch different than the allocation of FTRs, the net residual will be positive.

C. FTR Auction in a Network

The previous FTR examples assume the results of the FTR allocation in order to allow for simple comparison with the case of the TCCs. However, if we took the same bids as in the TCC case, but applied them to the more general auction of FTRs, the awards would change in more than assumed in the simple illustration.

This section provides another set of FTRs and the associated evaluation of the revenue adequacy of the results. In this case, the FTRs are found by actually solving for the accompanying optimal FTR awards given the accompanying bids. Here the corresponding auction conditions look like those for the TCC auction above. The principle difference in the bids is the requirement to identify the unbalanced FTRs. Here, we assume that the unbalanced FTRs are offered at bus 1 at a price of $40.
Concurrent Auction of Financial Transmission Rights

(WITH LOSSES, IDENTICAL LINKS, CONSTRAINT ON LINE 1-3)

The solution of the FTR auction appears in the accompanying figure. Given the same bids as before, the balance shifts towards the FTRs from bus 1 to bus 3, and the cost of FTRs from bus 2 to bus 3 goes up compared to the TCC example.

The pattern of payments under a different dispatch can be examined as before. In the case with the actual dispatch constrained at higher prices, the corresponding summary of the payments appears in the accompanying table.

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10 The solution obtained using the DC load model with losses, adapted to have balanced FTRs for some of the bidder. (Model DCFTR 3.07)
Here the net payments are small, only $75, up to rounding error. However, as always with any simultaneously feasible allocation of FTRs, the system is revenue adequate at the market clearing spot prices.

V. CONCLUSIONS

If a form of loss hedges were to be implemented by the ISO, some variation on Design A would be the preferred approach. Designs B and C do not assure ISO revenue adequacy and under either design there is a potential for substantial ISO revenue shortfalls. Under Design C, in particular, higher-than-expected energy prices would render the ISO insolvent, while lower-than-expected energy prices could lead to large ISO surpluses. Design D avoids these problems but is more restrictive than Design A.

Designs A, B and D entail treating the supply of losses as an obligation regardless of the level of injections. They lock in the cost of losses in dollar terms given this obligation to supply losses, but it needs to be recognized that the cost of the obligation to supply losses is uncertain and does not depend on the actual use of the system by the FTR holder.