ELECTRICITY MARKET DESIGN
Optimization and Market Equilibrium

William W. Hogan
Mossavar-Rahmani Center for Business and Government
John F. Kennedy School of Government
Harvard University
Cambridge, Massachusetts 02138

Workshop on Optimization and Equilibrium in Energy Economics

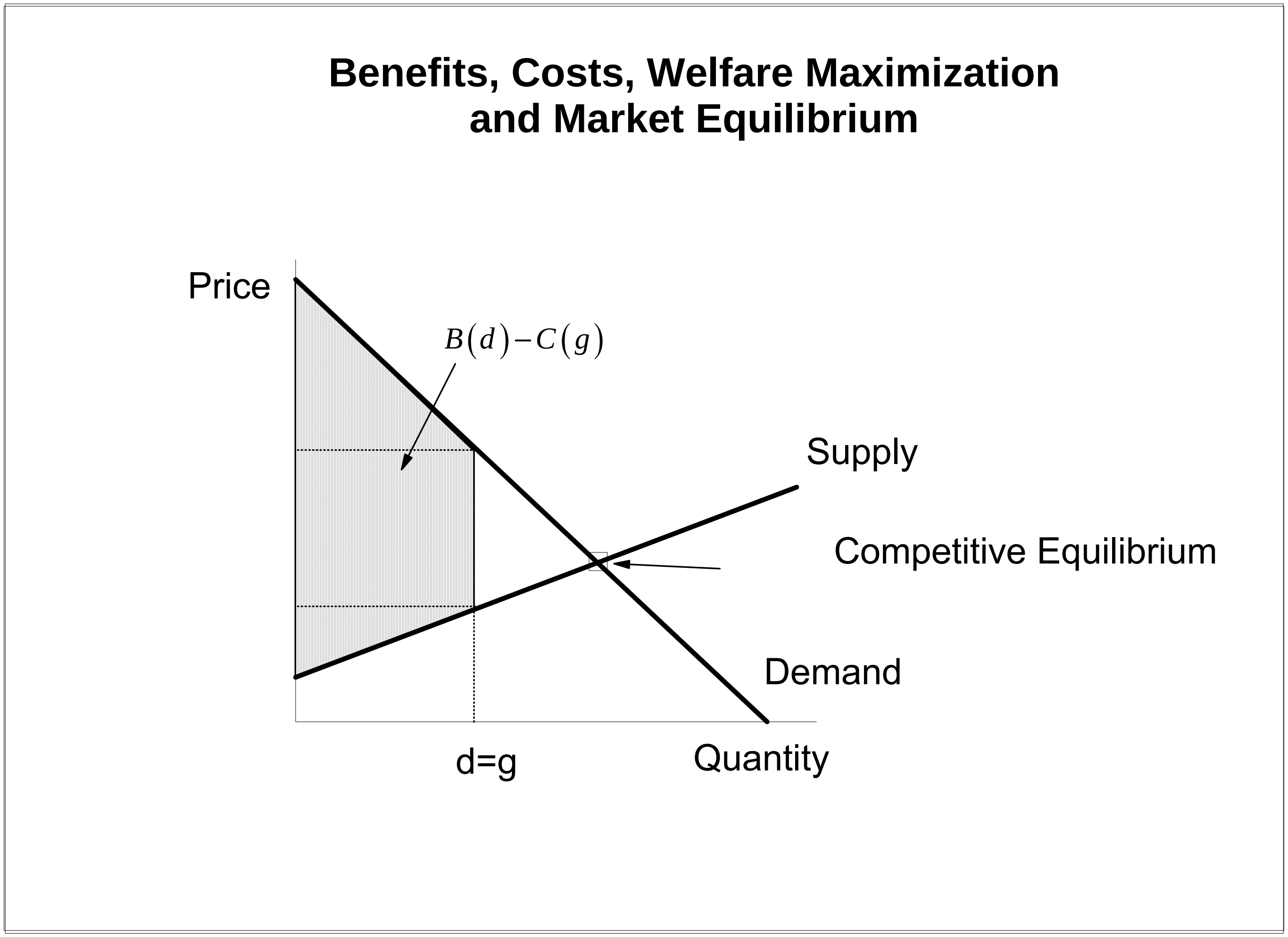
Institute for Pure and Applied Mathematics (IPAM), UCLA

January 13, 2016

The economic dispatch formulation stands at the core of electricity market design and implementation. Under certain conditions, the dispatch solution is a market equilibrium.

- **Deterministic**
  - Real-time spot market for physical dispatch and balancing settlements.
  - Day-ahead dispatch and scheduling.
- **Continuous convex economic dispatch**
  - Electric power systems are almost convex, and use convex approximations for dispatch. (Lavaei & Low, 2012),
  - System marginal costs provide locational, market-clearing, linear prices. (Schweppe, Caramanis, Tabors, & Bohn, 1988)
  - Linear prices support the economic dispatch.
  - Locational prices provide foundation for financial transmission right (FTRs).
- **Security conditions**
  - Contingency constraints.
  - Operating reserves.
- **Competitive assumption for market design**
  - Price-taking behavior by market participants.
  - Bid-based, security constrained, economic dispatch.
  - Market power mitigation with consistent offer caps.

The economic dispatch formulation applies bid-based supply and demand to produce a benefit-cost description. The economic dispatch maximizes the net benefits. For well-behaved supply offers and demand bids, this welfare maximizing economic dispatch is also a price taking, competitive market equilibrium.



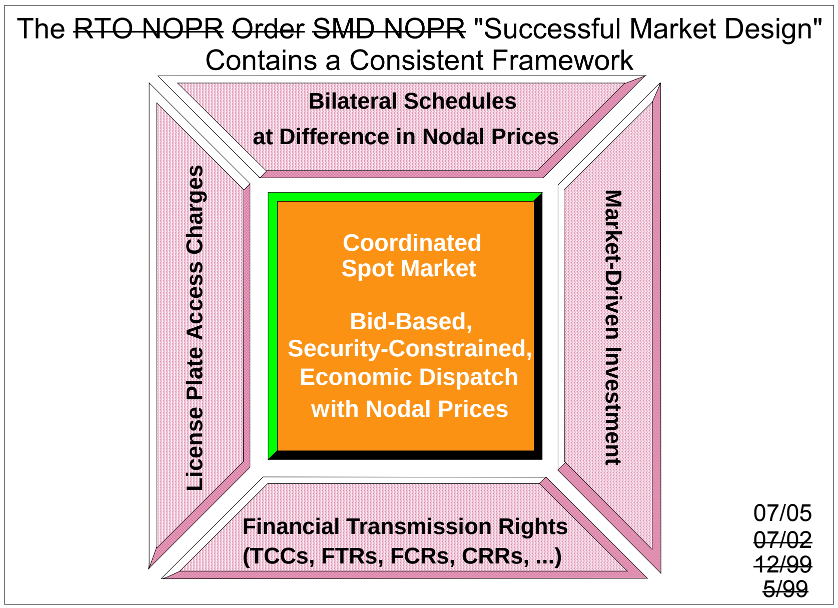
The basic security-constrained, economic dispatch formulation provides the foundation and the framework for real-time and day-ahead electricity spot market pricing. (Hogan, 1992)(Hogan, 2002)

Let $Benefits(d)$ define the benefits of bid-in load (d) and $Costs(g)$ the cost of generation (g) offers. Incorporate other relevant variables such as unit commitment decisions in the control variables in u. The net load at each location is defined as the vector $y = d - g$. Aggregate losses are $L(y,u)$. Finally the transmission constraints appear in the vector function $K(y,u)$. With these definitions, we treat the underlying security-constrained economic dispatch problem as

$$\begin{aligned}
&\underset{d\in D, g\in G, u\in U}{Max} \; Benefits(d) - Costs(g) \\
&\quad s.t. \\
&\qquad d - g = y, \\
&\qquad L(y,u) + \iota^t y = 0, \\
&\qquad K(y,u) \le 0.
\end{aligned}$$

This is a complicated problem with a large number of variables and constraints. With thousands of locations and thousands of transmission lines, the complete statement of the problem can run into millions of variables and millions of constraints. Fortunately, system operators are familiar with this model and have workable methods using a blend of optimization tools and operator judgment to approximate an economic dispatch solution.

The example of successful central coordination, ~~CRT, Regional Transmission Organization (RTO) Millennium Order (Order 2000) Standard Market Design (SMD) Notice of Proposed Rulemaking (NOPR),~~ "Successful Market Design" provides a workable market framework that is working in places like New York, PJM in the Mid-Atlantic Region, New England, the Midwest, California, SPP, and Texas. This efficient market design is under (constant) attack.



Poolco…OPCO…ISO…IMO…Transco…RTO… ITP…WMP…: "A rose by any other name …"

"Locational marginal pricing (LMP) is the electricity spot pricing model that serves as the benchmark for market design – the textbook ideal that should be the target for policy makers. A trading arrangement based on LMP takes all relevant generation and transmission costs appropriately into account and hence supports optimal investments." (International Energy Agency, 2007)

This is the only model that can meet the tests of open access and non-discrimination.

Anything that upsets this design will unravel the wholesale electricity market. The basic economic dispatch model accommodates the green energy agenda, as in the expanding California-Pacificorp Energy Imbalance Market (EIM).

All energy delivery takes place in the real-time market. Market participants will anticipate and make forward decisions based on expectations about real-time prices.

- **Real-Time Prices:** In a market where participants have discretion, the most important prices are those in real-time. "Despite the fact that quantities traded in the balancing markets are generally small, the prevailing balancing prices, or real-time prices, may have a strong impact on prices in the wholesale electricity markets. … No generator would want to sell on the wholesale market at a price lower than the expected real-time price, and no consumer would want to buy on the wholesale market at a price higher than the expected real-time price. As a consequence, any distortions in the real-time prices may filter through to the wholesale electricity prices." (Cervigni & Perekhodtsev, 2013)
- **Day-Ahead Prices:** Commitment decisions made day-ahead will be affected by the design of day-ahead pricing rules, but the energy component of day-ahead prices will be dominated by expectations about real-time prices.
- **Forward Prices:** Forward prices will look ahead to the real-time and day-ahead markets. Although forward prices are developed in advance, the last prices in real-time will drive the system.
- **Getting the Prices Right:** The last should be first. The most important focus should be on the models for real-time prices. Only after everything that can be done has been done, would it make sense to focus on out-of-market payments and forward market rules.

Anticipating a bid-based economic dispatch from a coordinated spot market, we formulate the benefit function for net loads as:

$$B(y) = \underset{d \in D, g \in G}{Max}\ Benefits(d) - Costs(g)$$
$$s.t.$$
$$d - g = y.$$

Under the usual convexity assumptions, the constraint multipliers for this optimization problem define a sub-gradient for this optimal value problem. For simplicity in the discussion here, we treat the sub-gradient as unique so that *B* is differentiable with gradient ∇B. This gives the right intuition for the resulting prices, with the locational prices of net loads at $p^t = \nabla B$. The revised formulation of the economic dispatch is:

$$\underset{y, u \in U}{Max}\ B(y)$$
$$s.t.$$
$$L(y,u) + \iota^t y = 0,$$
$$K(y,u) \le 0.$$

Although not true in general, assume the economic dispatch problem is well-behaved in the sense that when the economic dispatch problem solution $\left(y^*,u^*\right)$ satisfies the optimality conditions.

Optimality Conditions

$$\begin{aligned}
&\text{There exists}\left(y^*,u^*,\lambda,\eta\right),\ \text{such that}\\
&L\left(y^*,u^*\right)+\iota^t y^*=0,\\
&K\left(y^*,u^*\right)\leq 0,\quad \eta^t K\left(y^*,u^*\right)=0,\\
&\eta\geq 0,\quad u^*\in U,\\
&\left(y^*,u^*\right)\in \arg\max_{y,u\in U}\left[B\left(y\right)-\lambda\left(L\left(y,u\right)+\iota^t y\right)-\eta^t K\left(y,u\right)\right].
\end{aligned}$$

Hence, there is no duality gap.[1] The Lagrange multipliers provide the "shadow prices" for the constraints. The solution for the economic dispatch problem is also a solution for the corresponding dual function for this economic dispatch problem:

$$\underset{y,u\in U}{Max}\left[B\left(y\right)-\lambda\left(L\left(y,u\right)+\iota^t y\right)-\eta^t K\left(y,u\right)\right].$$

---

1 (Bertsekas, 1999), p. 427.

The optimality conditions lend themselves to an interpretation of the locational prices.

Assuming differentiability, the first order conditions for an optimum $\left(y^*,u^*\right)$ include:

$$\nabla B\left(y^*\right)-\lambda\left(\nabla L_y\left(y^*,u^*\right)+\iota^t\right)-\eta^t\nabla K_y\left(y^*,u^*\right)=0.$$

Hence, we have the locational prices as

$$p^t=\nabla B\left(y^*\right)=\lambda\iota^t+\lambda\nabla L_y\left(y^*,u^*\right)+\eta^t\nabla K_y\left(y^*,u^*\right).$$

The locational prices have the usual interpretation as the price of power at the swing bus

$$p_G=\lambda\ ,$$

the marginal cost of losses

$$p_L=\lambda\nabla L_y\left(y^*,u^*\right),$$

and the marginal cost of congestion

$$p_C=\eta^t\nabla K_y\left(y^*,u^*\right).$$

The market equilibrium interpretation of the economic dispatch is an important component of electricity market design. (Mas-Colell, Whinston, & Green, 1995) (Hogan, 2002) (Smeers, 2003)(Wang et al., 2012)

Assume that each market participant has an associated benefit function for electricity defined as $B_i(y_i)$, which is concave and continuously differentiable. In FERC terminology, the market participants are the transmission service customers. The customers' benefit functions can arise from a mixture of load or demand benefits and generation or supply costs. In this framework, the producing sector is the electricity transmission provider. The system operator receives and delivers power, coordinates a spot market, and provides transmission service across locations.

The competitive market equilibrium applied here is based on the conventional partial equilibrium framework that stands behind the typical supply and demand curve analysis. The market consists of the supply and demand of electric energy and transmission service plus an aggregate or numeraire "good" that represents the rest of the economy. Each customer is assumed to have an initial endowment $\tilde{w}_i$ of the numeraire good. In addition, each customer has an ownership share s_i in the profits "Π" of the electricity transmission provider, with $\sum_i s_i = 1$.

An assumption of the competitive model is that all customers are price takers. Hence, given market prices, p, customers choose the level of consumption of the aggregate good, c_i, and electric energy including the use of the transmission system according to the individual optimization problem maximizing benefits subject to an income constraint:

$$\begin{aligned} &\underset{y_i, c_i}{Max}\, B_i(y_i) + c_i \\ &s.t. \\ &\quad p^t y_i + c_i \le \tilde{w}_i + s_i \Pi. \end{aligned}$$

The market equilibrium interpretation separates transmission service from generation and load.

In this simple partial equilibrium model of the economy, there is only one producing entity, which is the system operator providing transmission service. Under the competitive market assumption, the producer is constrained to operate as a price taker who chooses inputs and outputs (y_i) that are feasible and that maximize profits. The profits amount to $\Pi = p^t \sum_i y_i$. Hence, the transmission system operator's problem is:

$$
\begin{aligned}
&\underset{\substack{y,u\in U\\ y_i}}{Max} \quad p^t y \\
&s.t. \\
&\quad y = \sum_i y_i, \\
&\quad L(y,u) + \iota^t y = 0, \\
&\quad K(y,u) \le 0.
\end{aligned}
$$

Given the initial endowment of goods $\tilde{w}_i$, and the ownership shares s_i, a competitive market equilibrium is defined as a vector of prices, p, profits, Π, controls, u, and a set of net loads, y_i, for all *i*, that simultaneously solves the transmission providers problem and the benefit maximization for the transmission service customers.

In this sense, the linear locational marginal prices p are said to “support” the solution. Under the optimality conditions assumed, the market equilibrium would satisfy the same local first-order necessary conditions as an optimal solution to the economic dispatch.

$$p^t = \nabla B\left(y^*\right) = \lambda \iota^t + \lambda \nabla L_y\left(y^*, u^*\right) + \eta^t \nabla K_y\left(y^*, u^*\right).$$

The prices are used for real-time settlements.

Before electricity restructuring, the economic dispatch formulation was familiar and commonly reported as standard practice. The big electricity market reform was to use the associated prices for market settlements.

- **Before: Power pool systems within and across vertically integrated utilities**
  - Cost-based, security-constrained, economic dispatch.
  - Split-savings methodologies for sharing the surplus.
  - Contract path methodologies for scheduling trading.

- **After: “Organized markets” under Regional Transmission Organizations**
  - Bid-based, security-constrained, economic dispatch.
  - Locational prices that support the economic dispatch.
  - Point-to-point scheduling and financial transmission rights.

The "split-savings" methodology played a major role in individual energy trading and in the traditional power pools. The focus was on allocating the economic rents, not on incentives.

The New England Power Pool (NEPOOL) calculates the savings of the actual hourly dispatch compared to each company's hypothetical individual "own load" dispatch and then distributes the savings according to a formula that NEPOOL chairman Robert Bigelow has described as "complicated enough that very few people understand it." (personal communication)

Split Savings

Sellers' share of savings = (1-t)(1/2(v* - c)) (MWh sold)

Where: t = Pool overhead tax, v* = Average Buyers' value, c =Individual Sellers' Cost

Buyers' share of savings = (1-t)(1/2(v - c*)) (MWh purchased)

Where: v = Individual Buyers' value, c* = Average Sellers' Cost

"This multi-level pricing structure means that, at any instant, there are different prices charged or paid for energy transferred through the pool dependent on each member's costs and the use of the energy by the member. This violates the "law of single price" for open markets that states that only one price can exist in an efficient open market otherwise there will be opportunities for arbitrage. If a pool were to operate as a market with split cost pricing then a "cheap" importer could purchase electricity from the pool at a lower price than an "expensive" importer. The "cheap" importer could then re-sell the purchased electricity to the "expensive" importer outside of the pool and at a profit. In this manner the multitude of prices creates opportunities for arbitrage on transactions conducted outside of the pool's jurisdiction." (Thomson, 1995)

The market equilibrium satisfies a "no arbitrage" condition.

At equilibrium, there are no feasible trades of electric loads that would be profitable at the prices *p*. Hence, let y^1 be any other feasible set of net loads, such that there is a u^1 with:

$$L\left(y^1, u^1\right) + \iota^t y^1 = 0,$$

$$K\left(y^1, u^1\right) \leq 0,$$

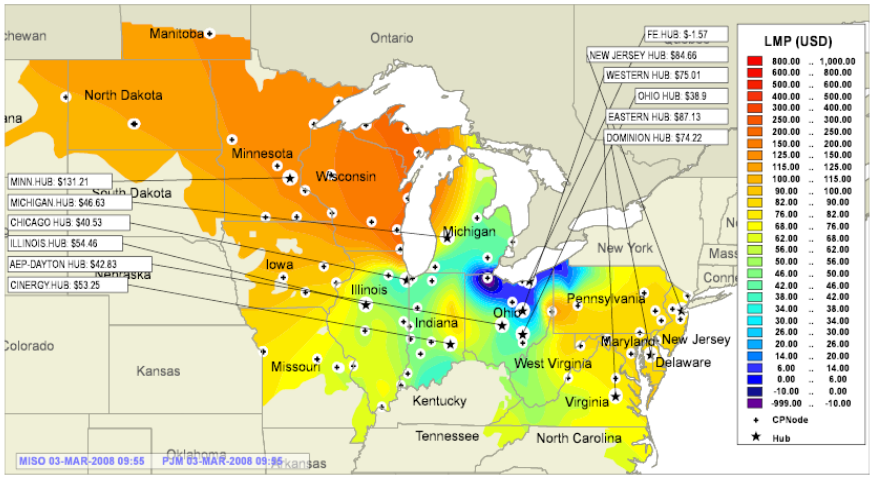
$$u^1 \in U.$$

Then, assuming concavity for the benefits function, we can show:

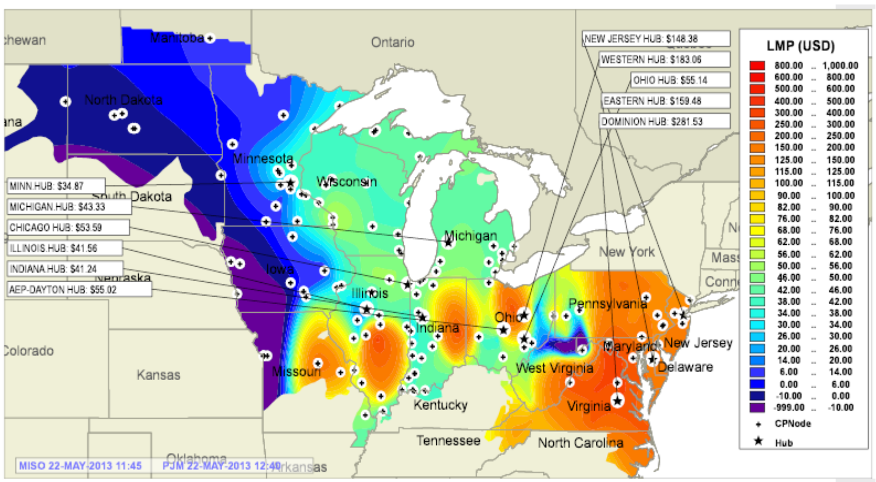
$$p^t\left(y^* - y^1\right) \geq 0.$$

This no-arbitrage condition will be important as part of the analysis of revenue adequacy in Financial Transmission Right (FTR) formulations. Importantly, the condition allows for the controls to change from the optimal value u^*. This implies a great degree of flexibility in changing the dispatch while maintaining the no-arbitrage condition for market equilibrium.

RTOs operate spot markets with locational prices. For example, PJM updates prices and dispatch every five minutes for over 10,000 locations. Locational spot prices for electricity exhibit substantial dynamic variability and persistent long-term average differences.



Minnesota Hub: $131.21/MWh. First Energy Hub: $-1.57/MWh. March 3, 2008, 9:55am

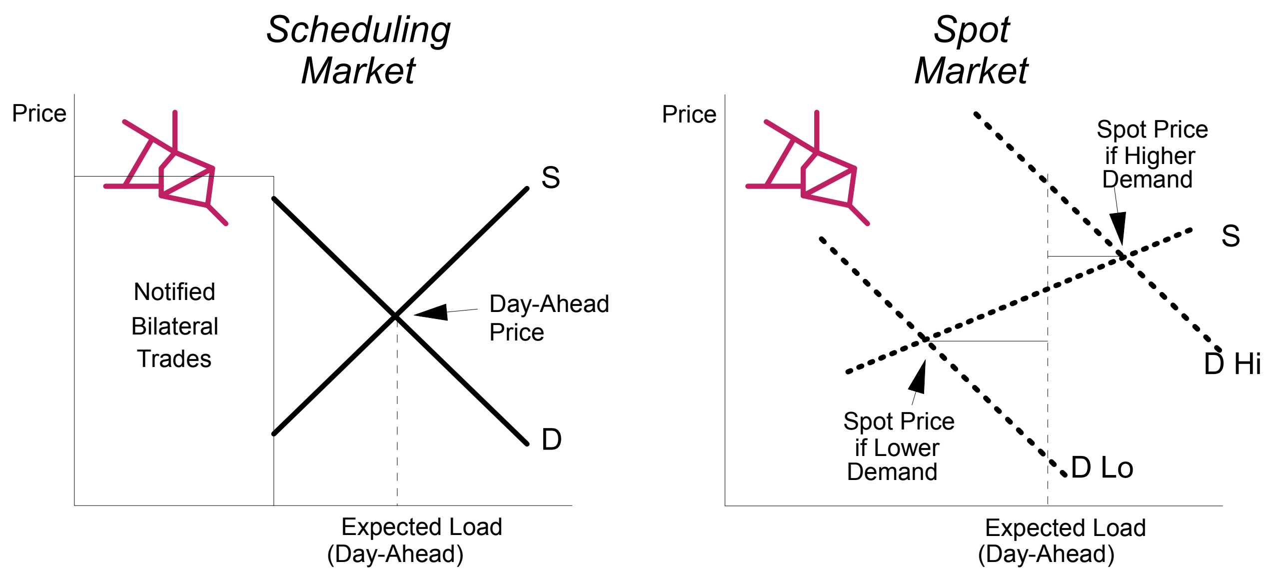


Missouri MPS -$71.25, Dominion Hub $281.53. May 22, 2013, 12:40pm.

From MISO-PJM Joint and Common Market, http://www.jointandcommon.com

The expected value of the real-time dispatch can differ from the day-ahead dispatch.

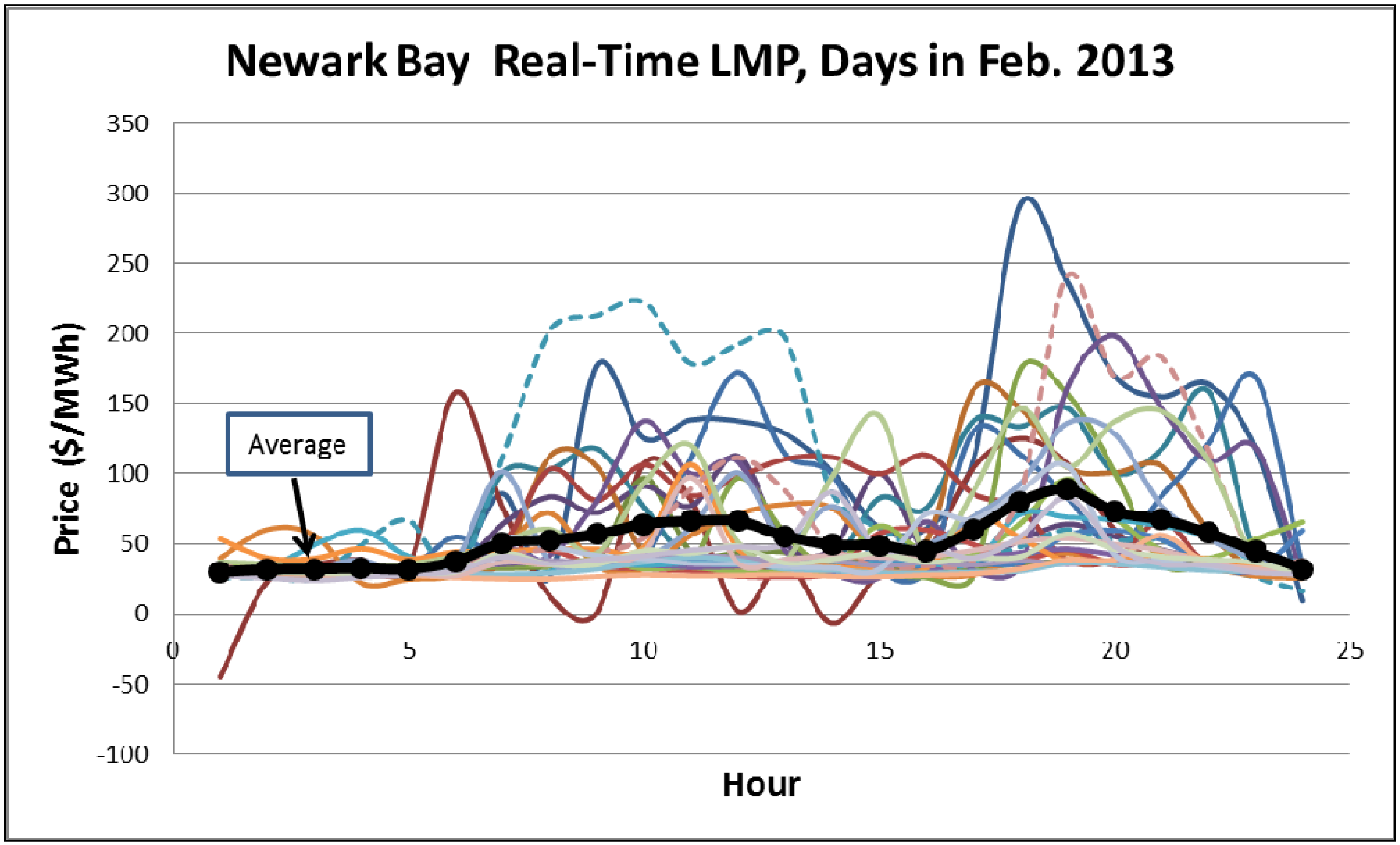
Spot Price Applies to Deviations from Scheduled Quantities



- Notified bilateral trades pay locational transmission charge but are settled outside pool
- Other trades settle at day-ahead price
- Locational congestion rents paid to transmission right holders along with excess congestion costs

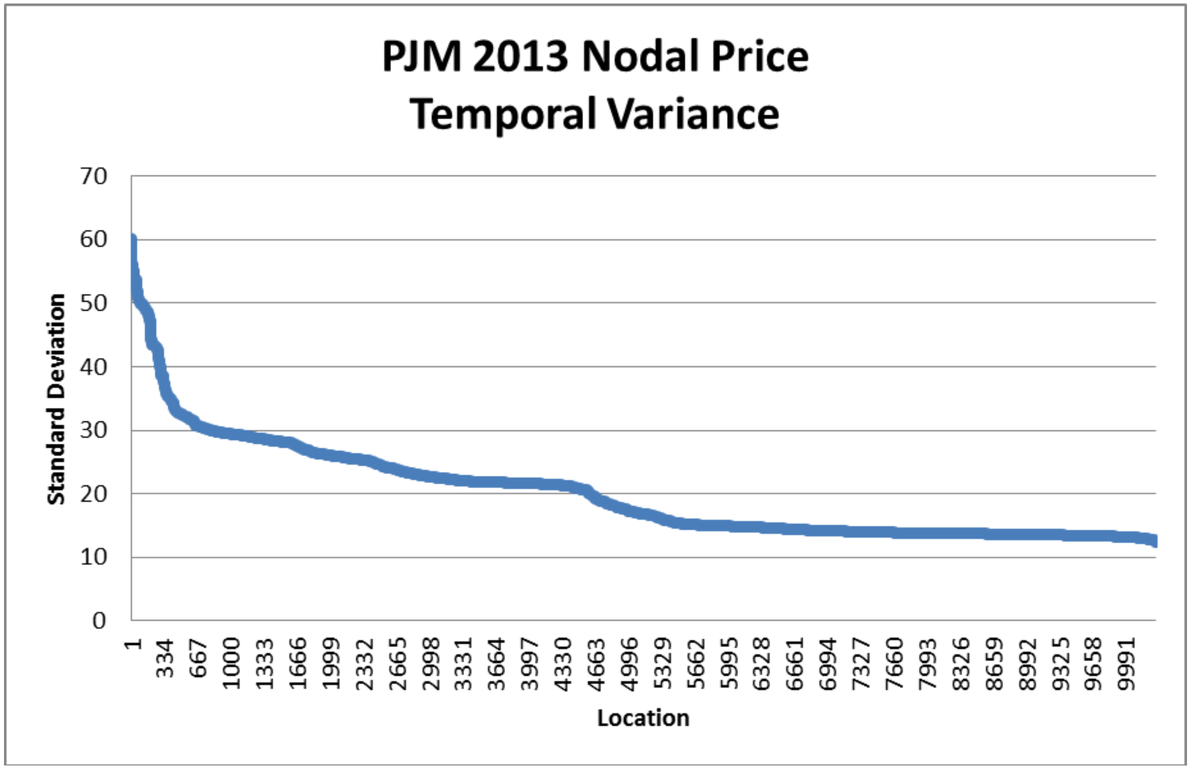
- Payments for deviations from day-ahead trades at spot price for all transactions
- Uplift covers ancillary services and other costs
- Excess congestion costs paid to holders of transmission rights

The hourly average prices capture very little of the total real-time price variation.



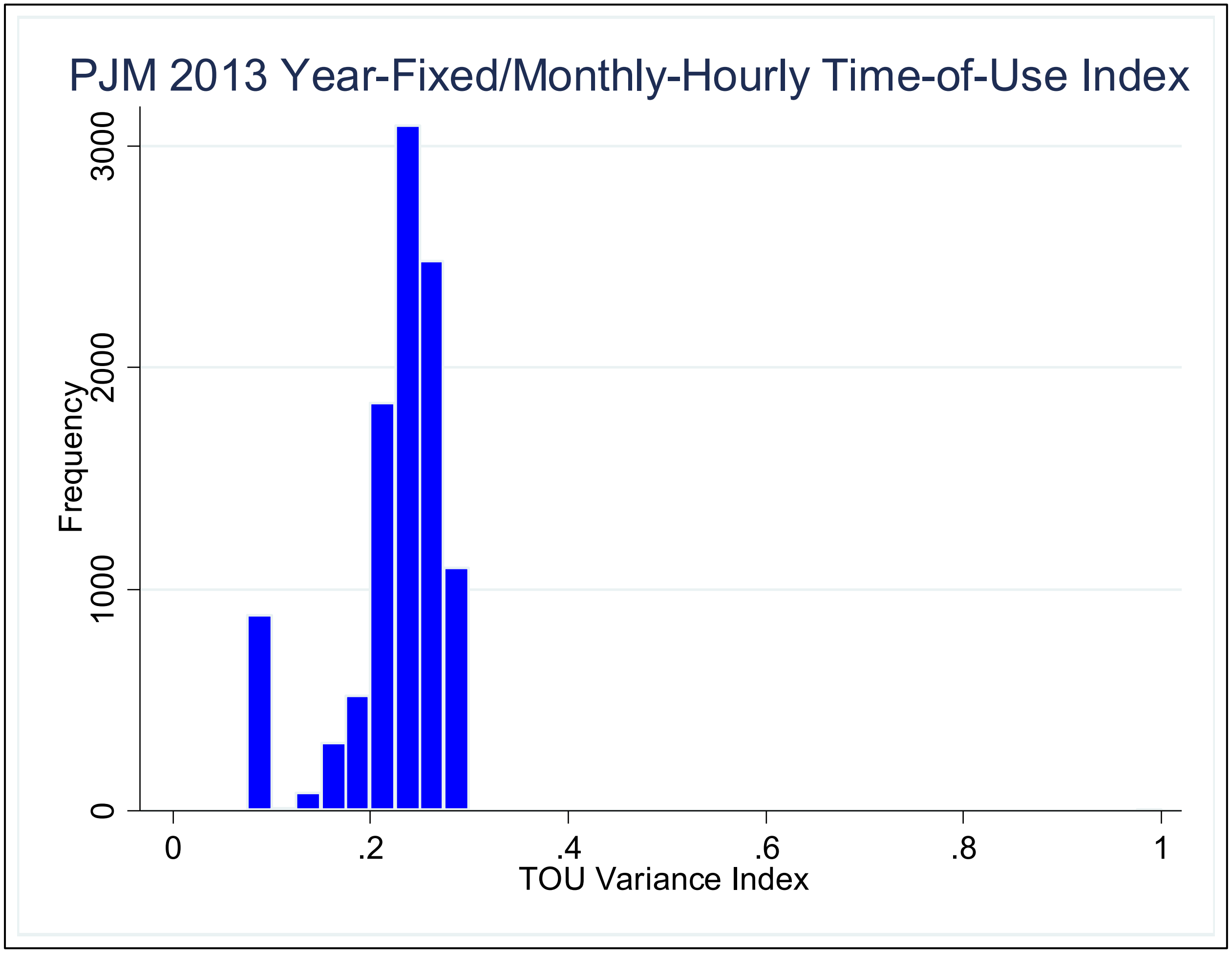
(Source: www.pjm.com) (W. Hogan, "Time-of-Use Rates and Real-Time Prices," August 23, 2014, www.whogan.com)

Volatile electricity prices provide an opportunity for inter-temporal price arbitrage.

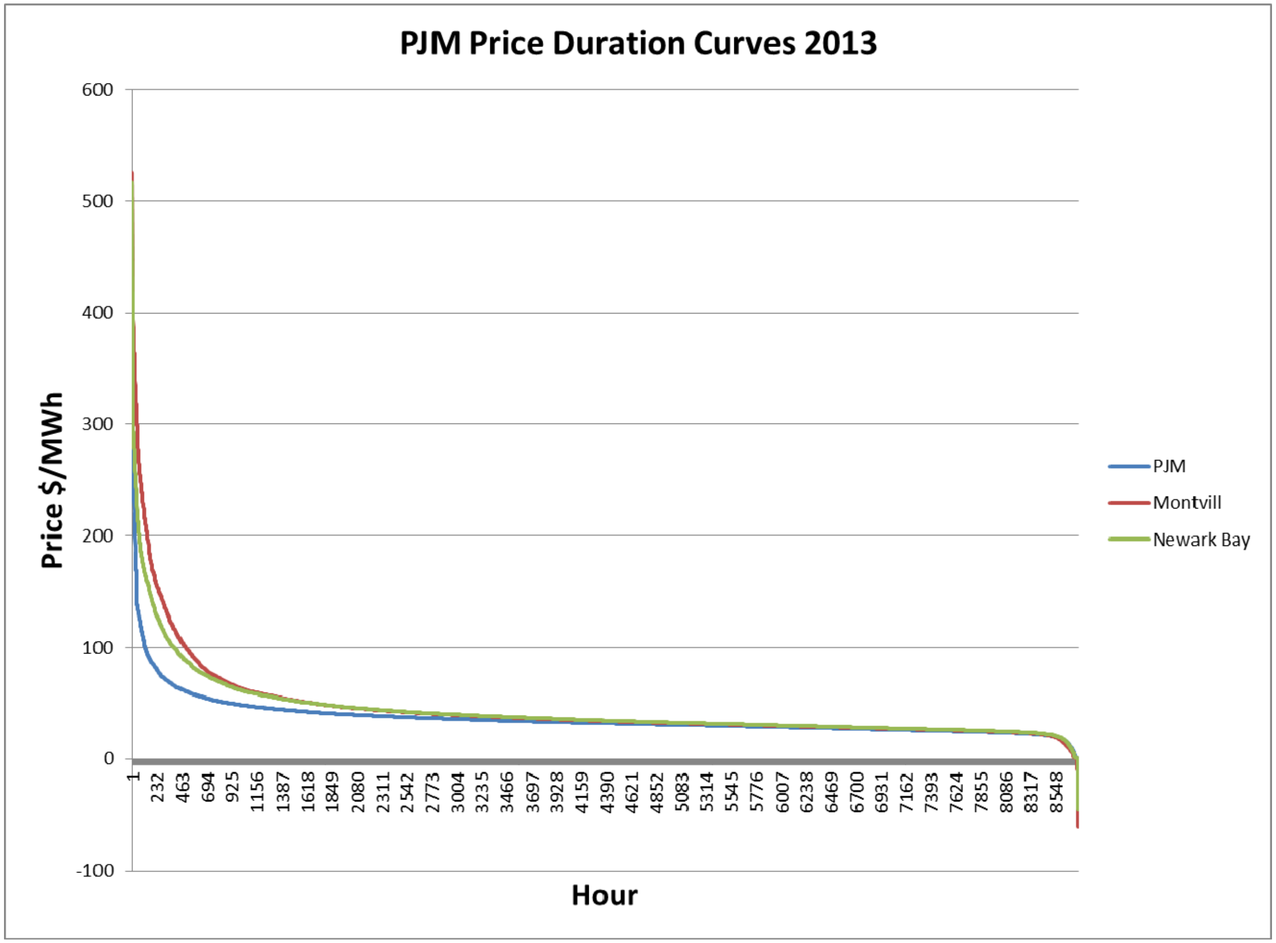


Source: Sandesh Kataria extract of PJM 2013 hourly price data for 10,296 locations, www.pjm.com.

Data for the approximately 10,000 pricing points in PJM measure the degree of real-time price variance explained by hourly TOU rates set at the monthly average for each hour. Most of the price variance remains unexplained. Hence, TOU rates are not a good proxy for real-time prices.

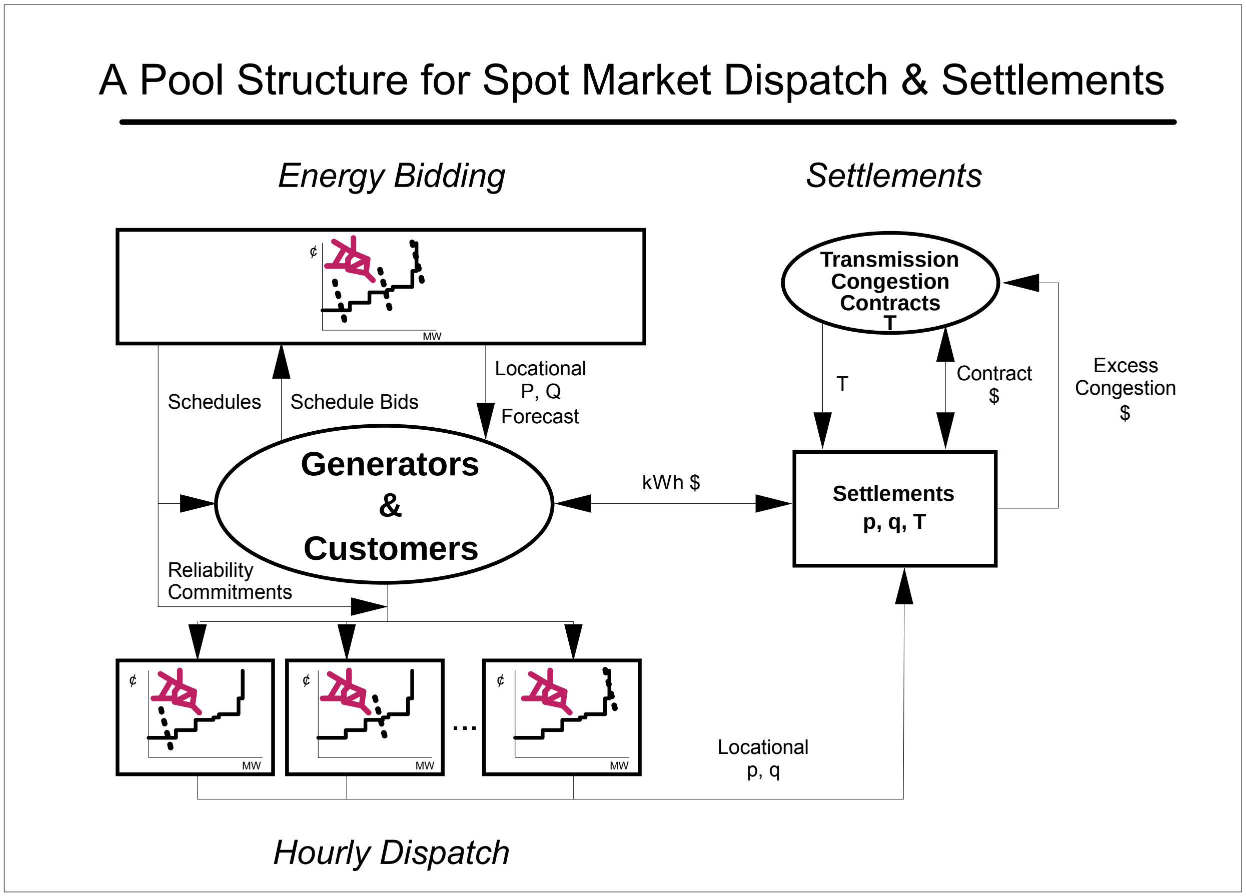


A price duration curve summarizes a great deal of information about the distribution of hourly prices. Data for PJM in 2013 show relatively few hours with very high or very low prices.



(PJM, www.pjm.com)

A large part of the motivation for the design of electricity spot markets is to provide a framework



Day-ahead markets provide a mechanism for short-term hedging of real-time prices. Most schedules and contracts are effectively financial contracts.

- **Net Pool: Settlements occur net of schedule commitments**
  - Schedules based on bilateral transactions or day-ahead dispatch.
  - Locational prices for deviations between schedule and real-time dispatch.
  - Central management of financial settlements.

- **Gross Pool: Settlements reflect gross transactions without netting**
  - Schedules based on bilateral transactions or day-ahead dispatch.
  - Locational prices for real-time dispatch.
  - Decentralized management of financial settlements through contracts for differences.

- **Virtual transactions.**
  - Day-ahead bids, offers and schedules that are explicitly treated as financial transactions.
  - Virtual transactions are settled at the real-time locational price.
  - Equivalent treatment under gross or net-pool.

With bid-based, security constrained economic dispatch and locational prices, any hybrid that mixes across these design elements can be made internally consistent.

Day-ahead markets provide a mechanism for short-term hedging of real-time prices. With virtual trading, and risk neutral market participants, the idealized day-ahead prices should equal the expected real-time price. Otherwise, traders are leaving money on the table.

$$P_{\text{Day Ahead}} \left| \text{Day Ahead Information} = E\left(P_{\text{Real Time}} \left| \text{Day Ahead Information} \right. \right) \right.$$

In the real system there are varying degrees of risk aversion and transaction costs. Market participants pay differing uplift charges depending on the nature of their virtual transactions. The details would depend on the nature of the risk aversion, commitment costs, and existing forward contracts. In addition to providing hedges, the day-ahead market determines an allocation of transmission capacity.

However, trading to capture arbitrage opportunities should produce a close connection between day-ahead and expected real-time prices. (PJM, 2015)

$$P_{\text{Day Ahead}} \left| \text{Day Ahead Information} \approx E\left(P_{\text{Real Time}} \left| \text{Day Ahead Information} \right. \right) + \text{ Transaction Costs} \right.$$

With virtual bids (offers) at P_V, the day-ahead dispatch becomes:

$$\begin{aligned} &\underset{v,d\in D,g\in G,u\in U}{Max} \; Benefits_{DA}\left(d\right) - Costs_{DA}\left(g\right) + P_V v \\ &s.t. \\ &\quad d - g + v = y, \\ &\quad L\left(y,u\right) + \iota^t y = 0, \\ &\quad K\left(y,u\right) \le 0. \end{aligned}$$

The expected value of the real-time dispatch can differ from the day-ahead dispatch. With enough uncertainty, the number of constraints that might be binding in real-time can exceed (number of nodes-1) and, therefore, the number of binding constraints in any day-ahead dispatch. Price arbitrage can make the day-ahead prices equal the expected real-time prices. The example illustrates the point.

Transmission Between Locations

P_A

$-K \le Flow \le K$

$$P_B = \begin{cases} P_B^{Hi} >> P_A, & \pi = 0.5 \\ P_B^{Lo} < P_A, & 1-\pi = 0.5 \end{cases}$$

Expected Values

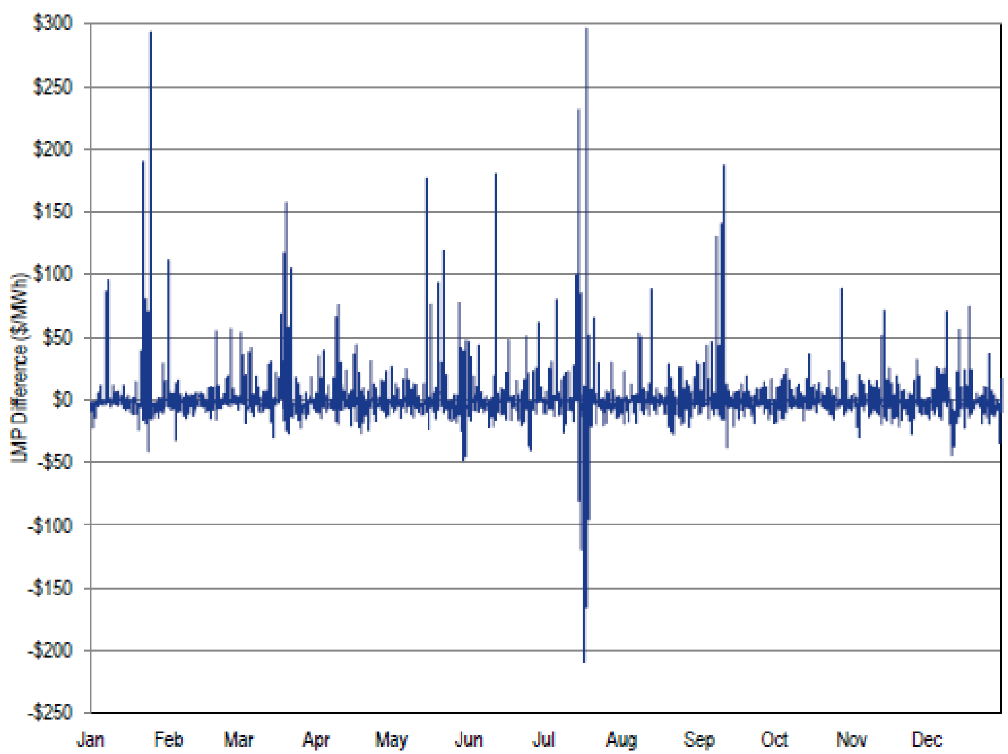
$$E(P_A) = P_A$$
$$E(\mu_{ab}) = 0.5\left(P_B^{Hi} - P_A\right)$$
$$E(\mu_{ba}) = 0.5\left(P_A - P_B^{Lo}\right)$$
$$E(P_B) = 0.5\left(P_B^{Lo} + P_B^{Hi}\right) > P_A$$
$$E(Flow) = 0.5(K) + 0.5(-K) = 0$$

Day-Ahead Dispatch

$$P_A^{DA} = P_A$$
$$P_B^{DA} = E(P_B)$$
$$\mu_{ab}^{DA} = E(P_B) - P_A < E(\mu_{ab})$$
$$\mu_{ba}^{DA} = 0 < E(\mu_{ba})$$
$$Flow^{DA} = K$$

Data for PJM are consistent with the arbitrage argument between day-ahead and real-time. For any given hour, there is a large variation in the difference between day-ahead and real-time prices. But the differences are centered around zero. The average difference for RT-DA was -$0.60/MWh, or 1.6%.

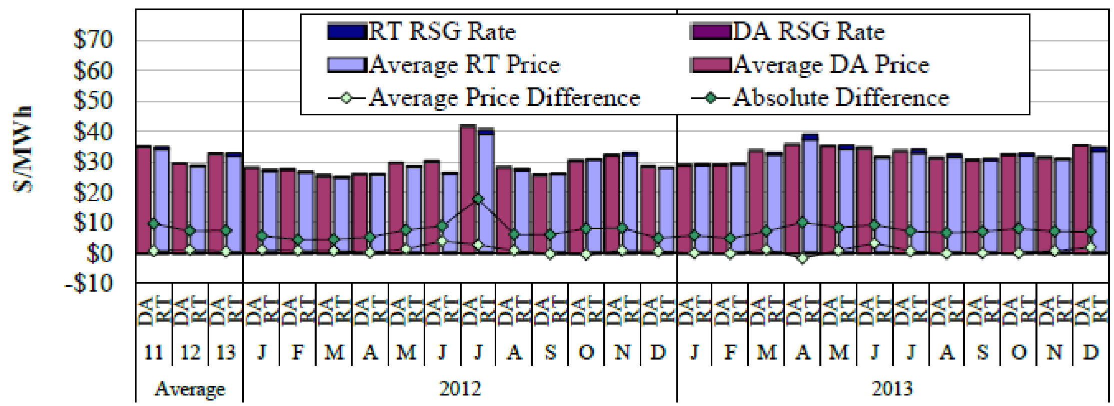
Figure 3-30 Real-time hourly LMP minus day-ahead hourly LMP: 2013[78]



(PJM, State of Market Report, 2013, Vol. 2, p. 111)

Data for the Midcontinent Independent System Operator (MISO) display similar results with variation across the region but small average differences across the full footprint.

Figure 9: Day-Ahead and Real-Time Prices
2012–2013



Average DA-RT Price Difference Excluding RSG (% of Real-Time Price)

	11	12	13	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D
	Average			2012												2013											
Cin/Indi Hub	2	4	2	4	3	3	1	5	15	7	3	-1	-1	3	2	0	-1	3	-5	2	10	2	0	0	0	2	6
Michigan Hub	2	4	2	5	2	-1	5	5	7	7	3	4	1	3	1	1	2	5	-5	1	7	-1	0	-2	0	5	7
Minnesota Hub	-2	-2	-1	3	-2	2	-12	-6	0	2	2	-7	3	-8	-1	3	4	4	-7	7	2	-2	-5	-1	-3	-9	-2
WUMS Area	3	1	2	3	0	4	0	-1	-1	2	3	-4	0	0	3	1	4	4	-5	1	6	2	-4	3	1	7	3

Average DA-RT Price Difference Including RSG (% of Real-Time Price)

	11	12	13	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D
Cin/Indi Hub	-1	2	-1	3	2	2	-2	3	12	3	1	-2	-2	0	1	-2	-2	1	-9	-1	9	-2	-4	-3	-2	0	1
Michigan Hub	-1	2	-1	4	2	-2	2	3	5	4	2	3	0	1	-1	-1	1	3	-10	-3	5	-5	-3	-4	-2	3	3
Minnesota Hub	-5	-4	-4	2	-2	1	-14	-8	-3	-2	1	-8	2	-10	-2	1	3	1	-11	2	0	-6	-8	-4	-6	-10	-6
WUMS Area	0	-1	-1	2	-1	3	-3	-3	-3	-2	1	-6	-1	-3	1	-1	3	1	-9	-3	4	-2	-6	0	-1	5	-1

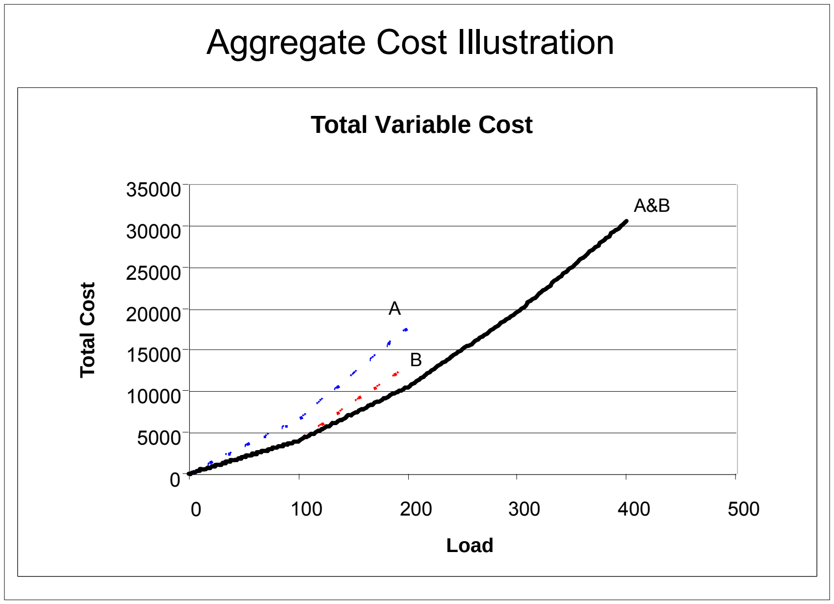
(Potomac Economics, 2013 State of the Market Report for the MISO Electricity Markets, June 2014, p.24.)

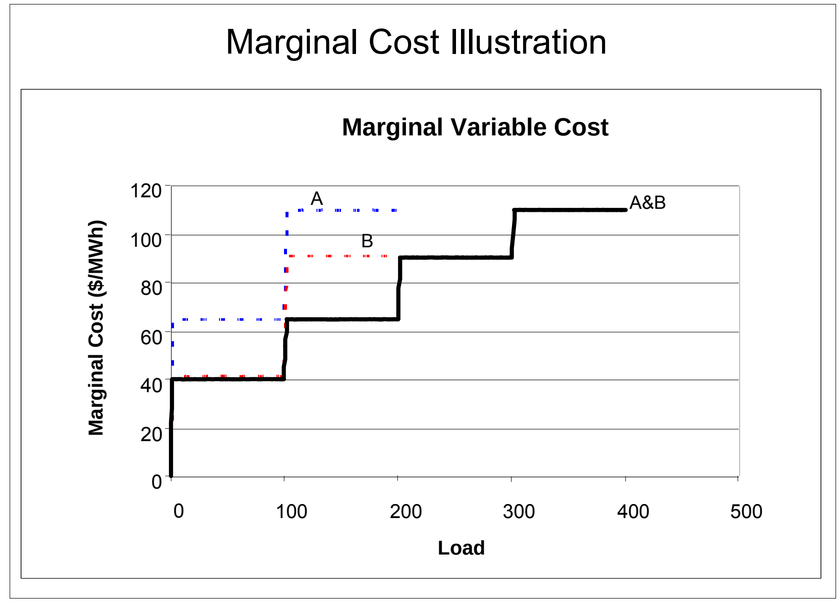
Econometric studies based on natural experiments tend to confirm the introduction of virtual trading supports better convergence between expected real-time and day-ahead prices. For the case of California, see (Jha & Wolak, 2015).

The day-ahead unit commitment problem presents both computational and market design problems.

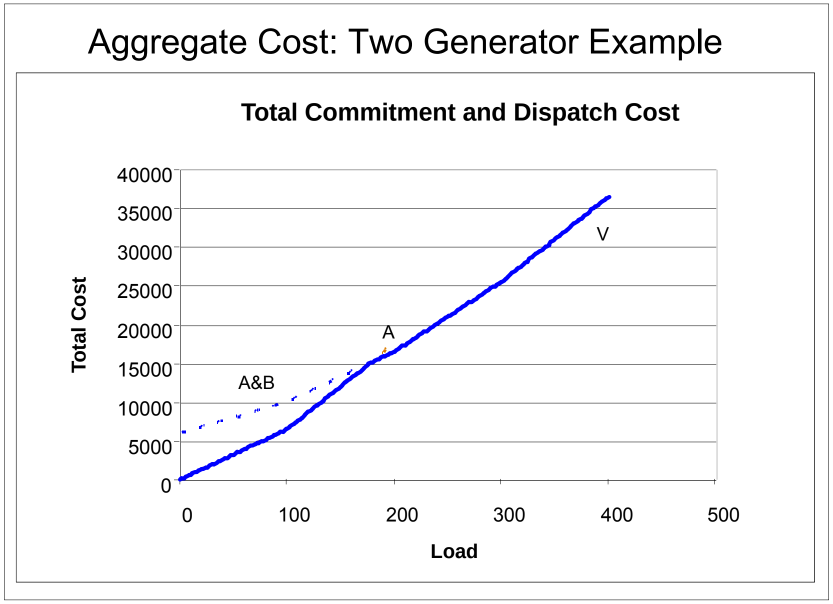
- **Computational.** The natural formulation of the unit commitment invokes a mixed-integer programming problem. This is more difficult than the familiar convex problem, but software is improving. (Hobbs, Rothkopf, & O'Neill, 2014) (Bixby, 2015)
- **Market design.** The rules for generation offers have important impacts.
  - **One-part bids**. Restrict offers for the dispatch to so-called "one-part" bids. Essentially restricts the structure of generation offers to convex functions. Requires bidders to internalize the costs. (Stoft, 2002)
  - **Multi-part bids**. Allow bids that approximate the generation technology characteristics, such as start-up and shut-down costs, ramping rates, minimum as well as maximum generation levels, minimum run-times, and so on. This produces non-convexities that can have impacts on cost and feasibility. (Sioshansi, O'Neill, & Oren, 2008)
- **Pricing Impacts.** The resulting dispatch-based prices have important implications for market clearing.
  - **One-part bids**. Dispatch-based prices appear simple, but these are not market-clearing prices. Bidders may have incentives not to follow the dispatch. This can create economic and reliability problems (e.g. NYISO).
  - **Multi-part bids**. The non-convex problem may have no linear market-clearing prices. This condition will be revealed by dispatch solution and will identify required side-payments ("uplift") to remove incentive to deviate from the economic dispatch.

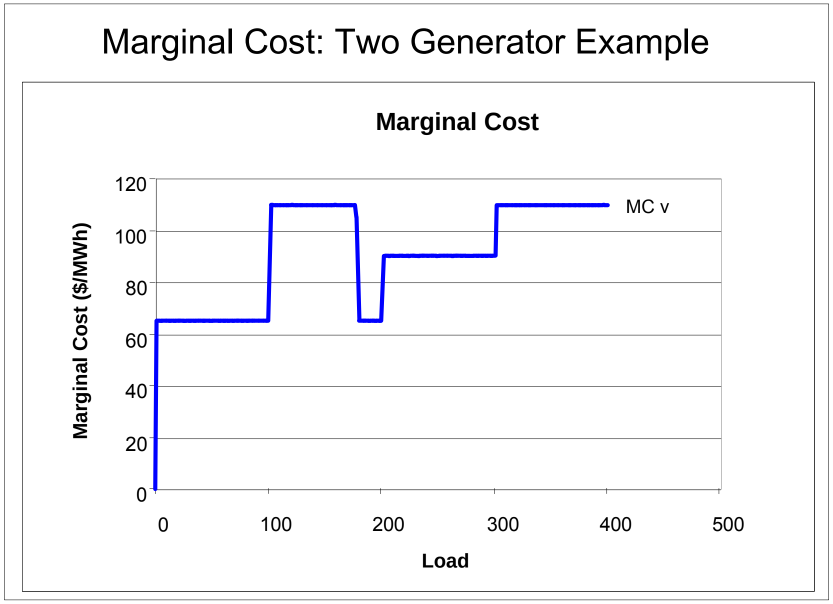
Energy dispatch is continuous, convex and yields linear prices. A simplified example with two generating units illustrates both total and marginal costs. (Gribik, Hogan, & Pope, 2007)





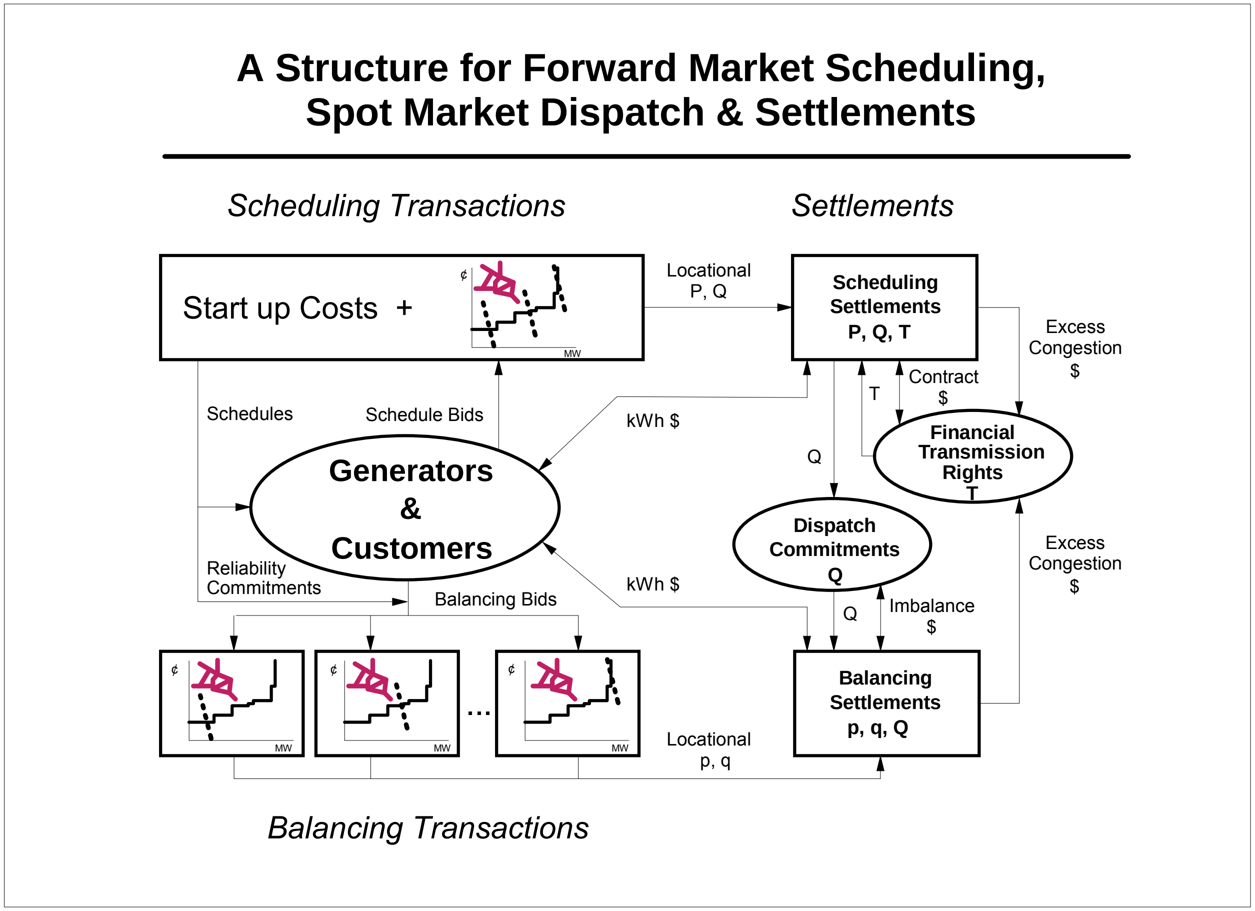
Unit commitment requires discrete decisions. Now the second unit (B) has a startup cost.





Marginal cost-based linear prices cannot support the commitment and dispatch. The solution has been to make "uplift" payments to assure reliable and economic unit commitment.

Organized electricity markets utilize day-ahead markets with bid-in loads and generation offers. In addition, day-ahead markets include a reliability commitment to ensure that adequate capacity will be available in real time to meet the actual load.



The unit commitment problem implies discrete choices that create non-convexities and computational problems. A stylized version of the unit commitment and dispatch problem for a fixed demand $\mathbf{y}$ as formulated in (Gribik et al., 2007)**:**

Constants:	Variables:
$\mathbf{y}_t =$ vector of nodal loads in period t	$start_{it} = \begin{cases} 0 \text{ if unit i is not started in period t} \\ 1 \text{ if unit i is started in period t} \end{cases}$
$m_{it} =$ minimum output from unit i in period t if unit is on	$on_{it} = \begin{cases} 0 \text{ if unit i is off in period t} \\ 1 \text{ if unit i is on in period t} \end{cases}$
$M_{it} =$ maximum output from unit i in period t if unit is on	$g_{it} =$ output of unit i in period t
$ramp_{it} =$ maximum ramp from unit i between period t-1 and period t	$\mathbf{d}_t =$ vector of nodal demands in period t.
$StartCost_{it} =$ Cost to start unit i in period t	
$NoLoad_{it} =$ No load cost for unit i in period t if unit is on	
$\bar{F}_{kt}^{\max} =$ Maximum flow on transmission constraint k in period t.	

$$v(\{\mathbf{y}_t\}) = \inf_{\text{g,d,on,start}} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}(g_{it}) \right)$$

subject to

$$\begin{aligned}
& m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
& -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
& start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
& start_{it} = 0 \text{ or } 1 && \forall i,t \\
& on_{it} = 0 \text{ or } 1 && \forall i,t \\
& \mathbf{e}^T(\mathbf{g}_t - \mathbf{d}_t) - LossFn_t(\mathbf{d}_t - \mathbf{g}_t) = 0 && \forall t \\
& Flow_{kt}(\mathbf{g}_t - \mathbf{d}_t) \le \bar{F}_{kt}^{\max} && \forall k,t \\
& \mathbf{d}_t = \mathbf{y}_t && \forall t.
\end{aligned}$$

The non-convex optimization problem introduces the problem of the duality gap. Recall the well-behaved convex economic dispatch problem solution y^*, u^* that satisfies the optimality conditions.

Optimality Conditions

$$\begin{aligned}
&\textit{There exists}\left(y^*, u^*, \lambda, \eta\right), \textit{such that}\\
&L\left(y^*, u^*\right) + \iota^t y^* = 0,\\
&K\left(y^*, u^*\right) \le 0, \quad \eta^t K\left(y^*, u^*\right) = 0,\\
&\eta \ge 0, \quad u^* \in U,\\
&\left(y^*, u^*\right) \in \arg\max_{y,u \in U}\left[B\left(y\right) - \lambda\left(L\left(y,u\right) + \iota^t y\right) - \eta^t K\left(y,u\right)\right].
\end{aligned}$$

Hence, there is no duality gap. But with the unit commitment problem, there may be no solution to the optimality conditions, and there is a duality gap where the optimal dual value is not equal to the optimal solution of the original primal economic dispatch problem:

$$\min_{\lambda,\eta}\left[\max_{y,u \in U}\left[B\left(y\right) - \lambda\left(L\left(y,u\right) + \iota^t y\right) - \eta^t K\left(y,u\right)\right]\right] > \mathop{Max}_{y,u \in U}\left[B\left(y\right) \middle| s.t.\ L\left(y,u\right) + \iota^t y = 0, K\left(y,u\right) \le 0.\right]$$

The possible duality gap presents both computational and pricing challenges. There may be no set of linear prices that support the economic commitment and dispatch solution. But linear prices continue to be important in market design.

Selecting the appropriate approximation model for defining energy and uplift prices involves practical tradeoffs. All involve "uplift" payments to guarantee payments for bid-based cost to participating bidders (generators and loads), to support the economic commitment and dispatch.

Uplift with Given Energy Prices=Optimal Profit – Actual Profit

- **Restricted Model (r)** (O'Neill, Sotkiewicz, Hobbs, Rothkopf, & Stewart, 2005)
    - Fix the unit commitment at the optimal solution.
    - Determine energy prices from the convex economic dispatch.
- **Dispatchable Model (d)**
    - Relax the discrete constraints and treat commitment decisions as continuous.
    - Determine energy prices from the relaxed, continuous, convex model.
- **Extended Locational Marginal Pricing (ELMP) Model (h)**
    - Equivalent formulations
        - Select the energy prices from the convex hull of the cost function.
        - Select the energy prices from the Lagrangean relaxation (i.e., usual dual problem for pricing the joint constraints).
    - Resulting energy prices minimize the total uplift.

A formulation that separates out the discrete variables (u) serves to distinguish the modeling approaches.[2] Here assume that the problem is convex but for the integer restriction on the commitment variables. Use least-cost dispatch perspective. The optimal commitment is u^0.

- **Unit commitment and dispatch**

$$\begin{aligned} v(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & u = 0,1. \end{aligned}$$

- **Restricted Model (r)**

$$\begin{aligned} v^r(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & u = u^O. \end{aligned}$$

- **Dispatchable Model (d)**

$$\begin{aligned} v^d(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & 0 \le u \le 1. \end{aligned}$$

---

2 For a further discussion of dual price functions, see (Bjørndal & Jörnsten, 2008), “Equilibrium prices supported by dual price functions in markets with non-convexities.

Economic commitment and dispatch is a special case of a general optimization problem.

$$v(y) = \underset{x \in X}{Min} \quad f(x) \\ s.t. \quad g(x) = y.$$

From the perspective of a price-taking bidder, uplift is the difference between actual and optimal profits.

$$\text{Actual profits:} \quad \pi(p, y) = py - v(y)$$

$$\text{Optimal Profits:} \quad \pi^*(p) = \underset{z}{Max}\{pz - v(z)\}$$

$$Uplift(p, y) = \pi^*(p) - \pi(p, y)$$

Classical Lagrangean relaxation and pricing creates a familiar dual problem and a characterization of the convex hull $v^h(y)$.

$$L(y, x, p) = f(x) + p(y - g(x))$$

$$\hat{L}(y, p) = \underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}$$

$$v^h(y) = \underset{p}{Sup}\, \hat{L}(y, p) = \underset{p}{Sup}\left\{\underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}\right\}$$

The optimal dual solution minimizes the uplift, and the "duality gap" is equal to the minimum uplift. (Gribik et al., 2007).

$$v(y) - v^h(y) = \underset{p}{Inf}\; Uplift(p, y) = Uplift(p^h, y).$$

In general, the solutions can be such that:

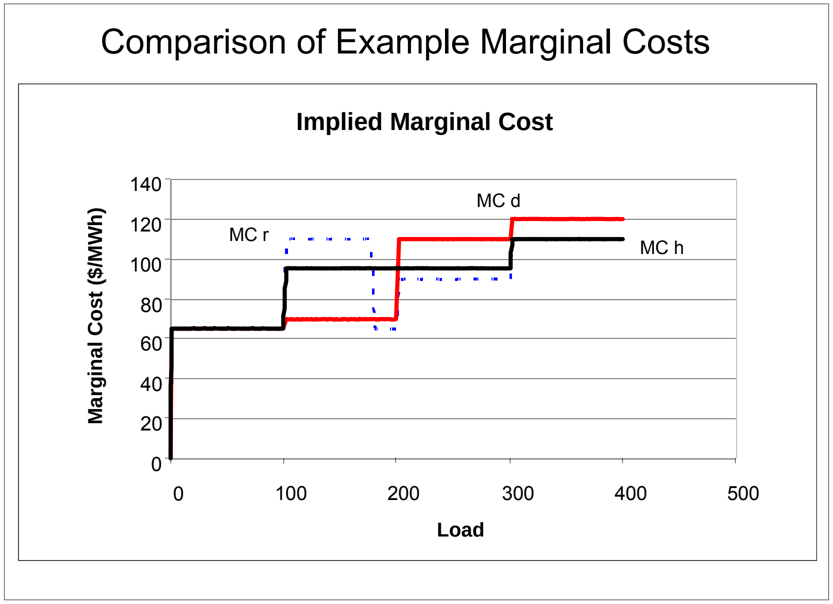
$$v^d(y) < v^h(y) < v^r(y) = v(y), \\ p^d \neq p^h \neq p^r.$$

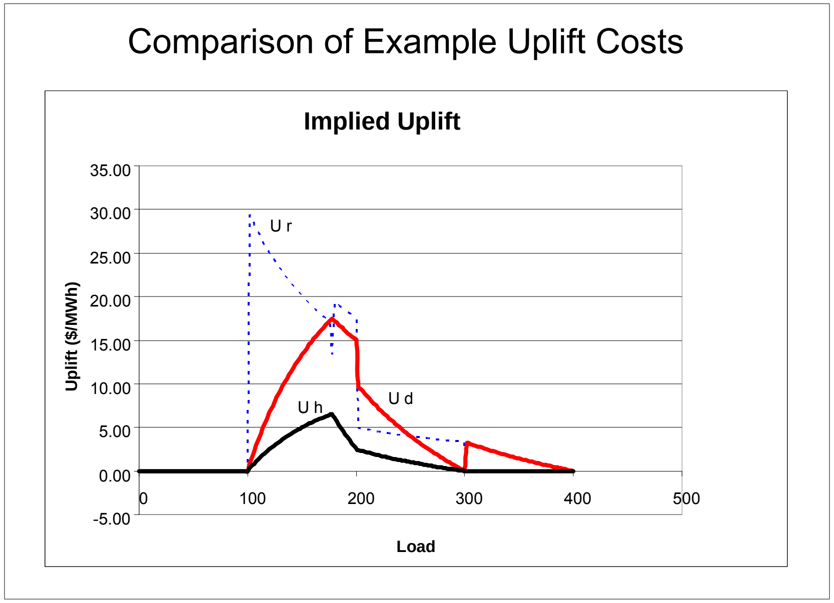
The ELMP model applied to the stylized unit commitment problem employs the dual prices from a particular Lagrangean relaxation.

$$v^h\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf_{\text{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \end{array} \right\} \end{array} \right\} \quad (1)$$

The ELMP price is determined for all periods as the pricing solution to this problem.

Comparing illustrative energy pricing and uplift models.





Both the relaxed dispatchable and ELMP models produce "standard" implied supply curve. The ELMP model produces the minimum uplift.

Alternative pricing models have different features and raise additional questions.

- **Computational Requirements.** Dispatchable model is the easiest case, ELMP model the hardest. But not likely to be a significant issue. Approximate solutions (e.g., NYISO model) may be workable. (Wang, Shanbhag, Zheng, Litvinov, & Meyn, 2013)

- **Network Application.** All models compatible with network pricing and reduce to standard LMP in the convex case.

- **Operating Reserve Demand.** All models compatible with existing and proposed operating reserve demand curves.

- **Solution Independence.** Restricted model sensitive to actual commitment. Relaxed and ELMP models (largely) independent of actual commitment and dispatch.

- **Financial Transmission Rights.** Transmission revenue collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. The FTR uplift amount included in the decomposition of the total uplift that is minimized by the ELMP prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under ELMP pricing.[3]

- **Day-ahead and real-time interaction.** With uncertainty in real-time and virtual bids, expected real-time price is important, and may be similar under all pricing models.

---

[3] (Cadwalader, Gribik, Hogan, & Pope, 2010), "Extended LMP and Financial Transmission Rights."

A "proof of concept" network example from the Midwest Independent System Operator (cont.).

Comparison of Convex Hull Prices and Current Prices

- Convex hull prices at reference node compared to current prices at reference node:

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Current Price	17.9	16.6	16.7	16.7	16.8	16.2	16.4	20.5	20.4	21.2	22.4	21.1
CHP	20.6	19.1	18.5	18.1	18.8	18.5	19.6	20.5	22.9	23.8	24.6	24.3
Difference	2.7	2.5	1.8	1.4	2.0	2.3	3.2	0.0	2.5	2.6	2.2	3.2

Hour	13	14	15	16	17	18	19	20	21	22	23	24
Current Price	20.8	20.5	20.4	20.6	20.5	23.2	48.3	40.0	26.3	21.3	21.1	18.4
CHP	23.1	22.2	20.5	20.2	21.0	25.1	57.0	35.9	29.0	25.5	24.2	23.2
Difference	2.3	1.7	0.1	-0.4	0.5	1.9	8.7	-4.1	2.7	4.2	3.1	4.8

Average
21.8
24.0
2.2

- The average convex hull price at the reference node is \$24.0/MWh compared to the average reference node price of \$21.8/MWh using current pricing structure.
  - The increase of \$2.2/MWh reflects the effects of including start-up and no-load costs in setting prices.

MidwestISO
Energizing the Heartland

13

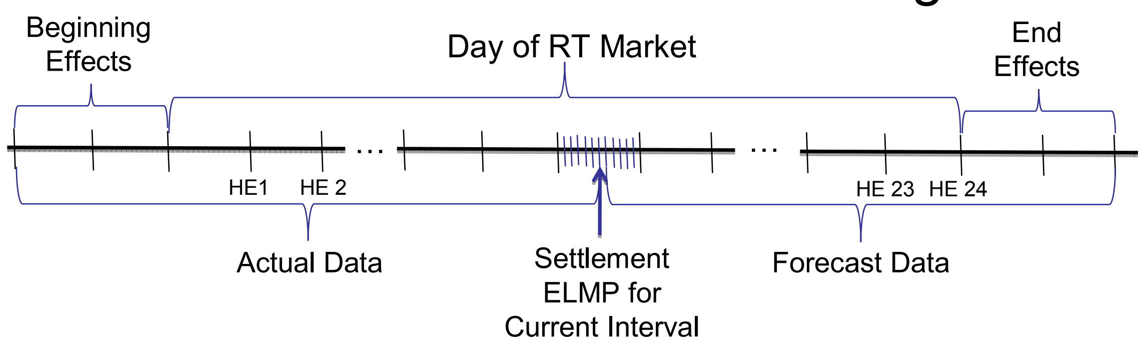
Source: Paul Gribik, "Investigation of Convex Hull Pricing at Midwest ISO," MISO, September 2009.

The day-ahead and real-time pricing present different problems.

- **Day-ahead.** In principle, the unit commitment, dispatch and pricing problems are relatively straightforward.
  - **Multi-period optimization.** The bids and offers are considered over a full day or 24 periods, with possible non-binding look-ahead to deal with end-point conditions.
  - **Virtual bidding.** The standard one-part formulation for virtual transactions adds a potentially large pool of convex offers that make the commitment and dispatch problem "closer" to the simpler convex formulation.
- **Real-time.** Unit commitment dispatch and pricing decisions may appear easier, but the fundamental issues remain.
  - **Unit commitment.** There are still binary commitment decisions, block loaded units, and minimum run-time constraints. The dispatch periods are not fully separable.
  - **No virtual bidding.** There are no virtual bids in the real-time dispatch.
  - **Rolling solutions.** The real-time dispatch model often includes a rolling solution (e.g. 5 minute dispatch) with substantial non-binding look ahead and longer settlement periods.
  - **Updated bidding.** Bids and offers can be revised to reflect changing conditions.

A real-time pricing model involves multiple periods and look ahead. Prices updated throughout the day on a rolling basis.

Calculation of RT ELMPs on Rolling Basis



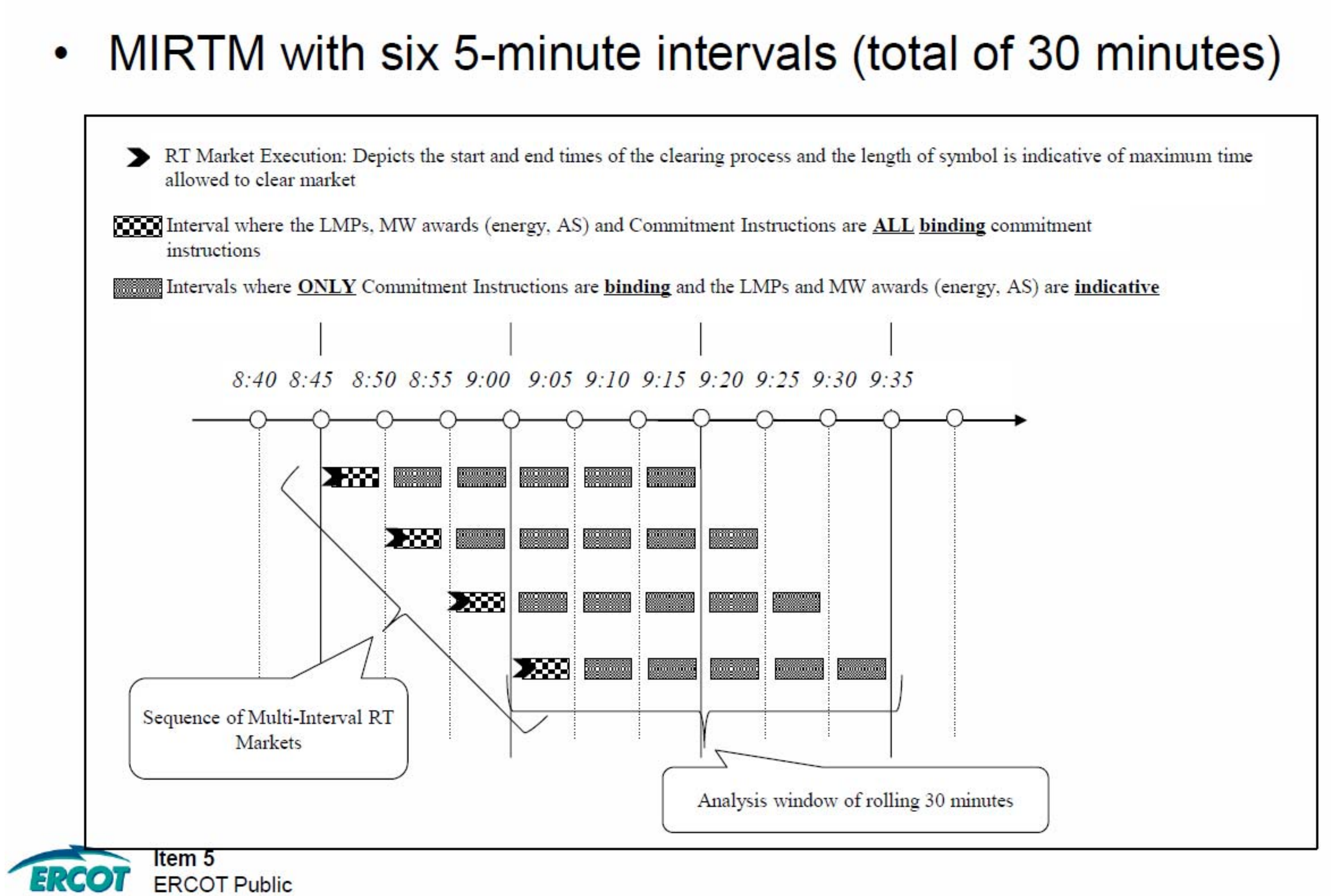
- Calculate ELMPs for a window containing the day of the RT Market.
- As day progresses, current hour is the hour of interest in which we model 5 minute intervals.
  - Other hours may be modeled as single hour long intervals.
  - Could model 5 minute intervals in next hour to provide 5-minute interval-level informational ELMPs.
- Use actual data for past intervals and forecast data for future intervals.
  - ELMPs calculated for current interval used for settlements.
  - ELMPs for other intervals are used for calculation only.
    - Past ELMPs frozen for settlement and future ELMPs are for informational purposes only.

MidwestISO Energizing the Heartland

89

Source: Paul Gribik, “Extended LMP (“ELMP”),” MISO, August 2010.

A real-time pricing model with multiple periods and look ahead is an approach found in most organized markets, or under consideration as in ERCOT:



Joel Mickey, "Multi-Interval Real-Time Market Overview," Board of Directors Meeting, ERCOT Public, October 13, 2015.

A real-time pricing model involves multiple periods and look ahead. Applying and ELMP framework involves choices about what is fixed and what is variable. Natural principles that have been suggested include:

- **Real-time price consistency.** Given that the actual conditions equal the forecast conditions, the methodology produces the same set of prices.
- **Real-time quantity anchor.** Conditioning to reflect evolving economic dispatch and commitment. For example, the pricing dispatch would account for ramping limits that constrain the degree that the pricing dispatch could deviate from the actual dispatch to ensure that the price market-clearing dispatch would always be feasible conditioned on the actual dispatch.
- **Real-time market-clearing quantity anchor.** Conditioning to reflect the evolving market-clearing solution to the pricing problem. With a duality gap, this dispatch deviates from the actual economic dispatch.

For actual physical commitment and dispatch, past decisions are sunk and real-time quantity anchors apply.

The pricing model could allow more flexibility. The Restricted model meets both conditions by always ignoring fixed costs. But ELMP in general incorporates intertemporal constraints and reflects fixed costs of units not committed.

The ELMP is a solution p for the dual or convex hull problem with the loss and transmission limits included as constraints. A “market-clearing” solution is a solution to the inner problem for given prices p.

$$
v^h\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\text{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \bar{F}_{kt}^{\max} & \forall k,t \end{array} \right\} \end{array} \right\}
$$

A sufficient condition for real-time price consistency in ELMP is that all commitment and dispatch variables that are in the economic dispatch or are assigned an uplift payment from the market-clearing solution be included in the pricing model. This allows for slowly pruning those offers that were not committed in either the economic commitment or the market-clearing commitment and are subsequently excluded from retroactive starts ($Excluded_\tau$ **). Hence, the conditional dual pricing model at time** τ **could take as determined prices the prior periods** $p_1^*, p_2^*, \cdots p_{\tau-1}^*$**:**

$$v^\tau\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\text{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\ start_{it} = 0, i \in Excluded_\tau & \end{array} \right\} \\ p_t = p_t^*, t \le \tau - 1 \end{array} \right\}$$

This model also has an interpretation as the ELMP model for a two stage dualization of the complicating constraints. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then dualize this reduced model to find the remaining prices. The corresponding statement of the conditional dispatch problem for which we find the ELMP going forward is:

$$
\begin{aligned}
& v^{\tau}\left(\{\mathbf{y}_t\}\right) \equiv \\
& \inf_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ - \sum_{t \le \tau - 1} \mathbf{p}_t^{*T} \left(\mathbf{d}_t - \mathbf{y}_t\right) \end{array} \right) \\
& \text{subject to} \\
& m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
& -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
& start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
& start_{it} = 0 \text{ or } 1 && \forall i,t \\
& on_{it} = 0 \text{ or } 1 && \forall i,t \\
& \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
& Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t \\
& start_{it} = 0, i \in Excluded_{\tau} \\
& \mathbf{d}_t = \mathbf{y}_t && \forall t \ge \tau.
\end{aligned}
$$

The dual of this problem reduces to a simple solution.

Two-Period Price Illustration

Dual Problem

$$
\begin{aligned}
v(y) = \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad & F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) \\
s.t.\ & \\
& 0 \le x_{a1} \le K_a u_{a1} \\
& 0 \le x_{a2} \le K_a \max(u_{a1},u_{a2}) \\
& 0 \le x_{b1} \le K_b \\
& 0 \le x_{b2} \le K_b \\
& u_{a1} = 0,1 \\
& u_{a2} = 0,1 \\
& x_{a1} + x_{b1} = y_1 \\
& x_{a2} + x_{b2} = y_2.
\end{aligned}
$$

$$
v^h(y) = \sup_p \left\{
\begin{aligned}
& py + \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2},d_1,d_2}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) - pd \\
& s.t. \\
& 0 \le x_{a1} \le K_a u_{a1} \\
& 0 \le x_{a2} \le K_a \max(u_{a1},u_{a2}) \\
& 0 \le x_{b1} \le K_b \\
& 0 \le x_{b2} \le K_b \\
& u_{a1} = 0,1 \\
& u_{a2} = 0,1 \\
& x_{a1} + x_{b1} = d_1 \\
& x_{a2} + x_{b2} = d_2.
\end{aligned}
\right\}
$$

$$
v^h(y) = \underset{p_1,p_2}{Max} \left(\begin{aligned} & p_1 y_1 + p_2 y_2 - \\ & \left[\begin{aligned} & Max\left((p_1 - c)K_b, 0\right) + \\ & Max\left((p_2 - c)K_b, 0\right) + \\ & Max\left((p_1 + p_2)K_a - F_a, 0\right) \end{aligned} \right] \end{aligned} \right)
$$

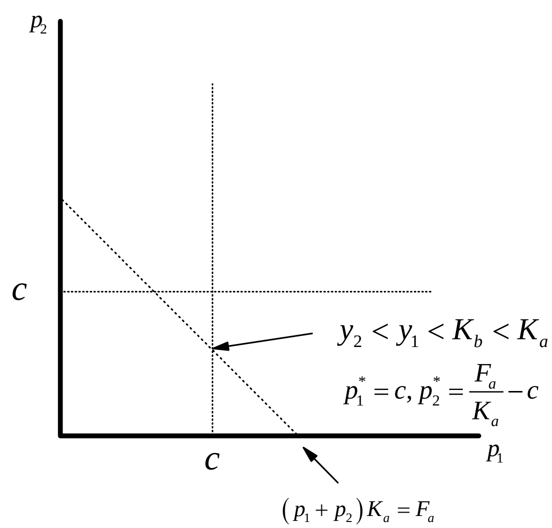
A two period example with low fixed costs illustrates the solution and properties of pricing model. The simplified structure with only fixed costs for one plant and variable costs for the other allows us to determine the solution from the graph of critical regions in the dual space of the prices.

Two-Period Price Illustration

Plant a has only fixed costs,
b has only variable costs.

$$v(y) = \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2})$$

$$\begin{aligned}
&s.t.\\
&0 \le x_{a1} \le K_a u_{a1}\\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2})\\
&0 \le x_{b1} \le K_b\\
&0 \le x_{b2} \le K_b\\
&u_{a1} = 0,1\\
&u_{a2} = 0,1\\
&x_{a1} + x_{b1} = y_1\\
&x_{a2} + x_{b2} = y_2.
\end{aligned}$$



Low Fixed Cost Case

Dual Problem

$$v^h(y) = \underset{p_1,p_2}{Max} \left(\begin{array}{l} p_1 y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\left((p_1 - c)K_b, 0\right) + \\ Max\left((p_2 - c)K_b, 0\right) + \\ Max\left((p_1 + p_2)K_a - F_a, 0\right) \end{array} \right] \end{array} \right)$$

The low fixed cost example may not change prices when the forecast changes.

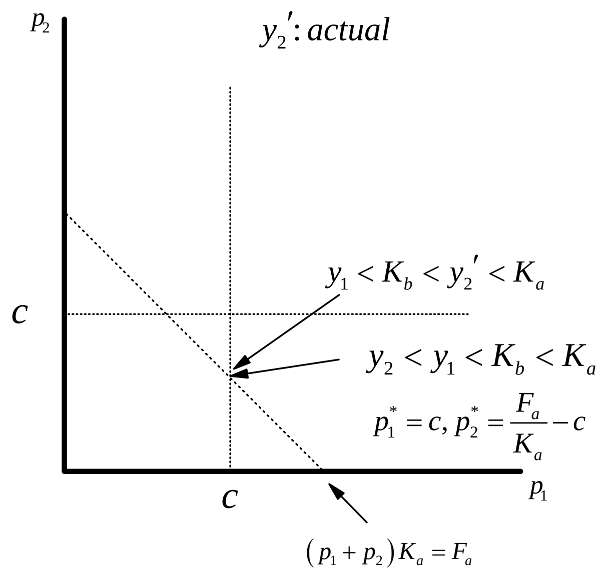
Two-Period Price Illustration

Conditional Price

Plant a has only fixed costs,
b has only variable costs.

y_2: *forecast*

y_2': *actual*



Low Fixed Cost Case

2nd Period Conditional Price Problem

$$v_p^\tau\left(y \middle| p_1^*\right) = \underset{p_2}{Max} \begin{pmatrix} p_1^* y_1 + p_2 y_2' - \\ \begin{bmatrix} Max\left(\left(p_1^* - c\right)K_b, 0\right) + \\ Max\left(\left(p_2 - c\right)K_b, 0\right) + \\ Max\left(\left(p_1^* + p_2\right)K_a - F_a, 0\right) \end{bmatrix} \end{pmatrix}$$

Dual Problem

$$v^h\left(y\right) = \underset{p_1, p_2}{Max} \begin{pmatrix} p_1 y_1 + p_2 y_2 - \\ \begin{bmatrix} Max\left(\left(p_1 - c\right)K_b, 0\right) + \\ Max\left(\left(p_2 - c\right)K_b, 0\right) + \\ Max\left(\left(p_1 + p_2\right)K_a - F_a, 0\right) \end{bmatrix} \end{pmatrix}$$

The two period example with high fixed costs gives has updated prices changing.

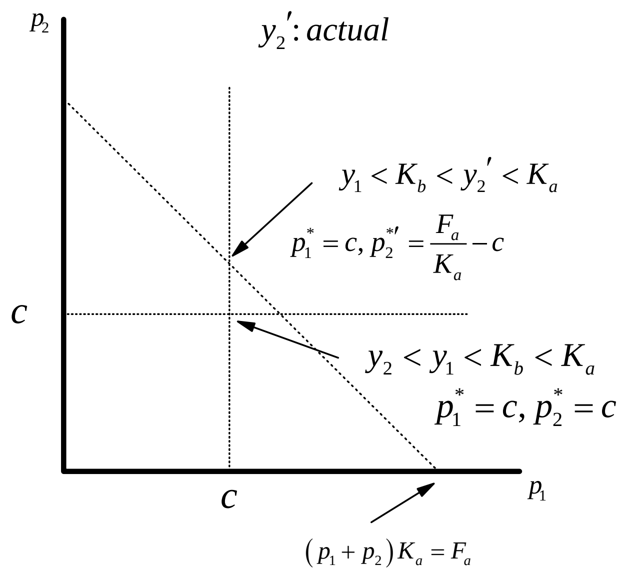
Two-Period Price Illustration

Conditional Price

Plant a has only fixed costs,
b has only variable costs.

y_2: *forecast*

y_2': *actual*



High Fixed Cost Case

2nd Period Conditional Price Problem

$$v_p^{\tau}\left(y \mid p_1^*\right) = \underset{p_2}{Max} \left(\begin{array}{l} p_1^* y_1 + p_2 y_2' - \\ \left[\begin{array}{l} Max\left(\left(p_1^* - c\right) K_b, 0\right) + \\ Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(\left(p_1^* + p_2\right) K_a - F_a, 0\right) \end{array} \right] \end{array} \right)$$

Dual Problem

$$v^h(y) = \underset{p_1, p_2}{Max} \left(\begin{array}{l} p_1 y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\left(\left(p_1 - c\right) K_b, 0\right) + \\ Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(\left(p_1 + p_2\right) K_a - F_a, 0\right) \end{array} \right] \end{array} \right)$$

A proposal for real-time price consistency in ELMP is to fix past decisions in the inner "market clearing" solution, as well as fixing the prices. Hence, the conditional market-clearing pricing model at time τ would take the determined prices $p_1^*, p_2^*, \cdots p_{\tau-1}^*$ and market clearing dispatch $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ for the prior periods as fixed and solve as the pricing model:

$$
v^\tau\left(\{\mathbf{y}_t\}\right) \equiv
\sup_p \left\{
\begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{
\begin{array}{ll}
\inf_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
z^{\tau-1} = z^{*\tau-1} &
\end{array}
\right\} \\
p_t = p_t^*, t \le \tau - 1
\end{array}
\right\}
$$

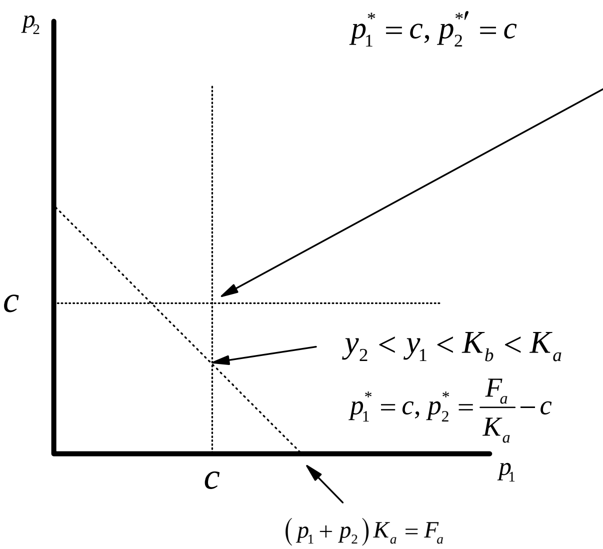
However, the hoped for price consistency depends on separability across periods. The general problem is not separable, and fixing $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ can create price inconsistency.

The conditional market-clearing solution can produce inconsistent prices even when the forecast equals the actual conditions.

Two-Period Price Illustration

Conditional Dispatch and Conditional Price

Plant a has only fixed costs,
b has only variable costs.



Low Fixed Cost Case

2nd Period Conditional Dispatch Problem

$$v_z^{\tau}\left(y \middle| \left(p_1^*, x_{a1}^*, x_{b1}^*, u_{a1}^*\right) = \left(c, 0, y_1, 0\right)\right) = \underset{p_2}{Max} \left(\begin{array}{l} p_1^* y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(p_2 K_a - F_a, 0\right) \end{array} \right] \end{array} \right)$$

2nd Period Conditional Price Problem

$$v_p^{\tau}\left(y \middle| p_1^*\right) = \underset{p_2}{Max} \left(\begin{array}{l} p_1^* y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\left(\left(p_1^* - c\right) K_b, 0\right) + \\ Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(\left(p_1^* + p_2\right) K_a - F_a, 0\right) \end{array} \right] \end{array} \right)$$

An adaptation of the sequential model to the relaxed approximation of the pricing problem presents a relative simply tool. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then we utilize the relaxed model to find the approximate prices:

$$
\begin{aligned}
& v^{r\tau}\left(\{\mathbf{y}_t\}\right) \equiv \\
& \inf_{\text{g,d,on,start}} \begin{pmatrix} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_{t \le \tau - 1} \mathbf{p}_t^{*T}\left(\mathbf{d}_t - \mathbf{y}_t\right) \end{pmatrix} \\
& \text{subject to} \\
& m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
& -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
& start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
& 0 \le start_{it} \le 1 && \forall i,t \\
& 0 \le on_{it} \le 1 && \forall i,t \\
& \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
& Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t \\
& start_{it} = 0, i \in Excluded_\tau \\
& \mathbf{d}_t = \mathbf{y}_t && \forall t \ge \tau.
\end{aligned}
$$

Approximations of an ELMP real-time pricing model would include.

- **Block Loaded Units.** Variabilize the average cost of units that are all on or off.
- **Fixed Cost Allocation.** The UK Pool solution with on and off peak periods.
- **Relaxation Variants.** Combine relaxed formulation with ad hoc fixed cost allocations.
- **Price Conditioning.** Set different windows and horizons for price consistency objective.

An objective is to obtain a workable pricing model.

- **Integrated With Day-Ahead.** Support a two-settlement system and virtual bids equating day-ahead and expected real-time prices.
- **Rolling Real-Time Pricing Updates:** Simplified separable approximations are easiest to implement. Intertemporal interactions and conditional price models present pother tradeoffs.
- **Stakeholder Testing and Verification.** Simple simulations to understand market impacts.[4]

[4] For further details, see the ISONE investigations, (Coutu & White, 2014).

.

References

Bertsekas, D. (1999). Nonlinear Programming. *Athena Scientific*, 1–780. doi:10.1137/1.9780898719383

Bixby, R. E. (2015). *Mixed Integer Programming:The State of the Art*. Retrieved from http://www.ksg.harvard.edu/hepg/Papers/2015/Panel 1_R Bixby.pdf

Bjørndal, M., & Jörnsten, K. (2008). Equilibrium prices supported by dual price functions in markets with non-convexities. *European Journal of Operational Research*, *190*(3), 768–789. Retrieved from http://www.sciencedirect.com/science/article/pii/S0377221707006340

Cadwalader, M., Gribik, P. R., Hogan, W. W., & Pope, S. (2010). Extended LMP and Financial Transmission Rights. Retrieved from http://www.hks.harvard.edu/fs/whogan/CHP_ELMP_FTR_060910.pdf

Cervigni, G., & Perekhodtsev, D. (2013). Wholesale Electricity Markets. In P. Rinci & G. Cervigni (Eds.), *The Economics of Electricity Markets : Theory and Policy*. Edward Elgar. Retrieved from http://www.e-elgar.com/bookentry_main.lasso?id=14440

Coutu, R., & White, M. (2014). *Real-Time Price Formation: Technical Session #5*. Retrieved from http://www.iso-ne.com/support/training/courses/energy_mkt_ancil_serv_top/price_information_technical_session5.pdf

Gribik, P. R., Hogan, W. W., & Pope, S. L. (2007). Market-Clearing Electricity Prices and Energy Uplift. Retrieved from http://www.hks.harvard.edu/fs/whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf

Hobbs, B. F., Rothkopf, M. H., & O'Neill, R. P. (2014). *The next generation of electric power unit commitment models* (Vol. 36). Springer. Retrieved from https://books-google-com.ezp-prod1.hul.harvard.edu/books?id=xx_6sgEACAAJ&lr&source=gbs_book_other_versions

Hogan, W. W. (1992). Contract networks for electric power transmission. *Journal of Regulatory Economics*, *4*(3), 211–242. Retrieved from http://ezp-prod1.hul.harvard.edu/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=bth&AN=16580807&site=ehost-live&scope=site

Hogan, W. W. (2002). Financial transmission right formulations. Retrieved from http://www.hks.harvard.edu/fs/whogan/FTR_Formulations_033102.pdf

International Energy Agency. (2007). *Tackling Investment Challenges in Power Generation in IEA Countries: Energy Market Experience*. *Energy*. Paris. Retrieved from http://www.iea.org/publications/freepublications/publication/tackling_investment.pdf

Jha, A., & Wolak, F. A. (2015). *Testing for Market Efficiency with Transactions Costs: An Application to Convergence Bidding in Wholesale Electricity Markets*. Retrieved from http://web.stanford.edu/group/fwolak/cgi-bin/sites/default/files/CAISO_VB_draft_VNBER_final.pdf

Lavaei, J., & Low, S. H. (2012). Zero duality gap in optimal power flow problem. *IEEE Transactions on Power Systems*, *27*(1), 92–107. doi:10.1109/TPWRS.2011.2160974

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press.

O'Neill, R. P., Sotkiewicz, P. M., Hobbs, B. F., Rothkopf, M. H., & Stewart, W. R. (2005). Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research*, *164*(1), 269–285. Retrieved from http://www.personal.psu.edu/sjh11/ACCTG597E/Class03/ONeillEtAlfEJoOR05.pdf

PJM. (2015). *Virtual Transactions in the PJM Energy Markets*. Retrieved from http://www.pjm.com/~/media/committees-groups/committees/mrc/20151022/20151022-item-01-virtual-transactions-in-the-pjm-energy-markets-whitepaper.ashx

Schweppe, F., Caramanis, M. C., Tabors, R. D., & Bohn, R. E. (1988). *Spot pricing of electricity*. Kluwer Academic Publishers. Retrieved from http://books.google.com/books?id=Sg5zRPWrZ_gC&pg=PA265&lpg=PA265&dq=spot+pricing+of+electricity+schweppe&source=bl&ots=1MIUfKBjBk&sig=FXe_GSyf_V_fcIuTmUtH7mKO_PM&hl=en&ei=Ovg7Tt66DO2x0AH50aGNCg&sa=X&oi=book_result&ct=result&resnum=3&ved=0CDYQ6AEwAg#v=onepage&q&f=false

Sioshansi, R., O'Neill, R. P., & Oren, S. S. (2008). Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets. *IEEE Transactions on Power Systems*, *23*(2), 344–352. doi:10.1109/TPWRS.2008.919246

Smeers, Y. (2003). Market incompleteness in regional electricity transmission. Part II: The forward and real time markets. *Networks and Spatial Economics*, *3*(2), 175–196. Retrieved from http://link.springer.com.ezp-prod1.hul.harvard.edu/article/10.1023/A:1023916120177

Stoft, S. E. (2002). *Power System Economics: Designing Markets for Electricity*. Wiley. Retrieved from https://books.google.com/books?id=DrTEsqJRKrYC

Thomson, R. G. (1995). *The electric power industry: deregulation and market structure* (No. 95-004WP). Retrieved from https://dspace.mit.edu/bitstream/handle/1721.1/50182/35719387.pdf?sequence=1

Wang, G., Negrete-pincetic, M., Kowli, A., Shafieepoorfard, E., Meyn, S., & Shanbhag, U. V. (2012). Dynamic Competitive Equilibria in Electricity Markets. In A. Chakrabortty & M. D. Ilić (Eds.), *Control and Optimization Methods for Electric Smart Grids* (pp. 35–62). Springer. doi:10.1007/978-1-4614-1605-0

Wang, G., Shanbhag, U. V., Zheng, T., Litvinov, E., & Meyn, S. (2013). An extreme-point subdifferential method for convex hull pricing in energy and reserve markets-Part II: Convergence analysis and numerical performance. *IEEE Transactions on Power Systems*, *28*, 2121–2127. doi:10.1109/TPWRS.2012.2229303

William W. Hogan is the Raymond Plank Professor of Global Energy Policy, John F. Kennedy School of Government, Harvard University. This paper draws on research for the Harvard Electricity Policy Group and for the Harvard-Japan Project on Energy and the Environment. The author is or has been a consultant on electric market reform and transmission issues for Allegheny Electric Global Market, American Electric Power, American National Power, Aquila, Atlantic Wind Connection, Australian Gas Light Company, Avista Corporation, Avista Utilities, Avista Energy, Barclays Bank PLC, Brazil Power Exchange Administrator (ASMAE), British National Grid Company, California Independent Energy Producers Association, California Independent System Operator, California Suppliers Group, Calpine Corporation, CAM Energy, Canadian Imperial Bank of Commerce, Centerpoint Energy, Central Maine Power Company, Chubu Electric Power Company, Citigroup, City Power Marketing LLC, Cobalt Capital Management LLC, Comision Reguladora De Energia (CRE, Mexico), Commonwealth Edison Company, COMPETE Coalition, Conectiv, Constellation Energy, Constellation Energy Commodities Group, Constellation Power Source, Coral Power, Credit First Suisse Boston, DC Energy, Detroit Edison Company, Deutsche Bank, Deutsche Bank Energy Trading LLC, Duquesne Light Company, Dyon LLC, Dynegy, Edison Electric Institute, Edison Mission Energy, Electricity Corporation of New Zealand, Electric Power Supply Association, El Paso Electric, Energy Endeavors LP, Exelon, Financial Marketers Coalition, FirstEnergy Corporation, FTI Consulting, GenOn Energy, GPU Inc. (and the Supporting Companies of PJM), GPU PowerNet Pty Ltd., GDF SUEZ Energy Resources NA, Great Bay Energy LLC, GWF Energy, Independent Energy Producers Assn, ISO New England, Koch Energy Trading, Inc., JP Morgan, LECG LLC, Luz del Sur, Maine Public Advocate, Maine Public Utilities Commission, Merrill Lynch, Midwest ISO, Mirant Corporation, MIT Grid Study, Monterey Enterprises LLC, MPS Merchant Services, Inc. (f/k/a Aquila Power Corporation), JP Morgan Ventures Energy Corp., Morgan Stanley Capital Group, National Independent Energy Producers, New England Power Company, New York Independent System Operator, New York Power Pool, New York Utilities Collaborative, Niagara Mohawk Corporation, NRG Energy, Inc., Ontario Attorney General, Ontario IMO, Ontario Ministries of Energy and Infrastructure, Pepco, Pinpoint Power, PJM Office of Interconnection, PJM Power Provider (P3) Group, Powerex Corp., Powhatan Energy Fund LLC, PPL Corporation, PPL Montana LLC, PPL EnergyPlus LLC, Public Service Company of Colorado, Public Service Electric & Gas Company, Public Service New Mexico, PSEG Companies, Red Wolf Energy Trading, Reliant Energy, Rhode Island Public Utilities Commission, Round Rock Energy LP, San Diego Gas & Electric Company, Secretaría de Energía (SENER, Mexico), Sempra Energy, SESCO LLC, Shell Energy North America (U.S.) L.P., SPP, Texas Genco, Texas Utilities Co, Tokyo Electric Power Company, Toronto Dominion Bank, Transalta, TransAlta Energy Marketing (California), TransAlta Energy Marketing (U.S.) Inc., Transcanada, TransCanada Energy LTD., TransÉnergie, Transpower of New Zealand, Tucson Electric Power, Twin Cities Power LLC, Vitol Inc., Westbrook Power, Western Power Trading Forum, Williams Energy Group, Wisconsin Electric Power Company, and XO Energy. The views presented here are not necessarily attributable to any of those mentioned, and any remaining errors are solely the responsibility of the author. (Related papers can be found on the web at www.whogan.com).