Pricing expropriation risk in natural resource contracts –
A real options approach

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Abstract
We develop a model for pricing expropriation risk in natural resource projects, in particular an oil field. The government is viewed as holding an American-style option to expropriate the oil field, but having higher production costs that a private firm. The dynamics of key variables – the spot price, futures prices and volatility – is described by a model proposed and estimated in Trolle and Schwartz (2007). We find that the value of the expropriation option increases with the spot price, the slope of the futures curve and futures (and spot) price volatility, while it decreases with the tax rate on corporate profits and with the production cost differential between the firm and the government. For reasonable parameter values and under market conditions not too different from what has been seen in recent years, the value of the expropriation option can be substantial.

JEL Classification: G13

Keywords: Real options, crude oil contracts, expropriation risk

Preliminary draft: September 2007

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1 Introduction

There are many dimensions to the study of expropriation risk in natural resources including political, environmental, sociological and economic issues. In this paper we abstract form most of these aspects and concentrate on some of the important economic tradeoffs that arise from the government having an option to expropriate the resource.

We show how to use the real options approach to value a natural resource project, in particular an oil field, exposed to expropriation risk.\footnote{For a comprehensive exposition of the real options approach to valuation, see Dixit and Pindyck (1994).} We view the government as holding an option to expropriate the oil field.\footnote{Throughout the paper, we consider the risk of the government taking over the entire oil field. The framework could be modified to consider the risk of a partial expropriation through a forced renegotiation of existing contracts involving, for instance, an increase in taxes or the government taking a certain stake in the oil field.} We assume that the government faces the following tradeoff: The benefit of expropriating is that it will receive all profits from the oil field rather than just a fraction of profits through taxes. The cost of expropriating is that a private firm can produce oil more cost-efficiently than the government and that the government pays a compensation to the firm when expropriating.\footnote{Other costs might include a reduction in the overall level of foreign direct investments and a higher cost of foreign capital due to reduced investor confidence. Later in the paper, we provide an example, where the fixed expropriation costs paid by the government are “dead-weight” costs which are not paid to the firm.}

The dynamics of the crude-oil spot price, futures prices and volatility that we use for the analysis is described by a model proposed in Trolle and Schwartz (2007). This model has several attractive features. Futures prices are driven by two factors, with one factor affecting the spot price of the commodity and another factor affecting the slope of the futures curve though the cost of carry. Futures (and spot) price volatility is stochastic and is driven by a third factor which implies that options are driven by three factors and that the model features “unspanned stochastic volatility” (that is, volatility risk cannot be completely hedged by trading in futures contracts) consistent with the data. The model has quasi-analytical prices of European-style options on futures contracts enabling fast calibration to liquid plain-vanilla exchange-traded derivatives. Finally, the dynamics of the futures curve can be described in terms of four state variables (three stochastic and one deterministic) jointly constituting an affine state vector which makes the model ideally suited for pricing complex commodity derivatives, including real options, such as the expropriation option, by simulation.

In Trolle and Schwartz (2007), the model was estimated and tested on NYMEX crude-oil
derivatives using an extensive panel data set of 45,517 futures prices and 233,104 option prices from January 2, 1990 until May 18, 2006, ensuring that the model provides a realistic description of the dynamics of the crude-oil market. This allows us to make not only qualitative, but also quantitative, predictions about the value of the expropriation option in various scenarios.

The expropriation option is an American-style option, since it can be exercised at any time during the life of the project. To value the option by simulation we use the Least Squares Monte Carlo (LSM) approach developed by Longstaff and Schwartz (2001). At every point in time, the government must compare the value of immediate exercise (expropriation) with the conditional expected value (under the risk neutral measure) from continuation. The conditional expected value of continuation, for each simulated path at each point in time, can be obtained from the fitted value of the linear regression of the discounted value (at the risk free rate) of the cash flows obtained from the simulation following the optimal exercise policy in the future, on a set of basis functions of the state variables. It is a recursive procedure starting from the maturity of the option and the outcome is the optimal exercise time for each path in the simulation. Knowing the optimal exercise time for each path, the expropriation option can then be easily valued. We can also estimate the value of the oil field to the government and to the firm both in the presence and absence of expropriation risk.

We find that, for a given contractual arrangement, the value of the expropriation option increases with the spot price, the slope of the futures curve and futures (and spot) price volatility. For a given set of state variables the value of the expropriation option decreases with the tax rate on corporate profits and with the production cost differential between the firm and the government. Under certain conditions, expropriation risk will have a substantial impact on the value of the oil field to the government and to the firm. From the firm’s point of view there is an “optimal” tax rate on corporate profits, that may be quite high in order to reduce the incentive for the government to expropriate.

The increase in the field’s value to the government due to expropriation risk is always smaller than the decrease in the field’s value to the firm, since oil is extracted at a higher cost when the government expropriates. In this sense, there is a “welfare loss” associated with the possibility of expropriation. The reduction in field’s value to the firm due to expropriation risk is exactly matched by a reduction in the amount that the firm will be willing to bid during the process when the government auctions off the lease for the field. Hence, the total value that the government can extract from the field is smaller in the presence of expropriation risk than in the absence of expropriation risk. It would therefore be optimal for the government
to commit itself to not expropriate the field, although such a commitment is not believable in countries without a credible legal framework to enforce contracts.

In our analysis, we will abstract from the various operational options that are typically imbedded in natural resource projects. These include options to adjust production as prices increase or decrease and the option to abandon the project if prices become too low. Due to the flexibility of the LSM approach, such options could be incorporated into the analysis. However, in the interest of parsimony, and because a wide variety of operational options have already been analyzed in the literature (see e.g. Brennan and Schwartz (1985) for an early paper), we do not include them here.

A couple of other papers also view expropriation risk through the lens of option pricing. These include Mahajan (1990) and Clark (2003) who both value an American-style expropriation option. However, in these papers the underlying models of uncertainty are highly stylized in order to obtain closed-form solutions for the expropriation option.

The structure of the paper is as follows. Section 2 discusses the pricing of expropriation risk. Section 3 analyzes a stylized illustrative example. Section 4 concludes. Appendix A briefly describes the model proposed and estimated in Trolle and Schwartz (2007). Appendix B describes the LSM procedure used for pricing the expropriation option.

2 Pricing expropriation risk

We assume that the oil field has a life of $T$ years. For the purpose of valuation we divide the $T$ years into $N$ periods, each with a length of $\Delta t = T/N$, and define $t_n = n\Delta t, n = 0, 1, \ldots, N$. We assume that oil produced during period $n$, i.e. from $t_{n-1}$ to $t_n$ is sold at the end of the period. The amount sold at time $t_n$ is denoted $Y(t_n)$.

Let $S(t_i)$ be the time-$t_i$ spot price of crude-oil and $F(t_i, t_n)$ be the time-$t_i$ price of a futures contract maturing at time $t_n$. The dynamics of $S(t_i)$ and $F(t_i, t_n)$ are given by the model proposed and estimated in Trolle and Schwartz (2007). For completeness, this model is summarized in Appendix A and estimates of its risk-neutral parameters (reproduced from Trolle and Schwartz (2007), Table 5) are given in Table 1.

Let $r$ be the (constant) interest rate, $\tau$ the tax rate on corporate profits and $C_{firm}$ and $C_{gov}$ the cost of producing one barrel of oil for the firm and the government, respectively. We assume that the firm produces oil more cost-efficiently that the government, i.e. $C_{firm} \leq C_{gov}$, and that the government will pay a compensation $K(t_i)$ to the firm if expropriating the oil...
field at time $t_i$.

Harrison and Kreps (1979), Harrison and Pliska (1981) and others have shown that the absence of arbitrage implies the existence of a probability distribution such that securities are priced based on their discounted (at the risk free rate) expected cash flows, where the expectation is taken under this risk-neutral probability measure (also called the “equivalent martingale measure”). When future contracts exist, futures prices are the expected spot prices at the maturities of the futures contracts under this risk-neutral measure.

If the government expropriates the oil field at time $t_i$ it will lose future tax receipts from the firm and pay the compensation. This has a present value of

$$V_{\text{cost}}(t_i) = \sum_{n=i+1}^{N} e^{-r(t_n-t_i)} (F(t_i,t_n) - C_{\text{firm}}) \times Y(t_n) \times \tau + K(t_i). \quad (1)$$

Instead it will receive all future profits from the field, which now has a higher production cost, with a present value of

$$V_{\text{gain}}(t_i) = \sum_{n=i+1}^{N} e^{-r(t_n-t_i)} (F(t_i,t_n) - C_{\text{gov}}) \times Y(t_n). \quad (2)$$

Let $\Pi(t_i)$ denote the (undiscounted) payoff from exercising the expropriation option at time $t_i$. It is given by

$$\Pi(t_i) = \max (V_{\text{gain}}(t_i) - V_{\text{cost}}(t_i), 0). \quad (3)$$

The option is of the American type since it can be exercised at any time before the oil field has been depleted. In particular, we assume that it can be exercised at time $t_n, n = 1, ..., N - 1$.

Let $P(t_i)$ denote the value of the option at time $t_i$, given that it has not already been exercised. At time $t_{N-1}$ the option value is simply

$$P(t_{N-1}) = \Pi(t_{N-1}). \quad (4)$$

At time $t_i, i = 1, ..., N - 2$ the value is given by

$$P(t_i) = \max_{n=i, ..., N-1} E_{t_i}^{\mathcal{Q}} \left[ e^{-r(t_n-t_i)} \Pi(t_n) \right]$$

$$= \max \left( \Pi(t_i), E_{t_i}^{\mathcal{Q}} \left[ e^{-r(t_{i+1}-t_i)} P(t_{i+1}) \right] \right). \quad (5)$$

\[4\]We exclude current time $t_0$ from the set of exercise opportunities. We also exclude $t_N$ since at that time the oil field has been depleted.
In other words, prior to $t_{N-1}$ the option value is equal to the maximum of exercising the option immediately and the (risk-neutral) expected discounted value of keeping the option alive. This means that we can value the option by a backward iterative procedure starting with (4) to obtain $P(t_{N-1})$, then applying (5) recursively to obtain $P(t_{N-2})$, $P(t_{N-3})$ and so on until we get to $P(t_1)$. Then $P(t_0) = e^{-r(t_1-t_0)}P(t_1)$.

The main problem is how to compute the conditional expectation in (5). This is a non-trivial matter with a high-dimensional state vector such as the one used in this paper. A simple and powerful procedure for pricing American options was suggested by Longstaff and Schwartz (2001). It is a simulation-based procedure called Least Squares Monte Carlo. We describe the procedure in more detail in Appendix B. It yields an estimate of the option price as well as an estimate of the optimal exercise strategy.

Suppose we simulate $M$ paths. Let $t_{Z_m}$ denote the estimated time of expropriation along path $m$ (with $Z_m = N$ if no expropriation takes place) and $S_m(t_n)$ the spot price at time $t_n$ along path $m$. Then, the value of the oil field to the firm in the presence of expropriation risk is

$$V_{firm}^{exp}(t_0) = \frac{1}{M} \sum_{m=1}^{M} \left( \sum_{n=1}^{Z_m} e^{-r(t_n-t_0)} (S_m(t_n) - C_{firm}) \times Y(t_n) \times (1 - \tau) + e^{-r(t_{Z_m}-t_0)} K(t_{Z_m}) \right),$$

(6)

while the value of the oil field to the government is

$$V_{gov}^{exp}(t_0) = \frac{1}{M} \sum_{m=1}^{M} \left( \sum_{n=1}^{Z_m} e^{-r(t_n-t_0)} (S_m(t_n) - C_{firm}) \times Y(t_n) \times \tau - e^{-r(t_{Z_m}-t_0)} K(t_{Z_m}) + \sum_{n=Z_m+1}^{N} e^{-r(t_n-t_0)} (S_m(t_n) - C_{gov}) \times Y(t_n) \right).$$

(7)

In the absence of expropriation risk, the value of the oil field to the firm and to the government are given analytically. The value to the firm is

$$V_{firm}(t_0) = \sum_{n=1}^{N} e^{-r(t_n-t_0)} (F(t_0,t_n) - C_{firm}) \times Y(t_n) \times (1 - \tau),$$

(8)

while the value to the government is

$$V_{gov}(t_0) = \sum_{n=1}^{N} e^{-r(t_n-t_0)} (F(t_0,t_n) - C_{firm}) \times Y(t_n) \times \tau.$$  

(9)

5Note that we may alternatively compute the value of the oil field to the government in the presence of expropriation risk as the value in the absence of expropriation risk plus the value of the expropriation option, i.e. $V_{gov}^{exp}(t_0) = V_{gov}(t_0) + P(t_0)$. 

5
3 Illustrative example

3.1 The base-line parameters

We consider a small-sized oil field with an annual production of 1.0 million barrels for ten years.\(^6\) In case of expropriation, we assume that the government pays the firm a compensation of 10.0 million USD times the remaining life of the oil field measured in years.\(^7\) We assume that the firm has a production cost of 10 USD/bl. In the base-line case, we assume that the government has a production cost of 20 USD/bl and that the tax rate is 60 percent (the sensitivity of the results to these assumptions will be examined later). We further assume an interest rate of 5 percent and that expropriation can take place monthly. In the notation of Section 2 we have \(T = 10\), \(N = 120\), \(Y(t_i) = \Delta t \times 1.0\) million, \(C_{\text{firm}} = 10\) USD/bl, \(C_{\text{gov}} = 20\) USD/bl, \(K(t_i) = (T - t_i) \times 10.0\) million USD, \(\tau = 0.60\) and \(r = 0.05\).

In the following we will focus on four statistics: 1) The dollar value of the expropriation option, i.e. \(P(t_0)\). 2) The “welfare loss”, i.e. \((V_{\text{gov}}(t_0) + V_{\text{firm}}(t_0)) - (V_{\text{exp}}^{\text{gov}}(t_0) + V_{\text{exp}}^{\text{firm}}(t_0))\). The “welfare loss” arises from the fact that the increase in the value to the government is always smaller than the decrease in the value to the firm, since oil is extracted at a higher cost when the government expropriates. 3) The percentage increase in the oil field’s value to the government when taking into account the expropriation possibility, i.e. \(V_{\text{exp}}^{\text{gov}}(t_0)/V_{\text{gov}}(t_0) - 1\). 4) The percentage decrease in the oil field’s value to the firm when taking into account the expropriation possibility, i.e. \(- (V_{\text{exp}}^{\text{firm}}(t_0)/V_{\text{firm}}(t_0) - 1)\).

3.2 Results

We first investigate how these key statistics depend on the state variables. Figure 1 shows the state variables over the period January 2, 1990 to May 18, 2006 as estimated by Trolle and Schwartz (2007). \(S(t)\) is the spot price, \(x(t)\) determines the slope of the futures curve through the cost of carry, \(v(t)\) determines futures (and spot) price volatility and \(\phi(t)\) is a locally deterministic state variable that enables the dynamics of the futures curve to be described by a Markov process.

We first vary \(S(t)\) and \(x(t)\), holding \(v(t)\) constant at its long-run mean of 2.79 over the

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\(^6\)For simplicity, we assume that production is constant for 10 years. However, the procedure can handle any production profile.

\(^7\)For instance, if the government expropriates the oil field after seven years the firm will receive a compensation of 30.0 million USD.
sample. Based on Figure 1, a realistic range for $S(t)$ is 20 USD/bl to 80 USD/bl while a realistic range for $x(t)$ is -3 to 3. To give a sense of how $S(t)$ and $x(t)$ impacts the futures curve, Figure 2 shows the futures curves at four extreme combinations of $S(t)$ and $x(t)$: (20 USD/bl, -3), (20 USD/bl, 3), (80 USD/bl, -3) and (80 USD/bl, 3). Clearly, the futures curve is strongly in backwardation when $x(t) = -3$, while the futures curve is strongly in contango when $x(t) = 3$. Since the correlation between innovations to $S(t)$ and $x(t)$ is strongly negative (the estimate of $\rho_{12}$ is -0.88 in Table 1) it is most likely that $x(t)$ decreases as $S(t)$ increases. That is, the futures curve is most likely to be in contango when the spot price is low and in backwardation when the spot price is high.

Figure 3 shows how the key statistics depend on $S(t)$ and $x(t)$. Not surprisingly, the dollar value of the expropriation option, the “welfare loss”, and the increase (decrease) of the oil field’s value to the government (firm) all rise with $S(t)$ and $x(t)$. When the spot price is low there is little incentive to expropriate given that the government has higher production costs and has to pay a fixed penalty. The incentive is even smaller if the futures curve is in backwardation, since in that case spot prices are expected (under the risk-neutral measure) to decrease in the future. In contrast, when the spot price is high and the futures curve is in contango, the incentive to expropriate is higher since the prospect of receiving all the profits rather than a fraction of the profits may dominate the increase in production costs and the fixed penalty. For high spot prices and upward-sloping futures curves, the value of the expropriation option and the “welfare loss” are very large. Keep in mind, though, that a scenario where the futures curve is strongly in contango at high spot prices is not very likely given the historical dynamics.

To give a sense of expropriation risk under reasonable market conditions we analyze the following three specific dates in the sample: October 11, 1990, December 21, 1998 and April 21, 2006 (in Figure 1 these dates are marked by vertical grey lines). The first date is the date when the spot price reaches its maximum during the first Gulf War and also corresponds to the date where $x(t)$ attains its minimum during the sample period. The second date is the date when the spot price reaches its minimum and $x(t)$ reaches its maximum during the sample period. These two dates clearly illustrates the inverse relationship that normally exist between the spot price and the slope of the futures curve. The last date corresponds to the date when the spot price reaches its maximum during the sample period. Note that the futures curve is not strongly backwardated at this date reflecting the fact that the slope of the futures curve also exhibits variation that is independent of the spot price. Table 2 shows the values of the
state variables and a number of statistics related to the possibility of expropriation on these three dates. On the first two dates, the value of the expropriation option and the “welfare loss” are very small. On the third date, however, expropriation risk is high. The value of the expropriation option is 29 million USD and the valuation of the field is significantly affected.\(^8\) The value of the field to the firm (government) is 198 (297) million USD in the absence of expropriation risk compared with 123 (326) million USD in the presence of expropriation risk. For the firm, this is a decrease in value of almost 38 percent. The “welfare loss” is almost 46 million USD.

We next vary \(S(t)\) and \(v(t)\), holding \(x(t)\) constant at zero (corresponding to a slightly upward-sloping futures curve). Figure 1 shows that \(v(t)\) reached almost 20 at the beginning of “Operation Desert Storm” in 1991 – a time of extreme market stress. Therefore we vary \(v(t)\) between 0 and 20. Figure 4 shows how the key statistics depend \(S(t)\) and \(v(t)\). Consistent with standard option pricing theory, the dollar value of the expropriation option, and therefore the increase in the oil field’s value to the government, rise with volatility. However, the oil field’s value to the firm and the “welfare loss” are not necessarily increasing in volatility. In fact, for high spot prices they are decreasing in volatility since the likelihood of expropriation may decrease.

We now investigate how, for a given set of state variables, the statistics depend on the tax rate and cost-differential. In this case we set \(S(t) = 50, x(t) = 0\) and \(v(t) = 2.79\). Figure 5 shows how the key statistics depend \(\tau\) and \(C_{gov}\), when we vary \(\tau\) between 20 percent and 100 percent and \(C_{gov}\) between 10 USD/bl (i.e. equal to \(C_{firm}\)) and 30 USD/bl. The dollar value of the expropriation option and the increase (decrease) of the oil field’s value to the government (firm) all fall with \(\tau\) and \(C_{gov}\). When the tax rate is 100 percent the government never expropriates, since it receives all profits from the oil field and cannot produce cheaper than the firm. For low tax rates, the possibility of expropriation has a very significant impact on the valuation of the oil field. For instance, when \(\tau = 20\) percent and \(C_{gov} = 20\) USD/bl, the oil field’s value to the firm (government) is 259 (65) million USD in the absence of expropriation risk compared with 109 (147) million USD in the presence of expropriation risk as the field will almost certainly get expropriated. The expropriation option is worth 82 million USD.

For a given \(C_{gov}\), the “welfare loss” decreases with \(\tau\) since the expropriation likelihood decreases with \(\tau\). However, for a given \(\tau\), the “welfare loss” as a function of \(C_{gov}\) is “humped-

\(^8\)The risk-neutral likelihood that the field will get expropriated during its 10 year lifespan is 85 percent.
shaped”. The reason is that there are two opposing forces: although the expropriation likelihood decreases with $C_{gov}$, the inefficiency is greater when expropriation occurs. If $\tau = 100$ percent (in which case the government never expropriates) or $C_{gov} = 10$ USD/bl (in which case the government produces as efficiently as the firm), the “welfare loss” is zero. In some cases, however, the “welfare loss” is substantial. For instance, when $\tau = 20$ percent and $C_{gov} = 20$ USD/bl, the “welfare loss” is almost 68 million USD.

In the absence of expropriation risk, it is optimal for the firm to negotiate the most favorable terms under which to exploit the oil field. In our case, this means obtaining the lowest possible tax rate. However, in the presence of expropriation risk, the firm must take into account that the incentive to expropriate is higher for lower tax rates. Figure 6, Panel A shows the oil field’s value to the firm as a function of $\tau$ and $C_{gov}$. For low values of $C_{gov}$ the “optimal” tax rate that maximizes the project’s value to the firm is quite high around 50 percent. As $C_{gov}$ increases the “optimal” tax rate decreases, since the incentive to expropriate decreases.

So far we have assumed that the fixed cost of expropriation paid by the government, $K$, is a compensation to the firm. Alternatively, we may assume that the expropriation cost is not paid to the firm but instead is a “dead-weight” cost which arises from other sources, such as a decrease in the overall level of foreign direct investments.\(^9\) This modification does not change the value of the expropriation option, but does affect the oil field’s value to the firm, as well as the “welfare loss”. Figure 6, Panel B shows the oil field’s value to the firm as a function of $\tau$ and $C_{gov}$ when $K$ is not received by the firm. In this case, there is a large decrease in firm value when expropriation is likely. Also, the “optimal” tax rate is even higher than before and can be as large as 80 percent for low values of $C_{gov}$.

Suppose $t_0$ is the time when the government auctions off the lease for the oil field. Furthermore, assume that extraction requires an initial investment of $I$. In the absence of expropriation risk, a firm would be willing to pay up to $V_{firm}(t_0) - I$ for the lease, while in the presence of expropriation risk, it would only be willing to pay up to $V_{firm}^{exp}(t_0) - I$.\(^10\) If the government could commit itself, ex-ante, not to expropriate, it would be able to extract a value of up to $V_{gov}(t_0) + V_{firm}(t_0) - I$ from the oil field. However, in the absence of a legal framework to enforce contracts, such a commitment is not credible, since, ex-post, after the firm has paid for the lease and made the initial investment, it will be optimal for the government to renege on its promise in some states of the world. A rational firm will anticipate this behavior and the

\(^9\)This implies that the term $K(t_{zm})$ disappears from equation (6) but remains in equation (7).

\(^10\)For simplicity, we abstract from the option to defer the initial investment.
maximum value that the government can extract from the field will be \( V^{\text{exp}}_{\text{gov}}(t_0) + V^{\text{exp}}_{\text{firm}}(t_0) - I \). Ultimately, therefore, it is the government that bears the “welfare loss” associated with the possibility of expropriation.

4 Conclusion

In this paper we develop a model for pricing expropriation risk in natural resource projects in general and an oil field in particular. The government is viewed as holding an American-style option to expropriate the oil field, but having higher production costs than a private firm. The dynamics of key variables – the spot price, futures prices and volatility – is described by a model proposed and estimated in Trolle and Schwartz (2007). The expropriation option is valued by simulations using the LSM approach developed by Longstaff and Schwartz (2001).

We find that the value of the expropriation option increases with the spot price, the slope of the futures curve and futures (and spot) price volatility, while it decreases with the tax rate on corporate profits and with the production cost differential between the firm and the government. For reasonable parameter values and under market conditions not too different from what has been seen in recent years, the value of the expropriation option can be substantial. In order to reduce the incentive for the government to expropriate, from the firm’s point of view there is an “optimal” tax rate on corporate profits which may be quite high. Furthermore, when firms act rationally, the possibility of expropriation leads to a decrease in the total value that the government can initially extract from the field.

The framework that we develop could easily incorporate more complex tax structures. For example we might include a royalty tax on the quantity (or value) of the output.

To keep the analysis tractable, we have not considered various operational options that are typically imbedded in natural resource projects. However, such options could be incorporated into the analysis, given the flexibility of the LSM approach. It would be interesting to investigate how the values of these options are affected by the possibility of expropriation. For instance, in the absence of expropriation risk an option to make an additional investment to expand the scale of the oil field would become more valuable as the spot price increases. However, in the presence of expropriation risk, this might not be the case since the expropriation likelihood also increases with the spot price.
Appendices

Appendix A: The Trolle and Schwartz (2007) model

Here, we briefly review the model of Trolle and Schwartz (2007). The model is based on the Heath, Jarrow, and Morton (1992) framework for interest rate dynamics and takes the initial futures curve or, equivalently, the initial cost of carry curve as given. $S(t)$ denotes the time-$t$ spot price of the commodity, $\delta(t)$ denotes the time-$t$ instantaneous spot cost of carry, $y(t,T)$ denote the time-$t$ instantaneous forward cost of carry at time $T$ and $v(t)$ denotes the volatility state variable. The general specification of the model is given by

$$\frac{dS(t)}{S(t)} = \delta(t)dt + \sigma_S \sqrt{v(t)}dW_1^Q(t)$$
$$dy(t,T) = \mu_y(t,T)dt + \sigma_y(t,T) \sqrt{v(t)}dW_2^Q(t)$$
$$dv(t) = \kappa_v(\theta - v(t))dt + \sigma_v \sqrt{v(t)}dW_3^Q(t),$$

where $W_1^Q(t)$, $W_2^Q(t)$ and $W_3^Q(t)$ denote correlated Wiener processes under the risk-neutral measure with pairwise correlations given by $\rho_{12}$, $\rho_{13}$ and $\rho_{123}$, and the drift term in (11) is given by

$$\mu_y(t,T) = -v(t)\sigma_y(t,T)\left(\rho_{12}\sigma_S + \int_t^T \sigma_y(t,u)du\right)$$

to ensure that the model is arbitrage-free.

We obtain a highly tractable model by specifying $\sigma_y(t,T)$ as

$$\sigma_y(t,T) = \alpha e^{-\gamma(T-t)}.$$  

In this case futures prices are exponentially affine in three state variables $s(t) \equiv \log(S(t))$, $x(t)$ and $\phi(t)$ which, along with $v(t)$ jointly constitute an affine state vector. In general, the model is time-inhomogeneous but we obtain a time-homogeneous model by assuming that the initial forward cost of carry curve, $y(0,t)$, is flat and equal to a constant, $\varphi$. Let $F(t,T)$ denote the time-$t$ price of a futures contract maturing at time $T$. Then we have

$$F(t,T) = \exp\left\{\varphi(T-t) + s(t) + \frac{\alpha}{\gamma} \left(1 - e^{-\gamma(T-t)}\right) x(t) + \frac{\alpha}{2\gamma} \left(1 - e^{-2\gamma(T-t)}\right) \phi(t)\right\},$$

\[11\]
where $s(t)$, $x(t)$ and $\phi(t)$ evolve according to

\[
\begin{align*}
 ds(t) &= \left( \varphi + \alpha x(t) + \alpha \phi(t) - \frac{1}{2} \sigma_S^2 v(t) \right) dt + \sigma_S \sqrt{v(t)} dW_Q^1(t) \\
 dx(t) &= \left( -\gamma x(t) - \left( \frac{\alpha}{\gamma} + \rho \sigma_S \right) v(t) \right) dt + \sqrt{v(t)} dW_Q^2(t) \\
 d\phi(t) &= \left( \frac{\alpha}{\gamma} v(t) - 2\gamma \phi(t) \right) dt.
\end{align*}
\]  

(16)  

(17)  

(18)

Appendix B: Pricing the expropriation option by the LSM algorithm

Here we briefly explain how to price the expropriation option using the LSM algorithm of Longstaff and Schwartz (2001).

1. Simulate $M$ paths of $X_t$. Let $X_m(t_i)$, $\hat{P}_m(t_i)$ and $\Pi_m(t_i)$ denote the value of the state vector, the estimated option value and the option payoff, respectively, at time $t_i$ along the $m$’th path. Furthermore, let $\mathcal{I}_i$ denote the subset of paths for which the option is in-the-money at time $t_i$.

2. At time $t_{N-1}$ the value of the option is equal to its immediate exercise value. Therefore, 

$$
\hat{P}_m(t_{N-1}) = \Pi_m(t_{N-1}), \ m = 1, ..., M.
$$

3. Apply backwards induction from $i = N - 2$ to $i = 1$.

- At time $t_i$ the value of the option is equal to the maximum of its immediate exercise value and its expected continuation value. Longstaff and Schwartz (2001) suggest approximating the expected continuation value by the fitted value of a cross-sectional least squares regression where the ex-post realized cash-flows from continuation are regressed on a set of basis functions of the state variables. In other words, we run the regression

$$
e^{-r(t_{i+1} - t_i)} \hat{P}_m(t_{i+1}) = \sum_{j=1}^{J} \beta_j \psi_j(X_m(t_i)) + \epsilon_m, m \in \mathcal{I}_i
$$  

(19)

where $\psi_j$ denotes the $j$’th basis function.\footnote{We use the following set of functions: 1, $s_m(t_i)$, $s_m(t_i)^2$, $x_m(t_i)$, $x_m(t_i)^2$, $v_m(t_i)$, $v_m(t_i)^2$, $s_m(t_i)x_m(t_i)$, $s_m(t_i)v_m(t_i)$ and $x_m(t_i)v_m(t_i)$. Adding higher-order monomials and cross-products does not change the results.}

Note that we only use the paths for which the option is in-the-money at time $t_i$ since it is only on these paths that the government
may choose to exercise. The fitted value of this regression

$$
\hat{C}_m(t_i) = \sum_{j=1}^{J} \hat{\beta}_j \psi_j(X_m(t_i)), \ m \in I_i
$$

(20)

is an efficient unbiased estimate of the expected continuation value.

- Update the estimated option value along each path as:

$$
\hat{P}_m(t_i) = \begin{cases} 
\Pi_m(t_i), & \Pi_m(t_i) \geq \hat{C}_m(t_i) \\
e^{-r(t_{i+1}-t_i)\hat{P}_m(t_{i+1})}, & \Pi_m(t_i) < \hat{C}_m(t_i), 
\end{cases}
$$

(21)

for \( m \in I_i \) and

$$
\hat{P}_m(t_i) = e^{-r(t_{i+1}-t_i)\hat{P}_m(t_{i+1})}
$$

(22)

for \( m \notin I_i \).

4. Compute the estimated option value as

$$
\hat{P}(t_0) = e^{-r(t_1-t_0)} \frac{1}{M} \sum_{m=1}^{M} \hat{P}_m(t_1).
$$

(23)

In general, we obtain a lower bound on the option value, since we approximate the continuation value. Clément, Lamberton, and Protter (2002) prove that the LSM algorithm converges to the true option price as \( M \to \infty \).

As part of the LSM algorithm we also obtain the early exercise strategy. Let \( t_{Z_m} \) denote the estimated time of expropriation along path \( m \). We start by setting \( Z_m = N \) for \( m = 1, ..., M \), corresponding to no expropriation along any of the paths. Then we move backwards from \( i = N - 1 \) to \( i = 1 \). If at time \( t_i \) along path \( m \) it is optimal to exercise the option we set \( Z_m = i \).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_v$</td>
<td>1.0125</td>
<td>0.0123</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>2.8226</td>
<td>0.0212</td>
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<td>$\alpha$</td>
<td>0.1365</td>
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<tr>
<td>$\gamma$</td>
<td>0.7796</td>
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<td>0.2275</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>-0.8797</td>
<td>0.0040</td>
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<tr>
<td>$\rho_{13}$</td>
<td>-0.0912</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\rho_{23}$</td>
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<td>0.0116</td>
</tr>
<tr>
<td>$\varphi$</td>
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<td>0.0001</td>
</tr>
</tbody>
</table>

Notes: The table shows maximum-likelihood estimates of the model in Appendix A based on daily data. For details we refer to Trolle and Schwartz (2007), Table 5.

Table 1: Parameter estimates
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
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<tbody>
<tr>
<td>$S(t_0)$</td>
<td>41.26</td>
<td>10.62</td>
</tr>
<tr>
<td>$x(t_0)$</td>
<td>-4.67</td>
<td>2.49</td>
</tr>
<tr>
<td>$\phi(t_0)$</td>
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<td>0.33</td>
</tr>
<tr>
<td>$v(t_0)$</td>
<td>13.72</td>
<td>7.11</td>
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<tr>
<td>$P(t_0)$, million USD</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_{gov}(t_0)$, million USD</td>
<td>55.49</td>
<td>29.70</td>
</tr>
<tr>
<td>$V_{gov}^{exp}(t_0)$, million USD</td>
<td>55.49</td>
<td>29.70</td>
</tr>
<tr>
<td>$V_{gov}^{exp}(t_0)/V_{gov}(t_0) - 1$, %</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_{firm}(t_0)$, million USD</td>
<td>36.99</td>
<td>19.80</td>
</tr>
<tr>
<td>$V_{firm}^{exp}(t_0)$, million USD</td>
<td>36.79</td>
<td>19.78</td>
</tr>
<tr>
<td>$V_{firm}^{exp}(t_0)/V_{firm}(t_0) - 1$, %</td>
<td>-0.54</td>
<td>-0.11</td>
</tr>
<tr>
<td>“Welfare loss”, million USD</td>
<td>0.21</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes: The table shows, on three specific dates, the values of the state variables, the value of the expropriation option ($P(t_0)$), the oil field’s value to the government without and with expropriation possibility and the percentage change in value ($V_{gov}(t_0)$, $V_{gov}^{exp}(t_0)$ and $V_{gov}^{exp}(t_0)/V_{gov}(t_0) - 1$, respectively), the oil field’s value to the firm without and with expropriation possibility and the percentage change in value ($V_{firm}(t_0)$, $V_{firm}^{exp}(t_0)$ and $V_{firm}^{exp}(t_0)/V_{firm}(t_0) - 1$, respectively), and the “welfare loss”, i.e. $(V_{gov}(t_0) + V_{firm}(t_0)) - (V_{gov}^{exp}(t_0) + V_{firm}^{exp}(t_0))$.

Table 2: Expropriation risk on three specific dates
Figure 1: Time series of state variables
The figure shows time series of the state variables as estimated by Trolle and Schwartz (2007), Figure 5. The vertical dotted lines mark the Iraqi invasion of Kuwait on August 2, 1990, the beginning of the US-led liberation of Kuwait (“Operation Desert Storm”) on January 17, 1991, the September 11, 2001 terrorist attacks and the US-led invasion of Iraq on March 20, 2003, respectively. The vertical grey lines mark the three dates that we investigate further in Table 2: October 11, 1990, December 21, 1998 and April 21, 2006. Each time series consists of 4082 daily observations from January 2, 1990 to May 18, 2006.
Figure 2: Futures curves

The figure shows the futures curves for different combinations of $S(t)$ and $x(t)$. ‘——’ corresponds to $S(t) = 20$ and $x(t) = -3$, ‘—–’ corresponds to $S(t) = 20$ and $x(t) = 3$, ‘···’ corresponds to $S(t) = 80$ and $x(t) = -3$, and ‘· · · · · ·’ corresponds to $S(t) = 80$ and $x(t) = 3$. 
Panel A shows the value of the expropriation option, i.e. $P(t_0)$. Panel B shows the “welfare loss”, i.e. $(V_{gov}(t_0) + V_{firm}(t_0)) − \left(\frac{V_{exp}^{gov}(t_0)}{V_{gov}(t_0)} + \frac{V_{exp}^{firm}(t_0)}{V_{firm}(t_0)}\right)$. Panel C shows the percentage increase in the oil field’s value to the government when taking into account the expropriation possibility, i.e. $\frac{V_{exp}^{gov}(t_0)}{V_{gov}(t_0)} - 1$. Panel D shows the percentage decrease in the oil field’s value to the firm when taking into account the expropriation possibility, i.e. $-\left(\frac{V_{exp}^{firm}(t_0)}{V_{firm}(t_0)} - 1\right)$.

Figure 3: Expropriation risk as a function of $S(t)$ and $x(t)$
Panel A: Option value

Panel B: “Welfare loss”

Panel C: Increase in gov. value

Panel D: Decrease in firm value

Figure 4: Expropriation risk as a function of $S(t)$ and $v(t)$

Panel A shows the value of the expropriation option, i.e. $P(t_0)$. Panel B shows the “welfare loss”, i.e. $(V_{gov}(t_0) + V_{firm}(t_0)) - (V_{exp}^{gov}(t_0) + V_{exp}^{firm}(t_0))$. Panel C shows the percentage increase in the oil field’s value to the government when taking into account the expropriation possibility, i.e. $V_{exp}^{gov}(t_0)/V_{gov}(t_0) - 1$. Panel D shows the percentage decrease in the oil field’s value to the firm when taking into account the expropriation possibility, i.e. $- (V_{exp}^{firm}(t_0)/V_{firm}(t_0) - 1)$. 
Panel A: Option value

Panel B: "Welfare loss"

Panel C: Increase in gov. value

Panel D: Decrease in firm value

Figure 5: Expropriation risk as a function of tax rate and government production cost

Panel A shows the value of the expropriation option, i.e. $P(t_0)$. Panel B shows the “welfare loss”, i.e. $(V_{gov}(t_0) + V_{firm}(t_0)) - (V_{gov}^{exp}(t_0) + V_{firm}^{exp}(t_0))$. Panel C shows the percentage increase in the oil field’s value to the government when taking into account the expropriation possibility, i.e. $V_{gov}^{exp}(t_0)/V_{gov}(t_0) - 1$. Panel D shows the percentage decrease in the oil field’s value to the firm when taking into account the expropriation possibility, i.e. $- (V_{firm}^{exp}(t_0)/V_{firm}(t_0) - 1)$. 
Figure 6: Oil field’s value to the firm in the presence of expropriation risk

The figure shows the dollar value of the oil field to the firm in the presence of expropriation risk, i.e. $V_{\text{firm}}^{\text{exp}}(t_0)$. Panel A shows the case where the fixed cost of expropriation paid by the government, $K$, is a compensation to the firm. Panel B shows the case where the expropriation cost is not paid to the firm.
References


