MARKET COORDINATION OF TRANSMISSION LOADING RELIEF ACROSS MULTIPLE REGIONS

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Market coordination of transmission loading relief in an electric grid with multiple regions implies trading across boundaries. Starting with the early framework for transmission loading relief that relies primarily on administrative priorities, introduction of prices and explicit trading to redispatch across regions would provide market solutions for meeting transmission constraints.

INTRODUCTION

The present paper addresses related approaches to market-oriented coordination of multiple regions in an electric network. An interconnected electric transmission grid inherently requires coordination of its use. In some systems, a single system operator has responsibility for coordinating market participants and maintaining the reliability of the entire grid. In other larger networks, there may be multiple system operators with each responsible for a different area. Inevitably the multiple operators must have some procedure for exchanging information and making decisions that affect the patterns of use across the grid.

With the introduction of competition and greater regional trading, the North American Electric Reliability Council (NERC) assumed responsibility for developing new coordination mechanisms that include transmission loading relief (TLR) to curtail or redispatch scheduled

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transactions to keep use of the grid within its secure capacity. The early approaches were not market oriented but relied upon a set of administrative priorities. Despite the need for some form of TLR, the early implementations have been targeted as part of the explanation for market disruptions, and there has been a call for better integration of the market and transmission loading relief. The market approaches outlined here take key elements of the initial TLR framework and build on them to include price information and trading among regions. The call for improved TLR motivates the analysis, but a coordination mechanism could be viewed as a process for finding a market equilibrium within the constraints of the transmission grid. The purpose here is to identify problem structures and information requirements for coordination across multiple regions.

TRANSMISSION LOADING RELIEF

Power flows on a transmission grid must respect certain limits to preserve system reliability. In the days of vertically integrated utilities with limited inter-utility trading, there were many procedures and practices developed to control use of the grid, principally by adjusting the pattern of load and generation. As the industry moves to a competitive market, there must be new approaches to managing use of the grid.

In principle, there is at least one well-defined solution to the problem of managing system dispatch to support a competitive market within the limits of system constraints. This approach is found in the pool-based market where an independent system operator (ISO) has responsibility for ensuring reliable system balance.[2] The ISO accepts bilateral schedules and spot market bids. These are combined in a bid-based, security-constrained, economic dispatch framework to adjust the net dispatch to balance the system and meet the security requirements. The ISO could identify locational marginal costs that would be the equilibrium prices in a competitive market. These prices would be used for spot market transactions and to charge for transmission congestion. In this way, the ISO internalizes the complex network interactions while permitting the market participants to trade in a spot market.[3]

Recognition of a coordination role for the ISO has led to the regular call for implementation of the ISO model and the frequent assertion that an ISO should encompass as large an area as possible.[4] However, despite its attractions, the single ISO approach is not yet the complete answer, at least in the United States and Europe. For the foreseeable future there will be different ISOs in different regions within large systems, such as the Eastern and Western interconnections in the United States or the grid in Europe. For example, in the Eastern interconnection in the United States we will have different ISOs in New England, New York,

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2 The discussion of ISOs here does not depend on whether the ISO is independent of the owners of the transmission grid (the Gridcos) or combined with ownership of the lines in a Transco. That is an important but separate issue.

3 For example, this is the organization of the short-term market within the Pennsylvania-New Jersey-Maryland Interconnection (PJM).

4 For example, see Midwest ISO Participants, "Additional Comments of Midwest ISO Participants," Federal Energy Regulatory Commission, Docket Nos. PL98-5-001-007, ER98-1438, EC98-24, Washington DC, June 25, 1998, p. 8.

PJM, the Midwest, and so on. The need for a better framework for coordination among connected ISOs will remain.

The coordination problem is especially relevant in the debate in the United States. The short summary of the dilemma is that the principal regulatory rules and tariff approaches under the Federal Energy Regulatory Commission's (FERC) policies embody a deeply flawed model for electricity transactions—the infamous "contract path" approach. Although the FERC is aware of the fundamental disconnect between the contract-path approach and the reality of grid operations, the simple truth is that in the absence of consensus about a better approach, the rules under the FERC's Open Access Same Time Information System (OASIS) default to a design that is based on flawed assumptions. The contract-path approach assumes that it is possible to determine a path through the network along which power will flow from source to sink, and further assumes that along that path it is possible to identify the available transmission capacity that can be assigned to each transaction.[5]

Neither assumption is true, and frequently neither assumption is even close to being true. In fact, as is well known, the scheduled power will flow to some degree on every path between source and destination. Furthermore, the capability of the grid to accommodate any particular transaction depends in part on the complete pattern of use of the grid at that time. The contract path, therefore, is a fiction. It is a dangerous fiction because it creates conditions for power flows—scheduled according to the fictional system—to be in excess of the capability of the real system, violating security constraints and compromising the bedrock policy of maintaining the reliability of the grid.[6] Apparently FERC knew this would be a problem, but hoped the problem would be small and manageable, and hoped that someone would manage it.

The task has fallen to NERC as the principal group with responsibility for maintaining the reliability of the grid. Recognizing the reality and struggling against the fiction, NERC has been wrestling with the problem. Accepting the FERC regulatory policy as fixed, NERC's initial approach was not to replace the flawed regulatory model. Rather, the approach was to create a complementary NERC un-scheduling system to compensate for the weaknesses in the FERC OASIS scheduling system. The NERC rules adjust the pattern of transactions to fit within the real capabilities of the grid. In effect, when the contract path model would overload the real grid, the NERC approach to transmission loading relief would curtail the schedules to respect the security constraints in the grid.

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5 In its Order 888, the FERC explained the many problems of the contract path approach. However, at the time it had no available consensus on an alternative. Hence, in Order 889 establishing the OASIS framework, rules were put in place that essentially required the contract-path approach. Federal Energy Regulatory Commission, Promoting Wholesale Competition Through Open Access Non-discriminatory Transmission Services by Public Utilities & Recovery of Stranded Costs by Public Utilities and Transmitting Utilities, Docket No. RM95-8-000 and Docket No. RM94-7-001, Order No. 888, Washington, DC, April 24, 1996, pp. 93-98. The contract paths are redefined as "posted paths" in Federal Energy Regulatory Commission, Open Access Same-Time Information System (formerly Real-Time Information Networks) and Standards of Conduct, Order No. 889, Final Rule, Washington, DC, April 24, 1996, p. 66.

6 Scott M. Harvey, William W. Hogan, and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

Some such un-scheduling or TLR procedure is needed. Without it, the principal alternative to "lights out" would be to set the OASIS transmission limits at very conservative levels, low enough to ensure that the contract-path fiction would not overload the network reality. This conservatism would substantially reduce the effective capacity of the grid and restrict trading, working against the fundamental objectives of electricity restructuring. To some extent this conservatism may have already appeared in the posted transmission limits. The goal, however, should be to have a better TLR approach, not to preserve the shaky foundations of OASIS.

For the present discussion, the essential NERC TLR procedures include a few critical elements. First, the electric grid is divided into mutually exclusive and collectively exhaustive areas or regions, and each is overseen by a security coordinator. In the initial design for the eastern interconnected grid in the United States, there were twenty-three security coordinators. Second, through some process, the security coordinators must be informed of all transactions within the grid, including the ultimate source of the power injections and the destination of the power withdrawals. Third, the security coordinators must estimate the actual pattern of load flow on the system that would be created by the collective set of net loads at all locations. Finally, if the load flow calculation indicates that a constraint within an area will be violated, the respective security coordinator must invoke a set of rules for curtailing transactions. The curtailments extend across the grid until enough transactions have been eliminated to relieve the constraint and bring the pattern of flows within the physical limits of secure operation.

The details of the TLR rules go further, but these essential elements capture the framework.[7] The requirement for some form of TLR procedure is self-evident, and it is further obvious that the rules must cover the entire grid. The need for information about all transactions, including specifics on the origins and destinations, is less obvious but is related to the defects of the contract-path fiction. The initial schedules might be arranged according to the contract path, but the power will not flow as contracted. The power will flow from source to sink by dividing along every parallel path. In principle, therefore, every input or output anywhere in the grid can have an impact on the constraints. Further, since the principal means of controlling the flows on the grid is by altering the dispatch, it is necessary to have information about the net loads at every location. To the extent that the reporting system does not include all inputs and outputs, the security coordinators must estimate the missing information in order to have a usable method for determining the actual flows to compare with the transmission limits.[8]

The early procedures adopted by NERC include a balance of reporting and estimation, as well as conservative application of the transmission limits. To the extent that there is less than complete reporting of loads, estimation is required to determine the full effects on transmission constraints. To deal with the errors in estimation, NERC would apply conservative transmission limits. Although the reporting system has been subject to criticism,[9] some such reporting system

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[7] North American Electric Reliability Council, "Policy 9 – Security Coordinator Procedures," Draft for Board of Trustees Approval, July 14, 1998.

[8] Here we focus on anticipated usage that requires calculation of use of the transmission lines. Essentially similar issues would arise for true real time adjustments that could use measurements to supplement load flow analysis.

[9] The early criticism centered on the "tagging" requirements imposed to trace a chain of individual transactions to the original source and ultimate destination.

is necessary. The NERC reporting process is the subject of continuing attention and improvements to obtain more and better information. In the market-oriented systems outlined below, the information requirements include those in the early TLR design, and more.

By contrast to the improvements in the reporting rules, the NERC curtailment rules for TLR have been criticized as arbitrary and inefficient, disrupting the market and effectively decreasing the secure capacity of the transmission grid.[10] One interpretation of the curtailment rules finds an explanation in the constraining framework of the FERC transmission open access tariffs. The OASIS-based contract-path schedules include a number of priority classes according to degree of “firmness” and duration. No market-based transactions or adjustments are called for; the curtailment approach views the problem solely through the lens of reliability, separate from the market. Under the OASIS system, when transactions across a path would exceed transmission limits on that path, the transactions would be curtailed in order of priority. If the contract path were real, this approach would provide a simple method to ensure at least the reliability of the grid, albeit not the market efficiency of transactions.

Whatever the merits of this approach, it has little to do with what must actually happen on the grid. In reality, it could be that none of the power is scheduled across the constraint, and all the flow comes from schedules on parallel paths. Something else must be done to ensure reliability. In an effort to stay within the FERC framework, the initial TLR mechanism imposed curtailments driven by the priority classifications. However, the TLR approach also recognized that the transmission limit may be elsewhere, not on the contract path. Furthermore, the impact of individual transactions on the limit may be different, depending on the ultimate source and sink of the transaction. To deal with the flows off the path, the initial TLR procedure applies distribution factors to identify the impacts of each source and sink on each of the constraints, and then applies a complicated formula to assign curtailments to the individual transactions. The resulting rule creates a number of difficulties.[11] For example, under these rules it is entirely possible that curtailments of transactions could be many times larger than the offending flows. A 70 MW excess on a transmission limit might precipitate 700 MW of transaction curtailments. Furthermore, the curtailment rules appear to be motivated by a sense of equity and spreading the MW pain across many transactions, but without any explicit connection to the market or knowledge of the real commercial pain.

For example, consider the curtailment rule within priority categories. The impact of transactions on the constraints is measured by the power transfer distribution factor (PTDF). In essence, the PTDF identifies the fraction of the transaction that impacts the constraints. The initial TLR mechanism assigns the curtailments within a priority class in relation to the size of the PTDF, as shown in the left panel of the accompanying figure. For transactions that have a 5% or greater positive impact on the limit, the percent reduction is proportional to the impact.

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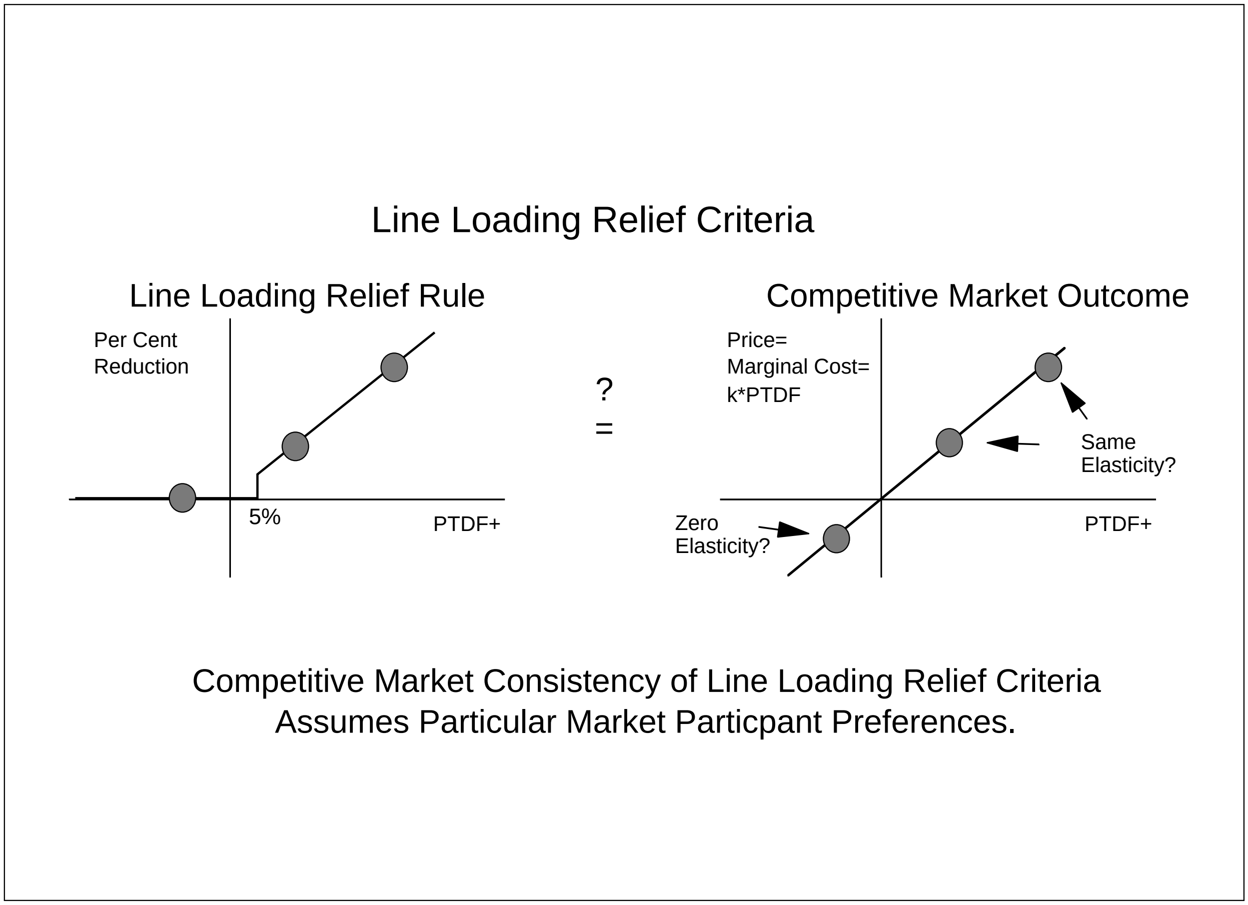
[10] For example, see “TAPS Slaps NERC Line Loading Relief,” Electricity Daily, Vol. 11, No. 7, July 10, 998, p. 1.

[11] For a more extensive critique of the initial TLR particulars, see Rajesh Rajaraman and Fernando L. Alvarado, "Inefficiencies of NERC's Transmission Loading Relief Procedures,” Electricity Journal, October 1998, pp. 47-54.

For transactions that have less than a 5% impact, or that have a negative impact and would relieve the constraint through counter-flow, there is no change.



By contrast, market incentives would signal the marginal cost of the flow on the transmission constraint. As we know, for given constraint costs this marginal cost would be a linear function of the PTDF, as shown in the right panel of the figure. [12] For the two results to be compatible, therefore, the implicit assumption would have to be that the transactions with a positive effect on the flow had a common price elasticity, but the transactions in the reverse direction had zero price elasticity. This is unlikely to be the case, and suggests that the TLR rules would be instructing market participants to make adjustments which would be inefficient and inconsistent with market outcomes.

As shown through further examples by Rajaraman and Alvarado, the deviation from efficient curtailment goes beyond this simple example.[13] In the event of even a single constraint, economic curtailment would include redispatch of many transactions. It would not be unusual for the efficient pattern of adjustment to involve simultaneous increases and decreases of output

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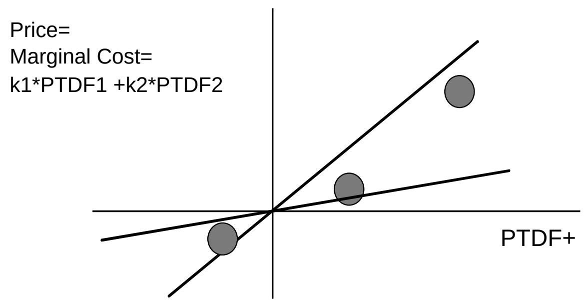
[12] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988.

[13] Rajesh Rajaraman and Fernando L. Alvarado, "Inefficiencies of NERC's Transmission Loading Relief Procedures," Electricity Journal, October 1998, pp. 47-54.

from generators at many different locations. And in the real system, there is no reason to expect that there will be only a single transmission constraint. In principle, therefore, under the initial TLR rules it would be possible for a single transaction to receive conflicting orders from different security coordinators. By contrast as illustrated in the accompanying figure, the market outcome would balance the marginal costs of many different interactions between transactions and transmission flows.

Adjustment Incentives Under Market Equilibrium and Economic Dispatch

Competitive Market Outcome With Multiple Constraints



Marginal Costs of Constraints Provide Price Incentives to Adjust Load and Generation to Relieve Transmission Constraints.

The missing ingredient, which is reflected in the complaints of the market participants, is an ability to use the market to support TLR, rather than using TLR to curtail the market. The objective would be to enhance the TLR procedure with market incentives and transactions that have the net effect of relieving the transmission constraints. The NERC people involved in the process know the limitations of TLR, and are working to address the problems. How far they can go is open to question. The limitations of the FERC framework are severe. These limitations need to be replaced with an alternative approach that recognizes and utilizes the interactions with the market.

A market-oriented TLR would still require coordination across multiple regions. There would still be a need for a flow of information about the expected uses of the system. But more would be required. The operation of a market would require information about prices. And if the price incentives are to be real, money would have to change hands. The challenge is to design a market approach that builds on the essential elements of the early TLR mechanism, with a minimum of additional requirements. The system should be able to accommodate multiple regions and be adaptable to aggregation as opportunities appear to integrate larger areas.

COORDINATION ON LOCATIONAL PRICES OF LOADS

Formally, the problem of coordinating across multiple regions has much in common with decomposition approaches for solving large optimization or equilibrium problems.[14] A coordinated solution of the smaller regional problems would provide a solution for the overall system. For a competitive electricity market, we can approach the task as determining either a market equilibrium or an economic dispatch based on the bids of the market participants. A competitive market equilibrium would be consistent with an economic dispatch based on the market preferences as expressed in the bids. The formulation is flexible, and would include both bilateral transactions and spot market transactions coordinated through the system operator.[15]

The connection between the competitive-market and the economic-dispatch frameworks is well known and has been exploited in the design of electricity markets with independent system operators.[16] There remains a debate about alternative ways to organize the solution of the problem for any given ISO,[17] but that is not the central point here. The bid-based economic dispatch approach has been widely and successfully implemented. And, as we shall see below, this approach could be adapted to an iterative approach to coordination across regions. If other methods of finding a satisfactory market equilibrium for a single ISO can be demonstrated, we could consider their application to the multiple ISO problem.

In principle, we start with the formulation of the problem for a single, system-wide ISO. Whatever the merits of having fewer and larger independent system operators, however, it is unlikely that the complete aggregation will occur soon, or ever. The task, therefore, is to develop a coordination mechanism for the system-wide problem that extends across multiple ISOs. These system operators can be viewed as the market-oriented security coordinators that could be the direction of development for the NERC framework.

Market Model with Full Grid

With the goal of describing coordination among ISOs, it is convenient to begin with an economic dispatch formulation that envisions full optimization across the entire system, but

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14 Arthur M. Geoffrion, "Elements of Large-Scale Mathematical Programming, Parts I and II," Management Science, Vol. 16, No. 11, July 1970, pp. 652-691. Kiyotaka Shimzu, Yo. Ishizuka, Jonathon F. Bard, Nondifferentiable and Two-Level Mathematical Programming, Kluwer Academic Publishers, Boston, 1997.

15 Scott M. Harvey, William W. Hogan, and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

16 For example, the market structures in Argentina, Australia, Chile, England and Wales, Norway and Sweden, New Zealand, and the Pennsylvania-New Jersey-Maryland Interconnection (PJM), among others, all exploit the connection between market equilibrium and economic dispatch.

17 Hung Po Chao, and Stephen Peck, "A Market Mechanism for Electric Power Transmission," Journal of Regulatory Economics, Vol. 10, No. 1, 1996, pp. 25-59. Tabors Caramanis Associates, "Auctionable Capacity Rights and Market-Based Pricing," March 24, 1997. The California ISO follows some of the principles, but is unusual in working under restrictions imposed to prevent it from employing a least-cost dispatch; see W. Hogan "WEPEX: What's Wrong With Least Cost?", Public Utilities Fortnightly, January 1, 1998. Laurence D. Kirsch, "ISO Economics: How California Flubbed it on Transmission Pricing," Public Utilities Fortnightly, October 15, 1998, pp. 24-32.

explicitly recognizes the existence of multiple regions. For this purpose, we define a model of the power system and a bid-based, security-constrained, economic dispatch.

Let:

y_i	the vector of net loads at the buses in region i, equal to demand minus generation at each bus, for regions i = 1, 2, …, n,
$B_i(y_i)$	the bid-based net benefit function for net loads in region i,
$L(y_1,\ldots,y_n)$	the system constraint on net loads at all buses to ensure balance with losses and generation,
$K_i\,(y_1,\ldots,y_n)$	the vector of constraints in the transmission grid in region i.

The net loads could be interpreted as for both real and reactive power in a full AC formulation of the optimal power flow or economic dispatch problem. However, nothing would be lost from the interpretation below if we think of the model in terms of real power only.

The regional net-benefit function B_i represents the benefits of load minus the costs of generation at each bus aggregated for the region. We can think of this as constructed in the usual way from the upward sloping supply bids and downward sloping demand bids of the market participants at each location.[18] Bilateral transactions would be included in the usual way as fixed schedules with or without increment and decrement bids that would be part of the benefit function.

The load balance constraints in L include the system wide requirement to balance loads, generation and losses. In the AC model there would be two elements in the vector, for real and reactive power. In a real-power only approximation, this would be the single load balancing constraint.

The constraints K_i include all the possible limitations on the flow of power in the grid, including thermal, voltage, stability, or any other limits.[19] The constraints are represented here as a function of the net loads at each bus. The formulation treats other variables, such as voltage magnitudes and angles, as intermediate values that are implicit in the problem but suppressed in the explicit model formulation. The transmission constraints include all limits that would arise in the event of a set of monitored contingencies. The number of constraints included in K_i could be quite large, but as is usual in these matters, the ultimate focus will be on the binding constraints in the ultimate dispatch.

Anticipating a later discussion, this approach to modeling the constraints is referred to as a full grid model in that the explicit variables are only the net loads. Hence, the model of constraints must have a characterization of the full grid, and it is assumed possible to determine the impact on any constraint from net load at any bus. Subsequently we will consider an

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[18] Hence the net-benefit function is concave and separable across locations. For simplicity, we assume that the function is differentiable, but this could be relaxed to include step functions without affecting the discussion conclusions here.

[19] The constraints could be many and complex, driven by the effects of Kirchoff's laws on power flows. As usual, we assume that the resulting constraint functions are differentiable

alternative model where only the regional grid need be considered, at the expense of introducing "dummy" connecting variables at the point of connection between regions.

The basic formulation of the problem that we would like to solve can be summarized as the same as the bid-based, security constrained, economic dispatch over the entire grid. The representation is at a high level of abstraction to emphasize the important details of the coordination problem. However, the representation is consistent with standard economic dispatch procedures. For a given dispatch hour, we choose the net loads to maximize the total sum of the net benefits over the entire grid:

$$\begin{aligned} &\underset{y_1,\cdots,y_n}{Max} \quad \sum_{i=1}^{n} B_i(y_i) \\ &subject\ to \\ &L(y_1,\cdots,y_n)=0\,, \\ &K_i(y_1,\cdots,y_n)\le 0, \quad i=1,2,\cdots,n. \end{aligned} \tag{1}$$

A solution to this optimization problem would give rise to constraint multipliers and a vector of locational market prices for each region that would satisfy the relation:[20]

$$p_i = \nabla B_i = \theta \nabla L_i + \sum_{j=1}^{n} \lambda_j \nabla K_{ji}\ . \tag{2}$$

Here the gradient ∇L_i is the marginal impact on generation and losses of an increase in load at the bus, and θ has an interpretation as the price of power at the reference bus selected for the load flow calculations. The matrix of gradients in ∇K_{ji} captures the impacts on the jth region's constraints from an increase in the loads of the buses in the ith region, and the variables λ_j represent the constraint prices or marginal values of the transmission limit. Note that most constraints will not be binding. Hence, λ_j will be zero for most constraints, excepting the binding constraints.[21]

In the terms of the TLR procedures, the gradients ∇K_{ji} are the distribution factors for the constraints assuming the change in net loads at the bus is balanced at the reference bus. In general, these distribution factors depend on the configuration of the net loads. In concept, the equivalent NERC PTDF for any other transaction between locations would equal the difference in the corresponding elements of ∇K_{ji}. Hence, these critical data are available and have a familiar interpretation.

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[20] For the real power case with the usual DC-load approximation, see F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988. M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "Optimal Spot Pricing: Practice and Theory," IEEE PAS, Volume PAS-101, No. 9, September 1982, develops optimal spot pricing for both real and reactive power.

[21] For a discussion of dispatch-based pricing, see William W. Hogan, E. Grant Read and Brendan J. Ring, "Using Mathematical Programming for Electricity Spot Pricing," Energy Models for Policy and Planning, International Transactions of Operational Research, Vol.3, No. 3/4, 1996.

Regional Decomposition

There is a close connection between the binding transmission constraints and the constraint prices. Anticipating the decomposition of the problem by regions, we recognize that for any region we could focus on the transmission constraints it monitors and "price out" the constraints in other regions. In terms of optimization theory, this is a selective dualization of the problem.[22] Hence, for region j it follows that if we know the constraint prices for the other regions (i≠j), a solution for the economic dispatch problem in (1) would also be a solution for the dualized problem[23]:

$$\begin{aligned}&\underset{y_1,\cdots,y_n}{Max} \quad B_j(y_j)+\sum_{i\neq j} B_i(y_i)-\sum_{i\neq j}\lambda_i K_i(y_1,\cdots,y_n)\\ &\textit{subject to}\\ &L(y_1,\cdots,y_n)=0\,,\\ &K_j(y_1,\cdots,y_n)\le 0\,.\end{aligned} \tag{3}$$

The choice of region is arbitrary. Furthermore, in the context of TLR it is natural to think of the existing schedules as given and emphasize the changes in the schedules. Then we can restate (3) by viewing the net loads (*y*) as given and formulate the problem as the determination of the deviations (*x*) from the given schedules, as in:

$$\begin{aligned}&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j+x_j)+\sum_{i\neq j} B_i(y_i+x_i)-\sum_{i\neq j}\lambda_i K_i(y_1+x_1,\cdots,y_n+x_n)\\ &\textit{subject to}\\ &L(y_1+x_1,\cdots,y_n+x_n)=0\,,\\ &K_j(y_1+x_1,\cdots,y_n+x_n)\le 0\,.\end{aligned} \tag{4}$$

If the given loads and constraint prices in (4) are an optimal solution to the economic dispatch problem in (1), then an optimal solution for the deviations would be zero.

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22 Arthur M. Geoffrion, "Duality in Nonlinear Programming: A Simplified Applications-Oriented Devleopment," SIAM Review, Vol. 13, 1971, pp. 1-37.

23 Shmuel Oren noted that the argument is motivated by the completely convex case, such as with the DC-load model for real power. In the AC-model, even without global convexity, the same argument applies to the solution of the first order Kuhn-Tucker necessary conditions of an optimum. The constrained AC problem could be well behaved in the sense that a solution for the first order conditions provides a solution for the market equilibrium problem. If not, the difficulties would extend beyond the mechanics of decomposition to call into question the existence of a competitive market equilibrium and might point to a greater role for more direct management of the grid and less reliance on markets.

Our focus on market solutions and prices motivates another reformulation through linearization of the problem. However, we apply this linearization in terms of the deviations (*x*) as in:

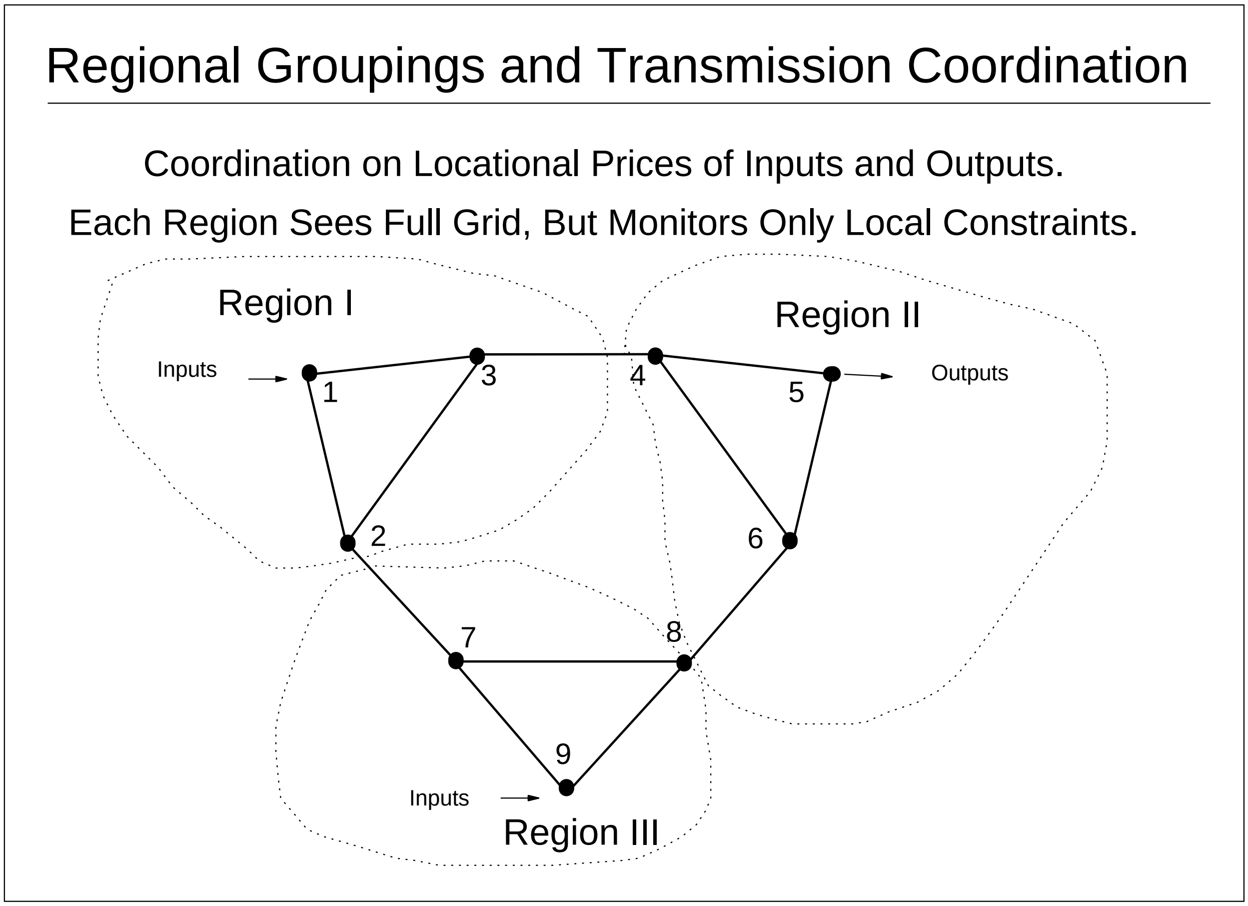
$$
\begin{aligned}
&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j + x_j) + \sum_{i \neq j} \nabla B_i x_i - \sum_i \sum_{k \neq j} \lambda_k \nabla K_{ki} x_i \\
&\textit{subject to} \\
&L(y_1 + x_1, \cdots, y_n + x_n) = 0\,, \\
&K_j(y_1 + x_1, \cdots, y_n + x_n) \leq 0\,.
\end{aligned} \tag{5}
$$

Here we have dropped constant terms from the objective function. If the constraint prices and net loads *y* in (5) provide a competitive market equilibrium and, therefore, a solution to the economic dispatch problem, then again zero would be an optimal solution for the deviations in this linearized problem.

An alternative way to write the linearized problem would be in terms of the market locational prices, where the prices at the buses are $p_i = \nabla B_i$ for an increment in net load. The components associated with the constraints are the locational congestion components of the prices, $\omega_{ki} = \lambda_k \nabla K_{ki}$, for an increment in the net load. In other words, ω_{ki} is the marginal congestion opportunity cost for loads in region i induced by the constraints in region k. Then (5) becomes:

$$
\begin{aligned}
&\underset{x_1,\cdots,x_n}{Max} \quad B_j(y_j + x_j) + \sum_{i \neq j} p_i x_i - \sum_i \left(\sum_{k \neq j} \omega_{ki}\right) x_i \\
&\textit{subject to} \\
&L(y_1 + x_1, \cdots, y_n + x_n) = 0\,, \\
&K_j(y_1 + x_1, \cdots, y_n + x_n) \leq 0\,.
\end{aligned} \tag{6}
$$

The accompanying figure illustrates the regional decomposition.



This formulation of the problem lends itself to a natural interpretation.

The formulation in (6) takes the perspective of an arbitrarily selected region. The function B_j defines the local net benefits based on the bids. The term $p_i x_i$ is the net profit from purchase and sale in region i, for each external region. The term $\omega_{ki} x_i$ is the congestion payment to region k for the change in net loads in region i. Hence, given the net load nominations made by all the other regions, and given the market clearing prices of net loads (p) and congestion (ω) elsewhere, the local problem for the regional ISO is to choose the deviations from the nominations across the grid, subject to load balancing and local constraints, so as to maximize the net local benefits plus the profits from purchase and sale elsewhere in the network, minus the cost of congestion. This still has the form of an economic dispatch problem. It is, for example, the type of problem solved by the PJM ISO, ignoring the prices and congestion costs in the external regions. However, including these prices and congestion costs would be easy, if they were available, and would not require any fundamental reformulation of the problem.

We could formulate this problem for each region. Each region would give explicit attention to its local constraints, but not to the constraints of other external regions. The congestion prices would capture the effects of external constraints. If the external prices and nominations are at the market equilibrium, then each region would have zero deviation as an optimal solution for its version of (6).

Given arbitrary nominations y and estimates of the market prices p and ω, the corresponding statement of the adjustment problem could now distinguish the solution as seen from the perspective of each region. Hence, if we identify x^j as the full system adjustment vector as seen from region j, $x^j = (x_1^j, \ldots, x_n^j)$, then we could restate (6) as in:

$$\begin{aligned}
&\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + \sum_{i \neq j} p_i x_i^j - \sum_i (\sum_{k \neq j} \omega_{ki}) x_i^j \\
&subject\ to \\
&L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0, \\
&K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0.
\end{aligned} \tag{7}$$

Heuristically, we could imagine a price coordination process that used (7) to generate a sequence of adjustments in the system nominations. One approach would be to update each region in turn. Another sequence would be to allow all the regions to update simultaneously.[24] Hence, we would start with an estimate of *y, p,* and ω. Each region would solve its version of (7) to generate its adjustment vector x^j. For the simultaneous update, the new estimates would be:

$$\begin{aligned}
y^{new} &= y^{previous} + \sum_{j=1}^{n} x^j / n, \\
p^{new} &= (p_1^{new}, \cdots, p_n^{new}) = (\nabla B_1(y_1^{previous} + x_1^1), \cdots, \nabla B_n(y_n^{previous} + x_n^n)), \\
\omega^{new} &= (\omega_1^{new}, \cdots, \omega_n^{new}),\ with \\
\omega_j^{new} &= (\omega_{j1}^{new}, \cdots, \omega_{jn}^{new}) = (\lambda_j^{new} \nabla K_{j1}(y_1^{previous} + x_1^j, \cdots, y_n^{previous} + x_n^j), \\
&\qquad \cdots, \lambda_j^{new} \nabla K_{jn}(y_1^{previous} + x_1^j, \cdots, y_n^{previous} + x_n^j)).
\end{aligned} \tag{8}$$

The motivation here is that a region's decisions should have the principal impact on its prices, both the estimate of the total prices at the buses and the implied cost of congestion for the constraints monitored by the region. The revised estimates of the local prices and system schedules would provide more and better information for all the other regions.

If this process converges, then we would have a competitive market equilibrium. This approach is similar to the market process outlined by Schweppe et al., in which a single auctioneer would update prices and the market participants would respond to these prices.[25] The coordination process here is different than that proposed by Schweppe et al., however, in that there is not just a single auctioneer (i.e., not just one ISO) announcing prices. Furthermore, the responding regions internalize some of the constraints to produce new estimates of the prices, not just new estimates of the net loads. A slightly more subtle point is that iteration through the regions guarantees that an overall solution would satisfy not only the first-order necessary

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[24] This is analogous to an application of Gauss-Seidel or Jacobi iteration on a set of non-linear equations. See J. M. Ortega and W. C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, 1970.

[25] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988.

conditions for optimality, but also the complementarity requirement that constraint prices are zero for non-binding constraints and positive only when the constraints are binding.

Use of the coordination problem within the context of an iterative approximation of the solution of the market equilibrium problem suggests the use of a slightly different formulation. With only an approximation of the prices, a solution for (7) may yield large, even unbounded changes in the adjustment variables. This is a familiar computational problem. It suggests the use of a well known approach of creating an auxiliary problem that is better behaved with approximate solutions but which is still consistent with the market equilibrium. Suppose, for example, that we have an estimate of the change in the price and congestion cost that would be induced by a change in the net loads. Let this be ε_i. Then the auxiliary problem with an approximate linear demand curve could be:

$$\begin{aligned} &\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + \sum_{i \neq j} (p_i - \varepsilon_i x_i^j) x_i^j - \sum_i (\sum_{k \neq j} \omega_{ki}) x_i^j \\ &\textit{subject to} \\ &L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0, \\ &K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0. \end{aligned} \tag{9}$$

An equilibrium solution with zero deviation for (6) or (7) would also be an equilibrium solution for (9). Importantly, at equilibrium all these formulations have the same first-order conditions in terms of the prices. However, at an approximate solution, (9) is more likely to be well behaved in the use of the estimates of how prices would change as the dispatch is adjusted.[26] Experience on related problems suggests that the convergence properties with reasonable starting estimates can be quite good.[27] Judicious choice of the approximation parameters ε helps. However, the equilibrium condition does not depend on the choice of ε, so it is not necessary to require that this information be exchanged among the regions.

Information Exchange and Payments

Coordination of multiple regions as envisioned here anticipates each region following certain rules and exchanging information. The coordination process does not quite require a coordinator. If each region follows the rules, the information need only be published on a bulletin board. However, reliance on market forces and coordination on prices requires that the prices provide meaningful incentives, so there would be payments made at the equilibrium prices.

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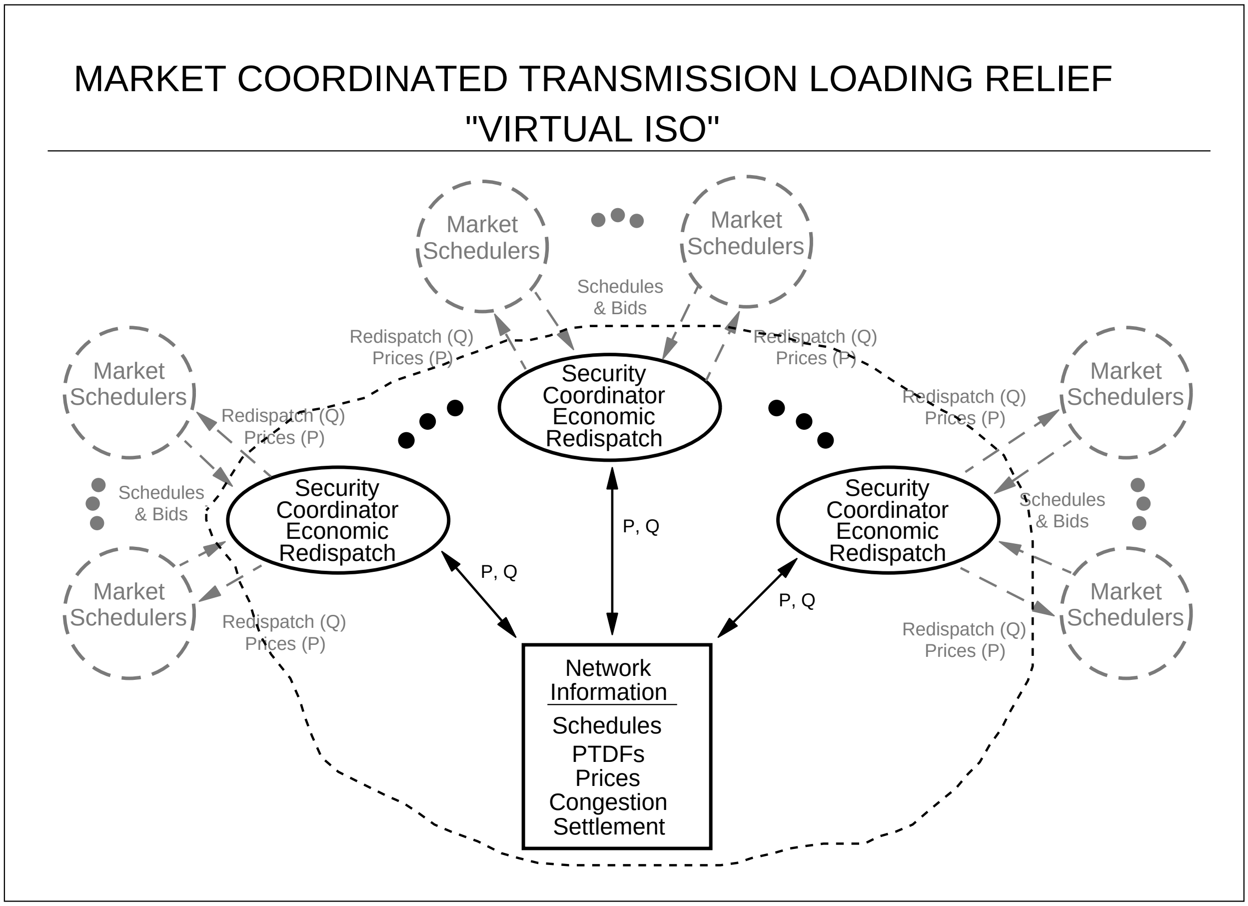
[26] The adjustment approximation might also apply to the congestion prices. However, it is harder to know how to make these adjustment estimates throughout the grid. The network interactions could be complicated, and part of the purpose of the decomposition is to avoid the requirement to know much about the nature of external constraints.

[27] Byong-Hun Ahn and William W. Hogan, "On Convergence of the PIES Algorithm for Computing Equilibria," Operations Reasearch, Vol, 30, No. 2, March-April 1983, pp. 281-300.

The market rules within a region are not the focus here. For simplicity, assume that each market follows rules within the region much like the rules in PJM. Hence, market participants have a deadline for submitting bilateral schedules and spot market bids. Once submitted, the ISO uses this information to find a bid-based, security-constrained, economic dispatch. This requires both analysis of the dispatch within the region, and iterative exchange of information with the other regional ISOs. The final equilibrium and associated prices are used in the settlement system. In effect, coordination among the ISOs produces a "virtual ISO" for the entire system, as illustrated in the figure.



The basic regional model in (9) is familiar as an economic dispatch formulation in terms of the net loads. The process starts with an initial set of schedules (y), with estimates of locational prices (p) and with estimates of congestion prices (ω) available for the entire grid. Given this information, each regional ISO solves its version of (9) and produces new estimates for its prices (p_j), for its congestion prices for the grid (ω_j) and for its schedule adjustments for the grid (x^j) according to (8). These in turn produce new estimates for the aggregate schedules, locational prices and locational congestion estimates. The process would continue further interaction among the ISOs until an acceptable solution is obtained.

Compared to the initial TLR procedures adopted by NERC, the new information is predominantly in the form of prices. The transaction reporting system already requires explicit reporting of inter-regional transactions, and implicit estimation of the balance of the intra-regional schedules, in order to calculate the impacts on transmission constraints. The prices would be a new reporting requirement, but would seem to be essential in some form to implement a market-oriented coordination system. The system would not require one ISO for the entire grid. Rather, each regional ISO could start with the information obtained from the others to produce its needed update on the price effects induced by the constraints the particular ISO is monitoring. Given the prices, each ISO needs keep track only of its own constraints.

The payments could take many forms. A natural organization would be to have users at each location treated as though they purchased or sold at their respective locational prices. Those buying and selling in the spot market would use the equilibrium prices (p). Those scheduling bilateral transactions would pay the difference in the locational prices at the source and destination.[28] As usual, the system could operate in a hub and spoke framework, decomposing transactions between locations as being to and from the hub.

The settlement system in aggregate would have the usual property that net payments by loads would be at least as large as the payments to generators. In other words, we always have $py \geq 0$. In the case of active transmission constraints, the net payments would be positive, or $py > 0$. The difference would reflect the value of the transmission congestion.

This happy result that avoids revenue deficits in the aggregate would not be guaranteed for each region. Region by region, we could not determine the sign of $p_i y_i$, only that the sum across all regions would be non-negative. For instance, if one region had all the generation and another had all the load, there could not be individual payments balance in each region. Hence, there would have to be a settlements system for the network as a whole. In other words, there would be payments to and from the various regions. The distribution of the surplus could be handled in various ways, such as in the creation of transmission congestion contracts.[29]

Coordination on Prices of Constraints

A variant of the coordination on locational prices is available from (9) by modifying the transfer to coordinate on the constraints prices λ rather than the locational congestion prices ω. As Stoft[30] explains, typically the number of binding constraints in the current dispatch will be (greatly) less than the number of constraints and the number of distinct locations.[31] Hence, the market might coordinate on the constraint prices.

The simplification would come at the cost of now requiring that the distribution factors ∇K_{ki} be published for each binding constraint.[32] To keep the arithmetic simple, suppose there are

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[28] An early implementation of this idea appears in the PJM proposal to allow transactions from outside the region avoid TLR curtailments from PJM by paying congestion cost at the difference in locational prices. PJM Energy Committee Minutes of September 9, 1998, pp. D-9-7&8.

[29] Scott M. Harvey, William W. Hogan, and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Center for Business and Government, Harvard University, June 6, 1996, (Revised March 8, 1997).

[30] Steven Stoft, "Congestion Prices with Fewer Prices than Zones," Electricity Journal, May 1998, pp. 23-31.

[31] This small number of binding constraints should apply for an individual dispatch. Note that this would not be true in the long run over a period covering different dispatches. Almost by definition, the potential binding constraints would be all the monitored contingency constraints, which is typically a very large number.

[32] Note that in the DC-load model approximation the distribution factors depend on the state of the grid, but not the dispatch. In the full AC-load model of the real, the distribution factors change with the dispatch.

a thousand nodes, three regions, and two binding constraints in each of the regions. Under the coordination via locational prices, each security coordinator would publish 1000 congestion prices for a total of 3000. These congestion prices would be additive, and immediately reduce to 1000 congestion prices posted for the current iteration.

Under coordination on prices of constraints, each security coordinator would publish two constraints prices and 2000 distribution factors. There would be a total of six constraint prices, which seems better than the 1000 locational congestion prices. But there would now be 6000 distribution factors published. Of course, these could be aggregated in the simple arithmetic to convert them into the locational congestion prices, but only at the cost of foregoing the narrow focus on the six prices.

The difference in approach is more apparent than real. It would be possible to do both—calculate the locational congestion prices and publish the computational components. Furthermore, as revealed by (9), use of the decomposition method to determine the congestion component still requires publication of the current estimate of the locational prices, at least to include marginal losses and disequilibrium effects.[33]

Other elements of the information exchange and payments would follow the discussion of coordination on locational prices. The emphasis on the constraint prices or the locational prices is a matter of choice, dealing fundamentally with the same basic formulation of the market model. In particular, the description of constraints and calculation of distribution factors assumes a grid-wide view of the impacts of net load at all locations. An alternative approach would reformulate the problem in terms of separate regions with connecting variables.

COORDINATION ON PRICES OF CONNECTING VARIABLES

A related approach to decomposition is found in the work of Kim and Baldick.[34] They present a formulation of the economic dispatch problem with an eye towards computer decomposition using parallel processors. However, as noted by Kim and Baldick, as well as by Boucher et al.,[35] this approach has an interpretation as a method for decentralized coordination to achieve a market equilibrium. By contrast with the market mechanism specified above, this approach combines a price adjustment process with a separable formulation of the network interactions.

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33 Even in a loss-less formulation with iterative coordination, there would be a need for locational prices. Here the current estimate of the locational price is a function of the previous estimate of the congestion prices, not the current estimate of the congestion prices. In equilibrium, the distinctions would disappear, but during the adjustments the difference would reflect the disequilibrium estimates of the congestion costs. Ignoring losses, however, it would be possible to use the current estimates of congestion as the estimate of the locational prices, which would give the objective function of the coordination problem the interpretation of paying for congestion on external constraints and collecting for congestion on internal constraints. See the appendix.

34 Balho H. Kim and Ross Baldick,"Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, pp. 932-939.

35 Jacqueline Boucher, Jean Dekelver and Yves Smeers, "The Restructuring of the European Electricity System," Electrabel, Brussels, Belgium, April 1998.

Market Model with Regional Grids and Connecting Variables

The separable formulation envisions a network with multiple regions. Each transmission line between regions is "conceptually divided into two lines by adding a 'dummy bus' at the border between the two regions. Real and reactive power flow variables and voltage magnitude and angle variables are defined for the dummy bus and these four variables are duplicated, with one copy assigned to each region."[36] This in turn requires the duplicated variables to have the same value at the border.

In terms of the present notation, we could introduce:

z_i	the vector of variables at the dummy buses in region i, including real and reactive power, voltage magnitudes and angles, for regions i = 1, 2, …, n,
$L_i(y_i,z_i)$	the constraint on net loads at all buses to ensure balance with losses and generation,
$K^*_i(y_i,z_i)$	the vector of constraints in transmission grid in region i.

The net loads in *y* can be defined once for the bus in terms of real and reactive power. There is an important feature here with the introduction of the dummy variables *z*. Ideally, we would like to define the connecting variables as the power flows on the lines. However, this would not be sufficient. To see the problem, imagine that each region defined a locally balanced set of inputs and outputs with no flows on the connecting lines. In general, when linking the separate regions, the combined grid would rearrange the patterns of flow. The fact that as separate grids there would be no flow is not enough to guarantee a consistent combined solution with no flow. To achieve a consistent decomposition, the dummy variables must include not only the line flows but also the voltage magnitude and angles.

Furthermore, Kim and Baldick focus on the "non-contingency constrained" economic dispatch problem.[37] Apparently, to achieve the decomposition of the regions in a contingency constrained formulation, the dummy variables must be defined separately for each contingency. The local flows on the regional grid would differ in the event of a contingency where there are external loops, as is the general case. This information is implicit in the full grid formulation of the constraints as above, but must become explicit in the formulation of regional grids with connecting variables. Strictly speaking, therefore, the number of variables at each dummy bus in a region is four times the number of monitored contingencies in all regions. Since there may be hundreds or thousands of monitored contingencies, these coupling variables could represent a substantial reformulation of the economic dispatch problem. The benefit, as shown below, is in the isolation of the local problem to consideration of only the local variables.

With these definitions, the model of Kim and Baldick could be cast as:

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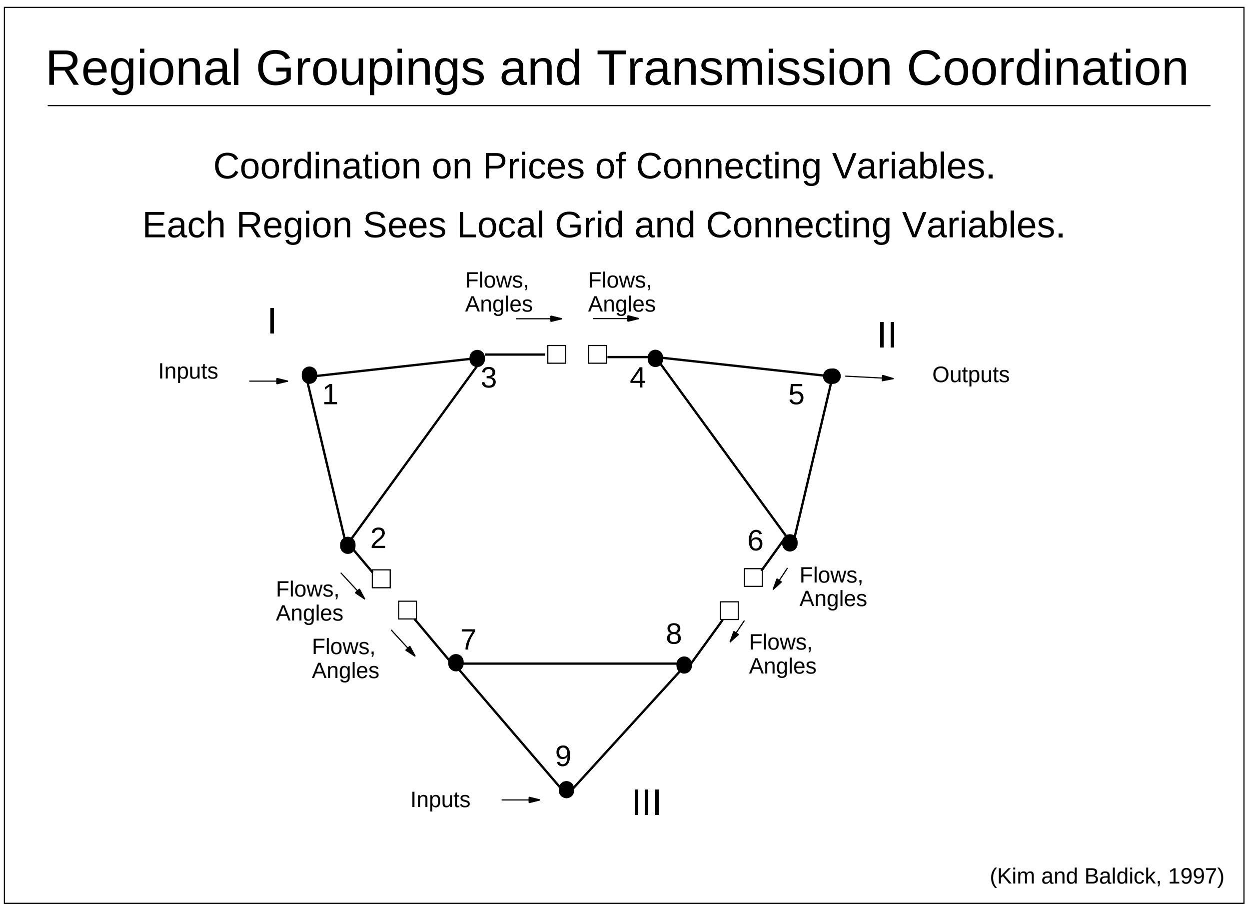
[36] Ross Baldick, Balho H. Kim, Craig Chase and Yufeng Luo, "A Fast Distributed Implementation of Optimal Power Flow," (To appear in IEEE Transactions on Power Systems) p. 1.

[37] Balho H. Kim and Ross Baldick,"Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, p. 932.

$$
\begin{aligned}
&\underset{\substack{y_1,\cdots,y_n \\ z_1,\cdots,z_n}}{Max} \quad \sum_{i=1}^{n} B_i(y_i) \\
&s.t. \\
&L_i(y_i, z_i) = 0\,, \quad i = 1, 2, \cdots, n, \\
&K_i^*(y_i, z_i) \leq 0, \quad i = 1, 2, \cdots, n, \\
&Az = 0 \quad .
\end{aligned} \tag{10}
$$

The final set of constraints, Az=0, would embody the simple linear condition that the dummy variable values for one region must equal the corresponding dummy variable values for the adjacent regions. The load balancing and transmission constraints L_i and K_i^* are defined only in terms of the variables of the immediate region.

The accompanying figure illustrates the regional decomposition.



This is the equivalent of the economic dispatch problem above in that a solution to either (1) or (10) would provide a solution to the other. The locational prices could be recovered from either approach.

Regional Decomposition

In essence, the approach of Kim and Baldick dualizes the linking constraints by introducing the vector of prices γ. Hence, given the proper value of these constraint prices, a solution of the economic dispatch problem would also be a solution for the dualized problem:

$$\begin{aligned}
&\underset{\substack{y_1,\cdots,y_n \\ z_1,\cdots,z_n}}{Max} \quad \sum_{i=1}^{n} B_i(y_i) + \sum_{i=1}^{n} (\gamma A)_i z_i \\
&subject\ to \\
&L_i(y_i, z_i) = 0, \quad i = 1, 2, \cdots, n, \\
&K_i^*(y_i, z_i) \le 0, \quad i = 1, 2, \cdots, n \quad .
\end{aligned} \tag{11}$$

As before, given the proper value of the constraint prices, the market equilibrium would also be a solution of this economic dispatch problem. If we let $\zeta_i = (\gamma A)_i$, we can see immediately the separability of this problem. Given the prices for the dummy variables, the optimization problem separates into independent optimization problems for each region in the form of:

$$\begin{aligned}
&\underset{y_i, z_i}{Max} \quad B_i(y_i) + \zeta_i z_i \\
&subject\ to \\
&L_i(y_i, z_i) = 0, \\
&K_i^*(y_i, z_i) \le 0 \quad .
\end{aligned} \tag{12}$$

The iterative approach in Kim and Baldick works with a version of (12) that is analogous to the approximation in (9). The details are important, but are not the focus here. In essence, each iteration involves obtaining a solution for the regional variables and then updating the prices ζ_i to the extent that the linking constraints are violated (i.e., $Az \neq 0$). A solution from this process (with $Az=0$) would provide a solution to the overall economic dispatch problem.

Information Exchange and Payments

As before, coordination of multiple regions through connecting variables anticipates each region following certain rules and exchanging critical information. Payments made through a settlements system give meaning to the pricing incentives.

The generic regional model in (12), deviates from the usual formulation of an economic dispatch in terms of net loads. Here the connecting variables, to include voltage magnitude and angles for each contingency, receive explicit attention in the objective function used by each ISO with the coordinating prices. This would require some change in perspective by system operators.

In essence, the process starts with an initial set of estimates of connection prices (ζ). Given this information, each regional ISO solves its version of (12) and produces new estimates for its connecting variables (z_i). A coordinator compares the paired estimates of the connecting variables and applies a rule to update the estimates of the connecting prices (ζ) for the next iteration. Strictly speaking, this requires slightly more of the coordinator than just acting as a bulletin board publishing information from each region.[38] However, this role for a central coordinator does not fundamentally change the regional decentralization of the process.

Given an equilibrium solution, each regional ISO has the net load schedules (y_i) and can obtain the locational prices (p_i). The loads and prices would be consistent with the market equilibrium and the overall economic dispatch. Payments would be made in terms of the net loads or for bilateral transactions, using the locational prices (p_i).

As before, payments at these locational prices would not be balanced region by region, but only in the aggregate. A settlements system could provide for payments to and from the regions. Spot market participants would conduct their transactions at locational prices. Bilateral transactions would pay the difference in locational prices at the source and destination. Any transmission constraints would lead to a net payment surplus that could be distributed, as with transmission congestion contracts.

However, the formulation of Kim and Baldick suggests an intriguing possibility. As discussed above, we would not have $p_i y_i \geq 0$ for each region. However, under certain assumptions, we would have $p_i y_i + \zeta_i z_i \geq 0$ for each region.[39] In other words, the regions would not be guaranteed at least a net payments balance in terms of the net loads alone, but each region would see at least a net payments balance if we included "payments" for the dummy variables. In effect, this would involve market trading or settlements across regions in terms of the dummy variables. This would seem natural enough in terms of the connecting power flows, buying and selling the power flowing in and out of a region. However, it seems less natural for the many voltage magnitudes and angles of the dummy variables at the border, particularly variables for different contingencies.

Nonetheless, it is an intriguing observation. And if there is a better trading and settlements system embedded in this observation, perhaps it could be adapted to the payments under price coordination across locations. The formulation in terms of connecting variables provides insight into what would be required for trading at the boundaries without a system-wide settlement. Since the underlying economic dispatch problems are the same, in principle all the detail of one could be recovered from the other.

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38 Kim and Baldick employed a linearized augmented Lagrangian formulation for updating the iterations. In personal communication, Baldick suggested that this could be important in their convergence results. This may be true, but it could be kept transparent from the perspective of the market participants and would not upset the basic interpretation of the coordination method.

39 For example, this would be true in general for the loss-less DC-load approximation because of (i) concavity of the net-benefit function, (ii) convexity of the feasible set in the DC-load model, and (ii) $(y_i, z_i) = 0$ would be a feasible solution for this approximation of the regional problem.

EXAMPLES AND CONVERGENCE TESTS

Part of the motivation for coordination approaches is that the methods would get close to a solution relatively quickly. Most real ISO processes will require some human intervention. The real security-constrained economic dispatch problem is too complicated to be fully automated. And without virtually complete automation, it would be of little use to have a coordination method that depended on hundreds or thousands of iterations to get reasonable answers.

From the perspective of convergence analysis, we want the adjustment method to behave like a contraction mapping. In other words, at each step we want substantial movement towards the solution. The details of the problem here are complicated, and the network interactions challenge our ability to be sure that the methods will converge at all, much less rapidly. However, our intuition is that the system should behave much like the analogous iteration methods on diagonal dominant systems of equations. If each region's principal impact is on its own prices and constraints, being not as highly coupled with the prices and constraints in other regions, we would expect the most important changes in the critical price variables to occur as a result of the local regional solution. If the price variables for other regions don't change as much, then there should be rapid convergence.

Kim and Baldick implemented their approach and reported exactly this type of favorable convergence experience for their test networks with specialized software for parallel computation. Their focus was on the implications for distributed computing, but their results have implications for market coordination. In their work, "the most significant feature … is that the solution converges within 3 or 4 iterations."[40] Whether or not this happy result would extend to larger, more realistic problems is an open question, but their experience gives reason for optimism.[41]

We do not have the similar tests of software for coordination on locational prices. To illustrate the operation of the method described above using (9), we summarize here a simple example on a DC-load model formulation without contingency constraints. Because of the nature of the locational price coordination mechanism, the addition of contingency constraints should have no effect on the convergence process, although it would complicate the illustration.

Here a test problem with 9 buses and 12 lines utilizes the stylized network in the accompanying figure. Each line is assumed to have the same impedance characteristics. In the loss-less DC load formulation, this simplifies the verification of the flows, where the sum of the line flows around any loop must equal zero.

In addition, each bus in the network has a generator or a load. The loads appear at buses 1, 5 and 9 with the indicated demand curves. The other buses are shown as generator buses with

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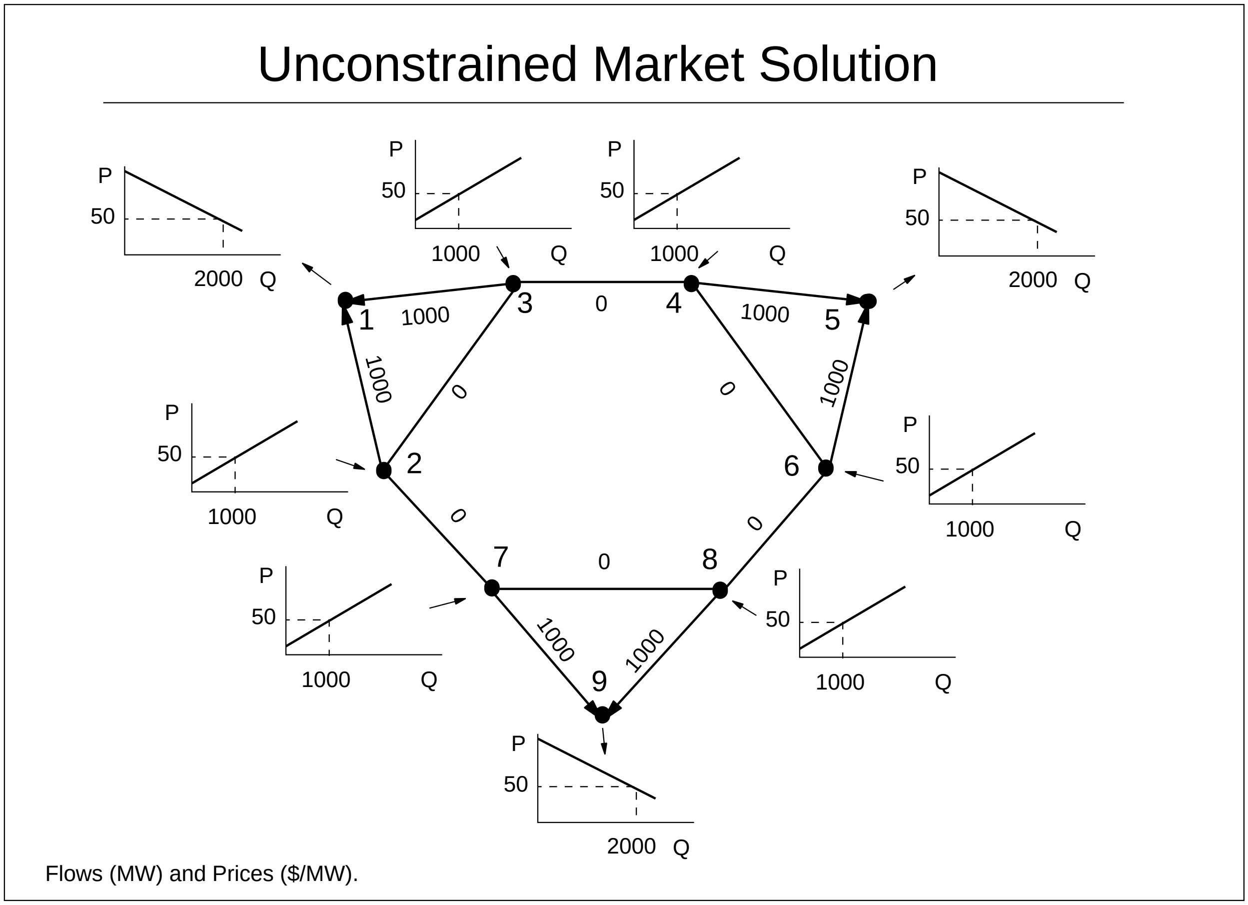
[40] Balho H. Kim and Ross Baldick,"Coarse-Grained Distributed Optimal Power Flow," IEEE Transactions on Power Systems, Vol. 12, No. 2, May 1997, p. 937.

[41] The method of Kim and Baldick has been tested on a 2587 line representation of ERCOT by Baldick et al., apparently without contingency constraints. There is no report of the number of iterations for this problem. However, they do describe substantial computational efficiencies measured in elapsed time which should imply rapid convergence in terms of the number of iterations: Ross Baldick, Balho H. Kim, Craig Chase and Yufeng Luo, "A Fast Distributed Implementation of Optimal Power Flow," (To appear in IEEE Transactions on Power Systems).

identical supply curves.[42] (The intent of all this symmetry is to make problem more transparent.) The figure shows the unconstrained solution, with a common system wide price of $50 per MW.



To introduce the regional decomposition, we convert the example by identifying three different regions with the associated lines and buses. Each of the groupings in regions I, II, and III has its own system operator with responsibility for solving its version of problem (9) and reporting the results.

In addition, the accompanying figure includes the description of the transmission constraints. For this illustration, all the constraints enter as thermal limits specifying the maximum allowable flow on each line. As can been seen by comparison with the previous figure, the unconstrained solution would violate these transmission limits. This gives rise to the need for coordinated transmission loading relief.

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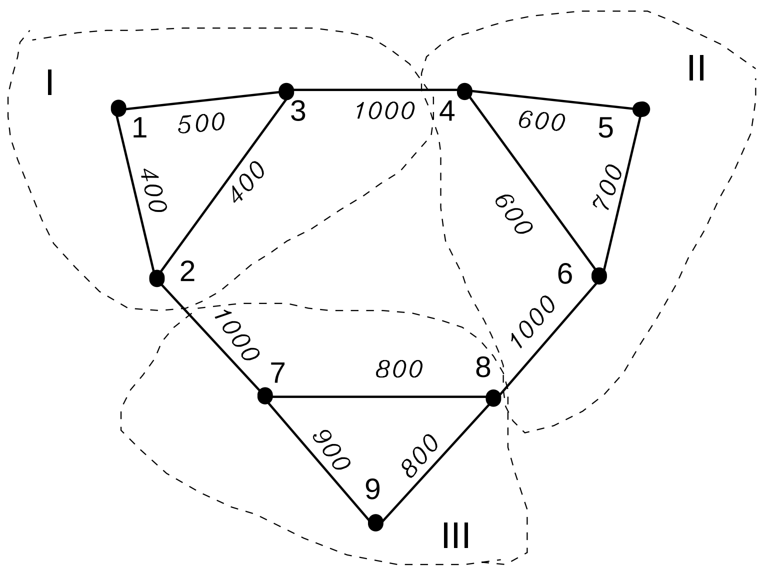
42 To reduce the clutter in the figure, only one point is shown for each curve. The intercepts are at $110 and $20 for the demand and supply curves, respectively. The slopes all have an absolute value of 0.03.

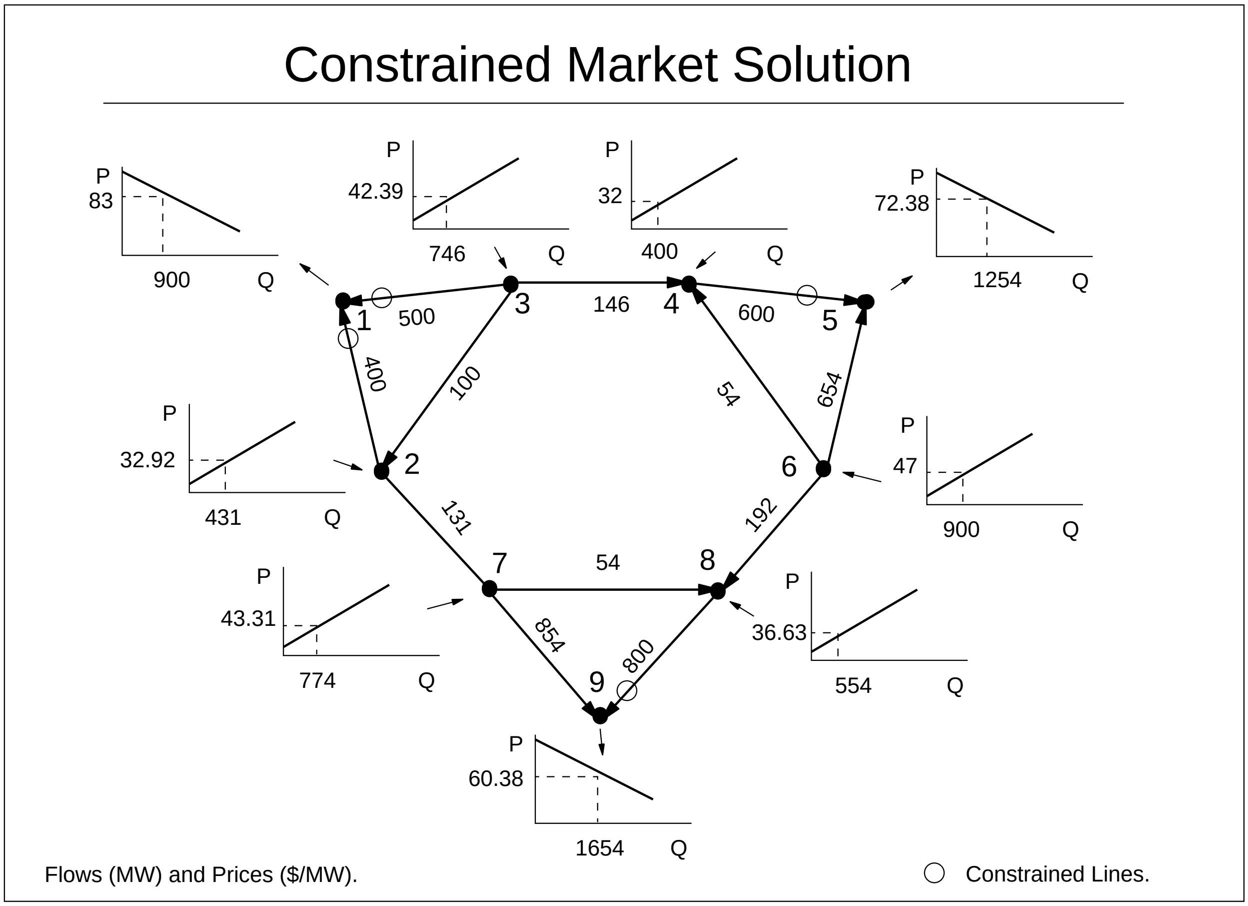
Regional Groupings and Transmission Limits



Thermal limits on transmission lines (MW).

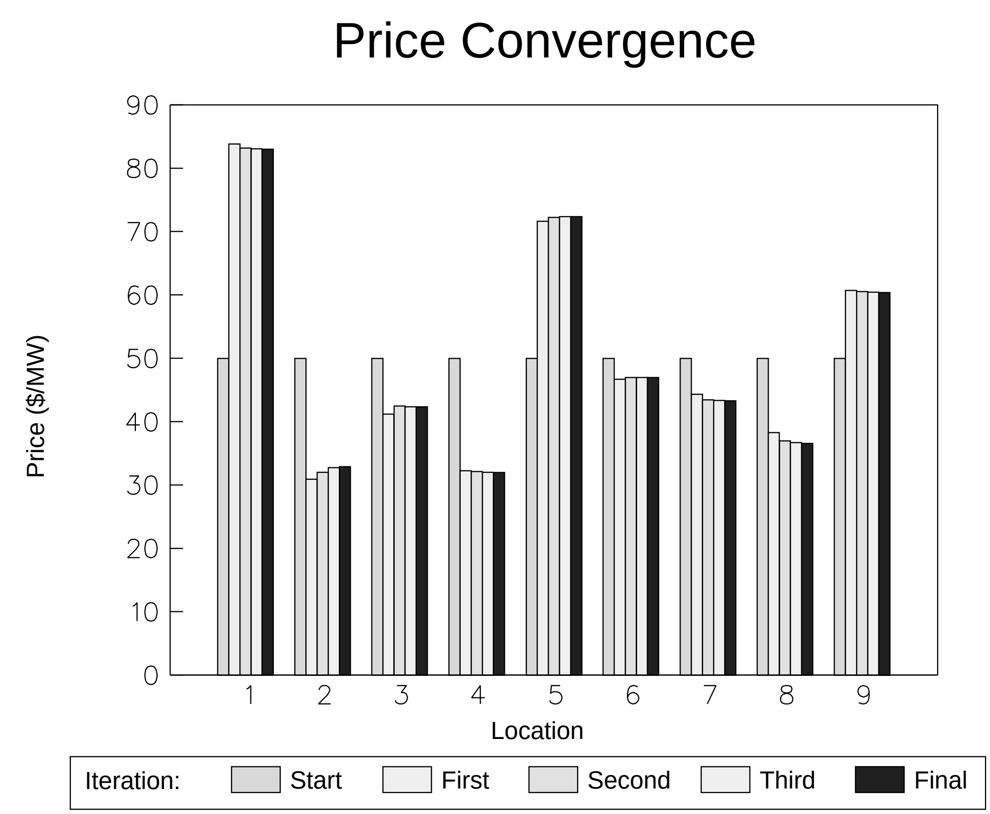
To find a solution for the constrained problem, we solved the local problem for each region, in turn, starting with region I. The initial estimate of bus prices is from the unconstrained case with the estimate of the congestion price as zero. For each regional problem, we took the previous estimates of the congestion prices for the other regions, and the latest estimate of the locational prices at the current schedules. One iteration if the coordination process consists of a full cycle through the three regions.

The process converged to the constrained solution as shown in the accompanying figure. This is the same solution obtained for the constrained network problem formulated as a single economic dispatch. It is a system-wide market equilibrium, and a region-by-region market equilibrium at the current prices.



The constrained locational prices range from a low of $32 per MW at bus 4 to a high of $83 at bus 1. There are four binding transmission constraints. In region I the lines between buses 1 and 2, and between buses 1 and 3, are at their limits. In region II, the line between buses 4 and 5 is constrained. In region III, the line between buses 8 and 9 is constrained.

Convergence to the constrained solution for this problem is fast. Starting with the unconstrained answer, the following figure shows the resulting locational prices after each full iteration. The prices adjusted almost to the constrained solution in the first full iteration through all the regions, and were essentially exact after three iterations. Further details of the adjustments appear in a summary table in the appendix.



Price coordination iteration for nine bus example.

The convergence speed was essentially the same for an alternative test starting in region III and visiting the regional calculations in reverse order. One interesting feature of this alternative sequence was that the line between buses 5 and 6 was temporarily constrained during the first full iteration, but not afterwards.

The example problem network is looped and the regions are reasonably coupled. The distribution factors relative to bus 1 are shown in the appendix. In effect, power moving from one bus to another affects the flow on every line in the system.

It is not known if this same early convergence would extend to a realistic problem or full AC implementation. Obviously, the larger the regions and the weaker the coupling, the better should be the early convergence. As for the AC model, reactive power is by its nature most affected by local variables, so the "dominant diagonal" property should be preserved. The related work of Kim and Baldick, and similar applications on other economic equilibrium models, suggests that good early convergence may be a viable conjecture.

IMPLEMENTATION ISSUES

The conceptual frameworks for market coordination and TLR provide a guide to the development of the information and institutional arrangements. The frameworks allow for aggregation of regions and provide paths toward a TLR arrangement with "fewer people and more knowledge."[43] Essentially, coordination on locational prices would require information very much like that in the early NERC systems for TLR. The principal addition would be the price and congestion cost estimates to be provided by the regional system operators or security coordinators. The method of coordination on connecting variables would require a different information structure and exchange, focusing on the connecting variables and prices, rather than system-wide schedules. However, this change in the system might have advantages in allowing the regional operator to concentrate on its own region.

Further consideration of these approaches for coordination among regions would need to address a number of implementation issues and questions. Here we outline a few of the matters for future investigation.

Drawing the Boundaries

Selection of the regions covered by the ISOs will be governed by many factors, covering everything from the historical starting point to regional politics. However, to the extent the simplicity of coordination and efficiency of the market matter, the analysis here suggests an approach for further research in providing guidance for defining the boundaries of regional aggregations.

The formulation of the regional problem and the discussion of convergence results were both motivated by the analogy to and experience with iterative solution of "dominant diagonal" systems. The regional ISO gives explicit attention to the constraints within its region, but relies on much more limited information about prices or connecting variables to capture the effect of constraints in other regions. If the constraints elsewhere have a small effect on the region's own prices, compared to the effect of the constraints within the region, then simplified representation of the external effects should lead to a good solution within the region. The internalization of the most important constraints, therefore, creates the dominant diagonal condition that is so important in rapid convergence. By contrast, if the regional ISO did not have explicit knowledge of the effects of very important and sensitive constraints, it would seem that it would be more difficult to converge to a solution through iteration that relied only on the prices.

The conjecture, therefore, is that a regional aggregation should be better when the interconnections are weaker in a particular sense. Not weaker in the sense that the connecting lines have only limited capacity, but weaker in the sense that the looped impacts across the boundaries are reduced relative to the looped impacts within the region. In the limit, obviously the best form of interconnection would be radial, where there would be no looped effects and no distant impacts on constraints and prices. The precise definition of weak loops is not clear, even in the simple example, but the goal is to have relatively little impact on the distant prices once a reasonable estimate of the prices is available.

Network Modeling

The characterization of the constraints in K_i necessarily involves some model of the entire grid, both internal to the region and external. It is well known that this is a difficult problem that

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[43] Paul McCoy suggested this summary statement of a goal for the TLR process.

often involves a degree of approximation.[44] Although this is not an insignificant matter, the problems are neither created by nor restricted to the application of a market-oriented coordination framework. It will be necessary to deal with the network approximations in any event.

The early TLR methods for calculating PTDF information confront the same issues of network modeling. The NERC use of a limited set of "flowgates" and PTDFs is a compromise that provides a starting point for improved methods of describing the network constraints. The current system used for TLR could apply equally well for an initial market coordination application.

The estimates of the constraints in K^*_i are less vulnerable to the problem of external modeling approximations. However, a similar difficulty arises in the need to get good estimates of the connecting variable values for every contingency. Here there may be an opportunity for approximation to simplify the problem. To the extent that the contingency effects have modest impacts on the external network and the associated connecting variables, there may be ways to use a subset of the variables, dropping those that would not change much in the contingency. In addition, there might be opportunities to build approximations of the external effects by including some approximations of K_i within K^*_i.[45] This is an area of conjecture and possible investigation, but it suggests at least three important ideas.

First, the two alternative approaches to network modeling are not incompatible. It should be possible, at least in principle, to have some regions and their constraints described according to a "full" network model for the set of included regions, while linked to other regions through the connecting variable formulation. This would suggest that there could also be mixing and matching of the market coordination protocols. It is hard to know which protocol or combination of protocols would be better.

Second, the use of linear approximations for all the constraints and a formulation in terms of real power only would be a natural simplification applicable to either problem. It would not be necessary to go all the way to the simplistic DC-load network formulation, which is handy for illustration but too gross an approximation for the real system. However, there is a great deal of experience in dealing with the linearizations of the grid constraints around a given load flow. This experience includes the associated construction of interface limits and other constraints to address the reactive power details that get dropped from the model. Tools like the GE MAPS model have been widely used for many years with network formulations including several thousand buses and many hundreds of contingency constraints. Conceptually, it would be a straightforward matter to adapt tools like this to the market coordination problem.

Finally, in a very real sense the approximations are the reality. If the regions are interconnected, there is no escape from the necessity to estimate the effects of external actions on each region and constraint. The more approximate the estimate, the more conservative must be the constraint limit needed in order to preserve system reliability. The security coordinators must do something, including making some approximation. Whatever approximations are used,

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[44] Shanyou Hao and Alex Papalexopoulos, "External Network Modeling for Optimal Power Flow Applications," IEEE Transactions on Power Systems, Vol. 20, No. 2, May 1995, pp. 825-837.

[45] Linear approximation of the contingency constraints was suggested by Ross Baldick in personal communication.

and whatever improvements are made, would be available just as well for the market-based coordination process as they would for a system of administrative curtailments. Gradual improvements could be included as available, without disrupting the market coordination framework.

Incompatible Systems and Transitions

The theory of decomposition and regional coordination assumes that each regional ISO is solving its version of (9) or (12). In this case, the coordinated regional market solutions would provide a solution for the market equilibrium in the full grid. It would be desirable to have each ISO working within the same framework. However, it would be likely that at least during a period of transition, one or more regions would not be following the same economic dispatch approach. The question of the robustness of the market coordination scheme then arises when one or more ISOs follows a different set of rules.

There are many possibilities and a number of questions. Could an ISO choose not to participate at all and not monitor or mange its constraints? Given the interdependence of the grid, this would seem to imply that other ISOs would have to take over responsibility for the constraints. This could be accomplished de facto by using an approximate and conservative representation of the constraints. This would limit capacity and trade compared to what would be possible if the presumably better informed ISO in the dissenting region cooperated in the market coordination.

Could an ISO continue to rely on administrative curtailment rules and ignore the prices used for market coordination? This would seem easy enough to for the regional ISO that prefers administrative rules. The only information needed from other regions would be the net loads in y as reported for the initial TLR system. The more serious difficulty would be in the absence of the regional information produced for the benefit of the market coordination process. In particular, the dissenting region's prices and congestion costs would not be available for coordination on locational prices. And for coordination on the connecting variables, the values of the connecting variables and prices would not be reported. These would have to be estimated somehow, a problem that would appear to be more serious for the coordination on the connecting variables.

For the coordination on locational prices, it would be possible for the market coordination process to operate in some regions, at least as long as the other regions provided information about their binding constraints. For example, suppose that the dissenting region with administrative curtailment rules is labeled "A", and all other regions are participating in the market coordination and settlements process. Suppose that after its application of administrative priorities for binding constraints, region A has a set of net loads across the grid that it agrees would be feasible, y^A. In addition, suppose region A reports the distribution factors for its binding constraints, ∇K_A. This would not be much more information than the initial NERC TLR required. For convenience, we pick one of the coordinating regions, and give it responsibility for serving as region A's proxy. We assume that at least some of the market participants can offer to buy or sell generation in region A that would be used in the market coordination process. To distinguish these bids, we identify the corresponding net benefit function as B_A. Since the regional ISO is not participating in the market coordination process, the bids from region A might be quite restricted, including the possibility of no market adjustments in region A.

With this information, we reformulate the selected coordinating region's problem in (9) as:

$$\begin{aligned}
&\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + B_A(y_A + x_A^j) + \sum_{i \neq j,A}(p_i - \varepsilon_i x_i^j)x_i^j - \sum_i(\sum_{k \neq j,A}\omega_{ki})x_i^j \\
&\textit{subject to} \\
&L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0, \qquad (13) \\
&K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0, \\
&(\nabla K_A)(y_1 + x_1^j - y_1^A, \cdots, y_n + x_n^j - y_n^A) \leq 0.
\end{aligned}$$

The final linearized constraint for region A, monitored by region j, is a standard technique for approximating a constraint relative to a known load flow. This constraint will ensure that the adjustments of the market coordination process would not later violate a limit that had been met through administrative curtailments directed by a region not participating in the process.[46] The selected coordinating region with responsibility for including the linearized constraint would report its price estimates and congestion costs for itself and for region A, incorporating the effect of the approximate estimate of region A's binding constraint. Note that it would not be necessary for every other coordinating region to include region A's approximate constraint. The effect of the constraints "outside" would be captured through the coordinating process, just as before.

Could a region refuse to participate in the settlements system? The money aversion built into the initial TLR process makes this an important question. At present, there are no prices and no congestion related settlement system across regions. In the worst case, like now, the problem could create an aggregate revenue deficiency, probably for congestion payments. It would be an easy matter to make sure that generation was purchased and paid for through the simple expedient of casting at least the net flows in and out of a region as bilateral transactions. The harder task would be to make sure that these transactions paid the associated congestion cost implicit in the difference between the locational prices. If people do not have to pay, the prices would provide no market incentive in support of the TLR process.

To go further with these questions, it would appear necessary to get more specific about what the regional ISOs can and cannot do, in order to design approximations that would capture some of the benefits of a market-oriented coordination system. However, it should be possible to make a gradual transition to an improved coordination and TLR process, starting with a few cooperating regions. The first steps should make things better, and the approach would proceed, region by region, without requiring a big-bang solution. However, the difficulties could be significant. It would be better if all were working from the same page.

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[46] If the linear approximation is inaccurate, or new constraints arise in region A, it would update its administrative curtailments and estimates of the distribution factors for binding constraints.

Gaming and Honest Revelation

The coordination frameworks outlined above assume that it is possible to find trial solutions and get meaningful estimates of the required schedule adjustments. In effect, the method assumes that it is possible to get the ISOs and, by implication, the markets participants to give honest answers during the iteration process. We are assuming that the regional ISOs do not game the process by providing misleading estimates or strategic changes in the proposed schedules.

For the case of the regional ISOs that follow the model of bid-based, security-constrained, economic dispatch, this would be a reasonable assumption. We would, of course, still have whatever limitations there are in strategic bids of the market participants. But this would be another matter. Once the bids were provided, the ISOs would be making the decisions needed to resolve the regional coordination and TLR issues based on the bids provided in advance by the market participants. The ISOs would be charged to seek an efficient solution as far as the coordination process goes, just as they are charged to seek an efficient dispatch. The market participants would not be asked to respond to the interim prices during the coordination iteration, so they would not be in a position to game the coordination process through strategic revelation of information.

This resolution of gaming problems would not be available for other market models that would give the market participants an opportunity to respond to the interim coordination price and congestion information. In effect, we would be involved in an iterative auction where the participants would not necessarily face any consequences for strategic schedules later withdrawn. Hence, for market models without regional dispatch by an ISO, there would have to be some rules designed to make interim bids meaningful and to move the market efficiently towards a solution. This could be a challenge. The complex network interactions make simple activity rules and bidding constraints inappropriate for the electric network coordination problem. For example, requiring monotonic bids from the market participants, a common feature of such rules in other settings, would not be consistent with the coordination process here. As we can see even from the simple example above, the prices do not move monotonically towards the solution, and the network interactions can have substantial indirect impacts on the efficient prices.

It remains to be seen if any method could be devised for regional coordination within the framework here that allowed for consideration of customer preferences through anything other than the standard model of the regional ISO running a bid-based dispatch.

CONCLUSION

Transmission usage must operate within the secure capabilities of the interconnected network. Given the strong interactions in the grid, some coordinated transmission loading relief mechanism is required. Administrative priority rules constrain the market and tend to reduce the effective capacity of the grid. A market-based coordination mechanism could improve utilization of the grid and match the potentially intricate changes in the system dispatch with the preferences of the market participants, supporting transmission loading relief. Alternative frameworks for network modeling could produce different mechanisms for market coordination. These frameworks help define both the ultimate goal and immediate steps that would better integrate the market and transmission usage in a grid with multiple regions.

APPENDIX

A Loss-Less Model Special Case

In a formulation ignoring losses, the iteration in (8) could be modified such that the estimate of prices is not the gradient of the net benefit functions, but anticipates (2) and sets the new estimate of the price as $p_i^{new} = \theta + \sum \omega_{ki}^{new}$. Since there are no losses, if we start with a balanced dispatch, the constraint L would guarantee that the deviations would sum to zero. Hence we could ignore θ and reformulate the regional coordination problem based on locational prices. In this case, the counterpart of (7) would become:

$$\begin{aligned} &\underset{x_1^j,\cdots,x_1^j}{Max} \quad B_j(y_j + x_j^j) + \sum_{i \neq j} \omega_{ji} x_i^j - (\sum_{k \neq j} \omega_{kj}) x_j^j \\ &\textit{subject to} \\ &L(y_1 + x_1^j, \cdots, y_n + x_n^j) = 0 , \\ &K_j(y_1 + x_1^j, \cdots, y_n + x_n^j) \leq 0 . \end{aligned} \tag{14}$$

This lends itself to the interpretation that the regional objective function chooses balanced grid-wide adjustments to maximize the net local benefits plus the congestion payments made on the local regional constraints minus the congestion payments for the external constraints. A corresponding adjustment would be made to the iteration approximation in (9).

Example Iterations

The following table summarizes key variables for the example application of the three region coordination on locational prices.

Multi-Regional Coordination of Constrained Equilibrium

		Start	Iteration 1	------------	Iteration 1	Iteration 2	------------	Iteration 2	Iteration 3	------------	Iteration 3	Final
Region			I	II	III	I	II	III	I	II	III	
Bus												
1	p	50.00	83.00	82.37	82.30	83.00	82.94	82.95	83.00	83.00	83.00	83.00
	q	2000	900	921	923	900	902	902	900	900	900	900
2	p	50.00	33.78	33.38	33.62	32.66	32.60	32.63	32.90	32.90	32.90	32.90
	q	-1000	-459	-446	-454	-422	-420	-421	-430	-430	-430	-430
3	p	50.00	43.46	42.63	42.24	42.82	42.75	42.75	42.38	42.38	42.38	42.38
	q	-1000	-782	-754	-741	-761	-758	-758	-746	-746	-746	-746
4	p	50.00	48.68	31.50	31.43	31.60	31.90	31.95	31.95	32.00	32.00	32.00
	q	-1000	-956	-383	-381	-387	-397	-398	-398	-400	-400	-400
5	p	50.00	48.57	71.54	71.32	71.32	72.39	72.43	72.44	72.38	72.38	72.38
	q	2000	2048	1282	1289	1289	1254	1252	1252	1254	1254	1254
6	p	50.00	48.46	48.28	47.93	47.77	46.93	46.97	46.98	46.99	47.00	47.00
	q	-1000	-949	-943	-931	-926	-898	-899	-899	-900	-900	-900
7	p	50.00	47.90	46.35	44.03	43.86	43.69	43.27	43.28	43.29	43.29	43.30
	q	-1000	-930	-878	-801	-795	-790	-776	-776	-776	-776	-777
8	p	50.00	48.13	47.19	36.91	36.86	36.81	36.64	36.63	36.63	36.64	36.65
	q	-1000	-938	-906	-564	-562	-560	-555	-554	-554	-555	-555
9	p	50.00	48.01	46.77	60.23	60.12	60.01	60.43	60.43	60.43	60.40	60.40
	q	2000	2066	2108	1659	1663	1666	1652	1652	1652	1653	1653
Line	λ_1	0.00	61.12	61.12	61.12	70.76	70.76	70.76	70.00	70.00	70.00	69.98
	λ_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_3	0.00	27.64	27.64	27.64	19.77	19.77	19.77	20.75	20.75	20.75	20.79
	λ_4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_5	0.00	0.00	63.26	63.26	63.26	65.95	65.95	65.95	65.76	65.76	65.73
	λ_6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_9	0.00	0.00	0.00	39.52	39.52	39.52	40.95	40.95	40.95	40.81	40.78
	λ_{10}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_{11}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	λ_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The following table reports the distribution factors. For each element, this is the impact on the flow on the corresponding line induced by an increase in net load at the bus balanced by a reduction in the net load at bus 1, which is the arbitrary reference bus for the transmission constraints.

Distribution Factors for Buses Relative to Bus 1

Bus	1	2	3	4	5	6	7	8	9
Line									
1 to 2	0	0.644	0.356	0.422	0.444	0.467	0.578	0.533	0.556
2 to 3	0	-0.289	0.289	0.156	0.111	0.067	-0.156	-0.067	-0.111
1 to 3	0	0.356	0.644	0.578	0.556	0.533	0.422	0.467	0.444
3 to 4	0	0.067	-0.067	0.733	0.667	0.600	0.267	0.400	0.333
4 to 5	0	0.022	-0.022	-0.089	0.556	0.200	0.089	0.133	0.111
5 to 6	0	0.044	-0.044	-0.178	0.111	0.400	0.178	0.267	0.222
4 to 6	0	0.022	-0.022	-0.089	-0.444	0.200	0.089	0.133	0.111
6 to 8	0	0.067	-0.067	-0.267	-0.333	-0.400	0.267	0.400	0.333
8 to 9	0	0.022	-0.022	-0.089	-0.111	-0.133	0.089	-0.200	0.444
7 to 8	0	-0.044	0.044	0.178	0.222	0.267	-0.178	0.400	0.111
7 to 9	0	-0.022	0.022	0.089	0.111	0.133	-0.089	0.200	0.556
2 to 7	0	-0.067	0.067	0.267	0.333	0.400	0.733	0.600	0.667