Modeling market liquidity in restructured power systems by stochastic Nash and Generalized Nash equilibrium

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Introduction

The restructuring of electricity markets has led to a new organization between the different segments

- Generation, transmission and retail

Congestion management plays a central role in the market design

1. Independent System Operator (ISO)
   - Responsible for energy and services
   - US-organization, nodal pricing

2. Transmission System Operator (TSO) and Power Exchange (PX)
   - TSO: services, PX: energy
   - EU-organization, zonal pricing

The wholesale market offers a mix of centralized (ISO or PX) and decentralized operations (bilateral contracts, OTC).
Introduction

The electricity prices are volatile and can spike to extremely high value.

There are incentives to hedge the risk by contracting financial derivatives:

- Contracts traded on exchange or OTC transactions
- Energy risk: futures, options (on futures) with varying delivery period
- Locational risk (congestion):
  - Financial Transmission rights (FTRs: US/discussion in EU)
  - Explicit auction of physical rights (EU today)
  - Contract for differences (Nordic market)
Introduction

Liquidity is an essential property of financial markets:

- Quickly buy/sell sufficient quantities of an asset without significantly affecting its price
- Key hypothesis for most derivative pricing models
- Difficult to measure because it embeds several dimensions

Measures of liquidity

- Volume traded, number of agents; bid-ask spread, depth, resiliency

- Depth: depends on the generators technologies

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Modeling market liquidity in power system
Introduction

Market reports usually conclude that the power derivatives are poorly liquid

- Main evidence for the instruments to hedge against congestion

E.G. NY-ISO (Siddiqui et al, 2005), NORDPOOL (Hagman and Bjorndalen, 2011)

Outline :

Analyzing market liquidity by Nash equilibrium:

1. Perfectly liquid market
   ▶ Incomplete and complete

2. Insufficiently liquid market
   ▶ Definition based on the volume, link to bid-ask spread

3. Illustration on an stylized example
Perfectly liquid market

Competitive equilibrium analyzed through a two-stage stochastic Nash equilibrium

- \((\Omega, P)\) finite probability space, \(\mathcal{Z}\): the space of all bounded measurable functions

\[
\begin{align*}
\text{t=0} & \quad \text{t=1} \\
\bullet \text{Trading on the financial market} & \quad \bullet \text{Electricity spot market} \quad \bullet \text{Settlement of the financial derivatives}
\end{align*}
\]

- Financial market with \(C\) contracts (not redundant)
  - \(p^f_c\): price of financial contract \(c\)
  - \(p^s_c(\omega)\): stochastic discounted pay-off (finite)

- \(N\) players on the market, hedging their spot profit
  - \(x_{\nu,c}\): position of player \(\nu\) in the contract \(c\)
  - \(\pi^s_\nu(\omega)\): random spot profit of player \(\nu\) (finite)

Perfect competition, spot equilibrium not influenced by forward decisions
Optimal hedging portfolio

Each player $\nu$ hedges its risks by choosing a portfolio minimizing a risk measure $\rho_\nu$ on the profit distribution.

$$A^{\nu}(p^f_c) \equiv \min_{x_\nu \in \mathbb{R}^C} \rho_\nu(\Pi_\nu)$$

$$\Pi_\nu(\omega) = \pi^s_\nu(\omega) + \sum_{c=1}^C x_{\nu,c}(p^s_c(\omega) - p^f_c)$$

The optimal value can be interpreted in terms of capital requirement. We assume that each player optimizes a convex risk measure (Föllmer and Schied, 2002)

- **Monotonicity:** $\forall Z_1, Z_2 \in \mathcal{Z}$: if $Z_1 \preceq Z_2$, then $\rho(Z_1) \geq \rho(Z_2)$
- **Cash invariance:** if $a \in \mathbb{R}$ then $\rho(Z + a) = \rho(Z) - a$
- **Convexity:** $\forall Z_1, Z_2 \in \mathcal{Z}, \forall t \in [0, 1] : \rho(tZ_1 + (1-t)Z_2) \leq t\rho(Z_1) + (1-t)\rho(Z_2)$

Best-known example: Entropic risk measure, CVaR,...
Optimal hedging portfolio

**Theorem** (Föllmer and Schield, 2002)

Any convex risk measure $\rho$ has a dual representation $(\mathcal{M}, \alpha)$:

$$\rho(Z) = \sup_{Q \in \mathcal{M}} E_Q[-Z] - \alpha(Q)$$

- $\mathcal{M} = \{Q \in \mathcal{P} : \alpha(Q) < +\infty\}$ is a convex set of probability measures
- $\alpha(\cdot)$ is a penalty function

Using the dual representation theorem, the hedging problem becomes

$$A^\nu(p^f) \equiv \min_{x_\nu \in \mathbb{R}^C} \left\{ \sup_{Q \in \mathcal{M}_\nu} E_Q[-\pi^s_\nu] - \sum_{c=1}^{C} x_{\nu,c} \left( E_Q[p^s_c] - p^f_c \right) - \alpha_\nu(Q) \right\}$$

The hedging problem defines the convex set of *not too attractive prices*

$$P_\nu = \left\{ p^f \in \mathbb{R}^C | \exists Q \in \mathcal{M}_\nu, p^f_c = E_Q[p^s_c] \right\}$$
Optimal hedging portfolio

Properties of $P_\nu$

- The problem is unbounded if $p^f \notin P_\nu$
  
  ▶ Similar (but not identical) to an arbitrage opportunity

- If $p^f \in P_\nu$, the problem becomes

\[
A_\nu(p^f) \equiv \sup_{Q \in \mathcal{M}_\nu} \mathbb{E}_Q[-\pi_\nu] - \alpha_\nu(Q)
\]

\[
p_c^f = \mathbb{E}_Q[p_c^s]
\]

\[(x_{\nu,c})\]

▶ $Q$ should price the financial contracts

- The set of optimal hedging positions $x_\nu$ is non-empty, compact and convex iff $p^f \in \text{int } P_\nu$
Nash Equilibrium in a liquid market

A stochastic Nash equilibrium in a liquid market is a tuple \([(x_{\nu,c})_{\nu=1}^{N}, (p^f_c)_{c=1}^{C}]\):

- \(x_{\nu,c}\) is an optimal solution of \(A^\nu(p^f_c)\)
- satisfying the clearing conditions: \(\sum_{\nu=1}^{N} x_{\nu,c} = 0\)

**Theorem**

If \(\overline{M} := \bigcap_{\nu=1}^{N} M_\nu\) has a non-empty interior, there exists a competitive equilibrium in a liquid market

- At equilibrium, the financial prices are not too attractive for the players

\[p^f \in \overline{P} = \bigcap_{\nu=1}^{N} P_\nu\]

- The set of Nash equilibria is bounded
Discussion on arbitrage opportunities

A convex risk measure is equivalent iff $\forall Q \in M_\nu : Q \sim P$

**Theorem**

*If the players risk measures are convex and equivalent, then an equilibrium is arbitrage free*

- The set of $\overline{P}$ defines prices bounds that are tighter than the usual arbitrage bounds
- Modeling risk aversion by the mean-variance may lead to arbitrage opportunities
The complete liquid market

**Theorem (Ralph and Smeers, 2011)**

*If the market is complete and the risk measures are coherent (Artzner et al., 1999), the equilibrium is obtained by solving:*

$$\max_{Q \in \mathcal{M}} \mathbb{E}_Q \left[ \sum_{\nu=1}^{N} -\pi^s_{\nu} \right]$$

*and the financial prices by* $p^f_c = \mathbb{E}_{Q^*}[p^s_c]$ *

**Interpretation:** A least risk averse agent pricing the system risk: $\sum_{\nu=1}^{N} \pi^s_{\nu}$

This allows a welfare notion $= \sum_{\nu=1}^{N} \mathbb{E}_{Q^*_\nu}[\pi^s_{\nu}]$
Outline

1. Perfectly liquid market
   ▶ Incomplete and complete

2. Insufficiently liquid market
   ▶ Definition based on the volume, link to bid-ask spread

3. Illustration on an stylized example
Modeling insufficient liquidity

- Illiquidity measured on the basis of total volume traded
- Liquidity bounds imposed
  \[
  \sum_{\nu=1}^{N} |x_{\nu,c}| \leq L_c
  \]
- Insufficient liquidity restricts the construction of agent's portfolio

A player \( \nu \) hedges its profit with illiquid contracts by solving:

\[
A^{\nu}(p_f, x_{-\nu}) \equiv \min_{x_{\nu} \in \mathbb{R}^C} \sup_{Q \in \mathcal{M}_\nu} \left\{ \mathbb{E}_Q[-\pi^s_{\nu}] - \sum_{c=1}^{C} x_{\nu,c}(p^f_c - \mathbb{E}_Q[p^s_c]) - \alpha_{\nu}(Q) \right\}
\]

s.t. \( |x_{\nu,c}| \leq L_c - \sum_{-\nu} |x_{-\nu,c}| \) \((\lambda_{\nu,c})\)

- The problem depends on strategies of other players \( x_{-\nu,c} \)
- The equilibrium problem becomes a Generalized Nash Equilibrium Problem (GNEP) with shared constraints
Generalized Nash Equilibrium in an illiquid market

An equilibrium in an illiquid market is a tuple \([(x^\nu_c)^N_{\nu=1}, (p^f_c)^C_{c=1}]\) such that

- \(x^\nu_c\) is an optimal solution of \(A^\nu(p^f_c, x-\nu)\)
- \(\sum_{\nu\in N} x_{\nu,c} = 0\)

Theorem (cf. Arrow and Debreu, 1952 on social equilibrium)

There exists an equilibrium in an illiquid market.

At equilibrium: \(p^f_c \in [\inf_{\omega \in \Omega} (p^s_c(\omega)) , \sup_{\omega \in \Omega} (p^s_c(\omega))]\)

- GNEP may lead to infinite set of solution
- Find a large set of equilibria to illustrate the inefficiencies
- Heuristics have recently been developed
GNE in an illiquid market

An equilibrium may contain arbitrage opportunities (but not exploitable). The optimality conditions for the player $\nu$ give

$$p^f_c = \mathbb{E} Q^*_\nu[p^s_c] + \text{sign}(x_{\nu,c}) \lambda_{\nu,c}$$

An interesting equilibrium is the normalized equilibrium (Rosen, 1965) where the shadow prices of the liquidity constraints are equal among players to $\gamma_c \geq 0$.

$$p^f_c + \gamma_c = \mathbb{E} Q^*_\nu[p^s_c] \quad \text{if} \quad x_{\nu,c} \geq 0$$
$$p^f_c - \gamma_c = \mathbb{E} Q^*_\nu[p^s_c] \quad \text{if} \quad x_{\nu,c} \leq 0$$

Interpretable as a market that endogenizes liquidity cost through a bid-ask spread
Outline

1. Perfectly liquid market
   ▶ Incomplete and complete
2. Insufficiently liquid market
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3. Illustration on an stylized example
Illustration: stylized example

- Perfectly competitive environment; Organized as a US-like market
- Example taken from (Chao et al., 1998)

- Market participants:
  - **Producers**: unlimited capacity
    \[ C_T^\nu(q_\nu) = a_\nu q_\nu + b_\nu \frac{q_\nu^2}{2} \]
  - **Retailers** (stylized): serve final consumers, offering different type of contracts, some with demand management. Sell at \( p^r_\nu \) and procure by bidding
    \[ P_\nu(q_\nu) = a_\nu - b_\nu q_\nu \]
  - **Independent System Operator (ISO)**: responsible for transmission and organizes the spot market.
Illustration: spot market

- The ISO collects the bids and maximizes the total welfare subject to network constraints:

\[
\max_{q_\nu \in \mathbb{R}_+^N} \left[ \sum_{\nu \in N_r} \int_0^{q_\nu} p_\nu(\xi_\nu) d\xi_\nu - \sum_{\nu \in N_p} \int_0^{q_\nu} C_\nu(\xi_\nu) d\xi_\nu \right]
\]

s.t.

\[
\sum_{\nu \in N} q_\nu = 0
\]

\[
-K_\ell \leq \sum_{\nu \in N} \text{PTDF}_{\nu,\ell} q_\nu \leq K_\ell
\]

(Kirkoff’s laws)

- Uncertainty and spot scenarios (#250)
  - Demand sensitive to weather variation ($\alpha_\nu$)
  - Transmission line outage (line 1-6)

- Price (nodal & transmission)

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[p^s]$</th>
<th>$\text{Var}[p^s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.72</td>
<td>3.08</td>
</tr>
<tr>
<td>6</td>
<td>53.05</td>
<td>76.7</td>
</tr>
<tr>
<td>1→6</td>
<td>28.3</td>
<td>85.2</td>
</tr>
</tbody>
</table>

- Risk exposure:

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[\pi^s_\nu]$</th>
<th>$\text{vol}(\pi^s_\nu)$</th>
<th>$-\text{CVaR}_{25%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2197</td>
<td>11%</td>
<td>1517</td>
</tr>
<tr>
<td>6</td>
<td>1979</td>
<td>87%</td>
<td>-309</td>
</tr>
</tbody>
</table>

G. de Maere and Y. Smeers, Modeling market liquidity in power system
Illustration: liquid market

- Two types of financial derivatives on energy and transmission
  - Futures (node 6) and FTRs (Point to Point contracts)
  - Possibility to hedge every price risks

- Producers and retailers hedge their stochastic profit using an E-CVaR (convex and equivalent)

\[ \text{E-CVaR}_{\alpha, \beta}(\Pi) = (1 - \beta) \mathbb{E}[-\Pi] + \beta \text{CVaR}_\alpha(\Pi) \]

- The optimal portfolio is obtained by solving a LP

\[
A(p^f) := \max \left\{ \beta t_\nu + \sum_{\omega \in \Omega} \text{prob}(\omega) \left( (1 - \beta) \Pi_\nu(\omega) - \beta \alpha^{-1} U_\nu(\omega) \right) \right\} \\
U_\nu(\omega) \geq 0 \\
U_\nu(\omega) \geq t_\nu - \Pi_\nu(\omega) \\
\Pi_\nu(\omega) = \sum_{c} x_{\nu,c} (p_{c}^s(\omega) - p_c^f) + \pi_{\nu}^s(\omega)
\]
Illustration: liquid market

The ISO is the ultimate counter party on the FTRs market

- Revenue adequacy requirement (implemented in all restructured US systems / problems in EU systems)
  - FTRs are restricted to be simultaneously feasible for the (N-1) configurations
  - Not allowed to trade energy futures
- The market is still liquid for the other players (netting is allowed)

Equilibrium properties

- The corresponding variational inequality problem has a monotone mapping
  - The Nash equilibrium has a convex non-empty solution set

- Algorithms to compute the equilibrium
  - Computational general equilibrium: "tatonnement", triangulation
  - Heuristic based on joint optimization (inspired by Negishi, 1960)
Illustration: liquid market

Trading leads to a significant reduction of risk

<table>
<thead>
<tr>
<th></th>
<th>$E[\pi^s_\nu]$</th>
<th>$\text{vol}(\pi^s_\nu)$</th>
<th>$E[\Pi]$</th>
<th>$\text{vol}(\Pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2197</td>
<td>11%</td>
<td>2198</td>
<td>1.8%</td>
</tr>
<tr>
<td>6</td>
<td>1979</td>
<td>87%</td>
<td>1890</td>
<td>48%</td>
</tr>
</tbody>
</table>

$E[\pi^s_\nu] = \pi^s_\nu \text{ vol}(\pi^s_\nu)$

Retailer business is more risky
The volumes of trades are important

<table>
<thead>
<tr>
<th></th>
<th>$p^f_C$</th>
<th>$p^f_C - E[p^s_C]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUTURE 6</td>
<td>53.5</td>
<td>0.45</td>
</tr>
<tr>
<td>FTR 1→6</td>
<td>28.9</td>
<td>0.58</td>
</tr>
</tbody>
</table>

G. de Maere and Y. Smeers
Modeling market liquidity in power system
Illustration: illiquid market

- Liquidity constraints imposed on futures and FTRs’ total volume
  - 33% of expected spot quantities for FUTURE 6
  - 50% of the total of network capacities for FTRs
- Equilibrium solved by a sampling procedure proposed by (Nabetani et al., 2008)
- 4000 equilibria computed: focus on the ”extreme” by excluding participants

<table>
<thead>
<tr>
<th>$p_c^f - \mathbb{E}[p_c^s]$</th>
<th>$\mathbb{E}[\Pi_\nu]$</th>
<th>vol(\Pi_\nu)</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUTURE 6</td>
<td>$[-2.95, 1.25]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTR 1→6</td>
<td>$[-2.92, 0.88]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$[2197, 2960]$</td>
<td>5%, 24%</td>
<td>$[0, 546]$</td>
</tr>
<tr>
<td>6</td>
<td>$[1853, 2655]$</td>
<td>38%, 87%</td>
<td>$[0, 541]$</td>
</tr>
</tbody>
</table>
Illustration: welfare

- Welfare: comparison to the complete market

<table>
<thead>
<tr>
<th></th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot market</td>
<td>12610</td>
</tr>
<tr>
<td>Complete financial market</td>
<td>18477</td>
</tr>
<tr>
<td>Liquid financial market</td>
<td>16923</td>
</tr>
<tr>
<td>Illiquid market</td>
<td>[15210, 16727]</td>
</tr>
<tr>
<td>Bid-ask spread</td>
<td>16714</td>
</tr>
</tbody>
</table>

- The completion of the market has a profound beneficial effect
  ▶ Possibility to trade volume risk
  ▶ No restriction for the ISO

- Endogenizing liquidity cost by a bid-ask spread leads to a welfare close to the upper bound of the illiquid market
Illustration: market interdependence

- Effect of restricting the volume of FTRs only

<table>
<thead>
<tr>
<th>Volume FTRs</th>
<th>Volume</th>
<th>FUTURE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>960</td>
<td>570</td>
<td>[52.9, 53.6]</td>
</tr>
<tr>
<td>720</td>
<td>478</td>
<td>[52.9, 53.7]</td>
</tr>
<tr>
<td>480</td>
<td>320</td>
<td>[52.9, 53.8]</td>
</tr>
<tr>
<td>240</td>
<td>269</td>
<td>[53.5, 54.0]</td>
</tr>
<tr>
<td>0</td>
<td>152</td>
<td>54.8</td>
</tr>
</tbody>
</table>

- Only the retailer at node 6 can perfectly hedge against price risks
- Have to pay a higher risk premium to incentivise the other players.
What has been done and what should be done

- What has been done: assume a model or class of models with exogenous liquidity.
- What should be done (1): calibrate/reverse engineer the model.
- What should be done (2): endogenize liquidity.
Reverse engineering: tie in with empirical measures of liquidity (1)

- Empirical analysis of the market studies find relations between liquidity and other characteristics of the market (e.g. Avsar and Goss (1999, 2006, 2009) on US electricity futures).
- Three variables: cost of liquidity (e.g. bid ask spread measured or estimated), volume, measure of volatility.
- Three relations between pairs of these variables.
- Can one relate this type of analysis to the model?
Reverse engineering: tie in with empirical measures of liquidity (2)

- We model liquidity by an overall constraint on transactions.
- We have price variance that can be interpreted as a measure of volatility.
- An we can derive a bid ask spread in some version of the model.
- Could we relate these model outputs through relations comparable to empirical relations?
Make liquidity endogenous (1)

Liquidity issue (e.g. bid ask spread) are related to different phenomena

- asymmetric information
- dealer’s risk which is a function of volatility
- market power
- direct cost of transactions
Make liquidity endogenous (2)

Liquidity issue (e.g. bid ask spread) are related to different phenomena

- modeling market power is deceptive
- asymmetry of information seems quite relevant: but can one model agents with different "statistical probabilities" (e.g. on operating costs)?
- could dealer’s risk and direct cost of transactions be used as residual parameters in calibration?
Conclusions

- Nash equilibrium in a liquid market
  - Existence, absence of arbitrage opportunities
  - Importance of convex risk measure
- Generalized Nash equilibrium in an illiquid market
  - Model based on the total volume traded
  - Possible remaining arbitrage
  - Particular equilibrium: bid-ask spread

- Future research: the determinant of illiquidity
  - Market power, asymmetry of information (Holstrom and Tirole, 2001)
  - Computable?