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# The Potential for Malicious Control in a Competitive Power Systems Environment

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In Proposition 1, on page 2, the existing phrase:

"There exits a set of complex vectors $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ and a **F** such that..."

should instead read:

"Given a set of complex vectors $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$, there exists an **F** such that..."

# THE POTENTIAL FOR MALICIOUS CONTROL IN A COMPETITIVE POWER SYSTEMS ENVIRONMENT

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## ABSTRACT

This paper examines the potential for coordinated control of a group of generators in a manner that destabilizes other machines in the system, while maintaining (nearly) completely satisfactory performance within this control group. In particular, this work demonstrates that for an arbitrary group of m control machines, linear feedback control exercised from their frequency measurements to their mechanical power inputs (governor control) can be constructed such that one mode of an otherwise stable, linearized system model is unstable. In the control group, m-1 machines can be guaranteed to show no participation in the unstable mode. As will be illustrated by example, the remaining one control machine can display small participation in the unstable mode relative to machines outside the control group. Hence, machines outside the control group might be forced to disconnect from the system first, due to growing oscillations in which they strongly participate. This result indicates the need for careful planning in restructuring utility regulation for a competitive environment, in order to avoid a potentially subtle form of anti-competitive behavior.

## I. INTRODUCTION AND MOTIVATION

Recent literature and regulatory policy statements [1] have operated upon the assumption that in the evolving utility environment, generating units will operate in a competitive manner. Most such works assume that effects related to dynamic control action operate on a time scale for which economic effects are less significant, and can be monitored through regional coordination of transmission groups. The role of coordinating transmission groups in monitoring the exact nature of dynamic control at generators remains a open issue, falling under the heading of "ancillary services." This paper is intended to serve as a cautionary note to indicate that groups of generators may, under certain scenarios, have the power to curtail other machines access to the network through exercise of dynamic control. We demonstrate that such control can destabilize a system swing mode in which these latter machines are the primary participants. The scenario is based upon simplified system models, and the suggested behavior is highly unlikely for any responsible generating entity. Nonetheless, in planning a more competitive operating environment for power systems in the U.S. and around the world, it may prove useful to recognize the technical potential for this form of anti-competitive behavior, in order to guard against it.

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This paper examines a linearized model of power systems swing dynamics based on classical models for synchronous machines. We further assume that a subset of m of these machines can be operated under coordinated control, and that their supervising entity can apply linear control from measurement of these machines' frequencies, to their mechanical power inputs (i.e., coupled governor control within the control group is allowed). This is a simplified model, and neglects nonlinear effects of turbine/governor dynamics, and possibly mitigating effects from auxiliary controls such as power systems stabilizers. Nevertheless, such simplified models have long proven useful in predicting electromechanical modes in power networks. It is our premise that more detailed system models are unlikely to change the basic conclusions of this work.

Within the context of this linearized model for the power system dynamics, this paper shows the following result. An *arbitrary* group of m machines may be chosen to form a control group. Assume that a linearized model for the nominal system, without feedback, has a minimum state representation with all of its eigenvalues strictly stable. The entity responsible for the control group is assumed to have knowledge of this linearized model. For the control group of machines, we will describe construction of a linear feedback having the following properties:

i) n-2 modes of the linearized system are unchanged by application of this feedback.

ii) In the system with feedback, one complex conjugate pair of eigenvalues of has positive real part, and hence predicts the presence of an unstable oscillatory mode with exponentially growing envelope.

iii) The eigenvectors associated with the unstable mode have zero components in those elements corresponding to m-1 of the control group machine frequencies, indicating that these machines have no participation in the unstable mode.

For the construction above, it is typical that one machine in the control group *will* show participation in the unstable mode. For purposes of discussion, we shall refer to this as the "sacrificial machine." For an arbitrary choice of the m control machines, the relative magnitude of its participation factor [9] relative to machines outside the control group is not constrained. However, intuition would suggest the following, as will be demonstrated in numerical examples to follow. Suppose the mode to be destabilized is selected such that the m control machines lie within a coherent group with respect to that mode (for a recent sampling methodologies for identifying coherent groups, see [4], [5], [6]). Then the participation of the sacrificial machine in the unstable mode is expected to be relatively small with respect to one or more of the machines outside the control group. This leads to a situation in which the affected machines outside the control group are forced to disconnect from the system first, due to their machine frequencies and angles experiencing growing oscillations.

## II. EIGENVALUE/VECTOR PLACEMENT

At the heart of the results of this paper is a simple application of the work of [2], coupled with the separation principle [7]. We first review the classic results of [2], which describe all eigenvalue and (right) eigenvector pairs achievable by linear state feedback in a controllable system. In particular, consider the closed loop linear state equation

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BF})\mathbf{x} \qquad (1)$$

associated with an open loop system

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \qquad (2)$$

to which a state feedback $\mathbf{u} = -\mathbf{Fx}$ is applied. Here

$$\mathbf{x} \in \mathcal{R}^n;\ \mathbf{u} \in \mathcal{R}^m;\ \mathbf{A} \in \mathcal{R}^{n\times n};\ \mathbf{B} \in \mathcal{R}^{n\times m};\ \mathbf{F} \in \mathcal{R}^{m\times n}$$

where $\mathcal{R}^n$ ($\mathcal{R}^{n\times m}$) denotes a vector (matrix) of n (nxm) real elements, and $\mathcal{C}^n$ a vector of n complex elements. Given a complex scalar $\lambda$, we define the Hautus matrix

$$\mathbf{S}_\lambda := [(\lambda\mathbf{I} - \mathbf{A})\ \mathbf{B}]\ ,\ \mathbf{S}_\lambda \in \mathcal{C}^{n\times(n+m)}. \qquad (3)$$

Also define the compatibly partitioned matrix

$$\mathbf{K}_\lambda := \begin{bmatrix} \mathbf{N}_\lambda \\ \mathbf{M}_\lambda \end{bmatrix},\ \mathbf{K}_\lambda \in \mathcal{C}^{(n+m)\times m}, \qquad (4)$$

where $\mathbf{K}_\lambda$ is composed of columns that span the null space of $\mathbf{S}_\lambda$; i.e., for any $\mathbf{h} \in \mathcal{C}^{m+n}$ such that

$$\mathbf{S}_\lambda\mathbf{h} = \mathbf{0},$$

there exists a $\mathbf{k} \in \mathcal{C}^m$ such that

$$\mathbf{h} = \mathbf{K}_\lambda\mathbf{k}.$$

With this notation developed, we restate Proposition 1 of [2] for completeness.

**Proposition 1** (from [2])

Let $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ be a self conjugate set of distinct complex numbers. There exits a set of complex vectors $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ and a $\mathbf{F}$ such that $\lambda_i\mathbf{v}_i = (\mathbf{A} + \mathbf{BF})\mathbf{v}_i$ for all $i \in \{1, 2, ..., n\}$ if and only if the following three conditions are satisfied for all $i \in \{1, 2, ..., n\}$.

i) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ form a linearly independent set within $\mathcal{C}^n$ (over the field of complex scalars).

ii) $v_i = v_j^*$ whenever $\lambda_i = \lambda_j^*$.

iii) $v_i \in \text{span}\{\mathbf{N}_{\lambda_i}\}$

Furthermore, if $\mathbf{F}$ exists and $\mathbf{B}$ is full column rank, then $\mathbf{F}$ is unique.

The application of these results to our problem is straightforward. First, observe that for any $(\lambda_i, v_i)$ that is in the spectrum of $\mathbf{A}$, conditions (i) and (ii) are trivially satisfied. Moreover, by definition, $v_i$ lies in the null space of $(\lambda_i\mathbf{I} - \mathbf{A})$, and hence in the span of $\mathbf{N}_{\lambda_i}$. We conclude that however one may attempt to place other eigenpairs satisfying (i)-(iii), it is always possible to leave an arbitrary number of eigenpairs invariant from their original, uncontrolled values.

Hence, part of our goal stated in the introduction, that of leaving n-2 eigenvalues and eigenvectors of the system unchanged by the proposed feedback, is easily achieved.

## III. CONSTRUCTION OF THE DESIRED STATE FEEDBACK

The construction of the desired malicious state feedback for the control group of generators requires knowledge of the linearized swing dynamics, with classical models for machines. Hence, the bulk of the information required is power flow data, which is required to be publicly available under proposed FERC rules. The only potentially proprietary data required for machines is machine inertias and transient reactances. An obvious point for future investigation will be the sensitivity of the unstable system eigenvalues and eigenvectors (after malicious feedback) to errors in the system data on which the design was based. A large number of works in the power systems literature address this type of eigen-sensitivity calculation.

Under the assumptions above, the state matrix description for the linearized swing dynamic equations is known, and is denoted **A**. For calculation, it will prove convenient to form a state representation that is not minimal, in which *all* machine angles are maintained as states. In this way the choice of a reference machine does not impact the state description. For a system having a total number of machines $\ell$, the state space is then composed of $\ell$ generator frequency deviations, $\omega$, and $\ell$ angle deviations, $\delta$. As appropriate to a linear model, all deviations are relative to the steady state equilibrium. Assume that **A** has distinct eigenvalues, $\{\lambda_1, \lambda_2, ..., \lambda_n\}$, and hence a full set of eigenvectors, $\{v_1, v_2, ..., v_n\}$, spanning $\mathcal{C}^n$.

For illustration, suppose the control machines were to be indexed 1 through m (note that our numerical examples to follow uses a fixed generator ordering, which are not re-arranged to place control machines first). Let $\mathbf{J}_c$ denote the diagonal matrix of per unit normalized generator inertias for the control group. Then $\mathbf{B} = [\mathbf{J}_c^{-1}\ \mathbf{0}]^T$ would define the control input matrix. By construction, **B** must have full column rank. For the purposes of initial development in this section, all states are assumed available for measurement. The following section describes how this assumption can be dropped, and the control designed to allow only measurement of the control group frequencies. Finally, we shall also assume that the system is controllable from the power inputs of the control machines. A linearized swing dynamic model is typically controllable from any one machine power input, so this property may be expected to hold for most system. This can be confirmed numerically for the examples to follow in section V.

The algorithm for construction of the state feedback matrix, **F**, is described below.

**F Construction-Step 1**: Select unstable eigenvalue locations, denoted as $\hat{\lambda}_1, \hat{\lambda}_2$. A reasonable (but certainly *not unique*) choice is given by:

$$\hat{\lambda}_1 := \lambda_1 + 2*|\mathcal{Re}\{\lambda_1\}|;\ \hat{\lambda}_2 := \lambda_2 + 2*|\mathcal{Re}\{\lambda_2\}|$$

**F Construction-Step 2**: Construct the matrix $\mathbf{K}_{\hat{\lambda}_1}$ of vectors spanning the null space of $\mathbf{S}_{\hat{\lambda}_1}$. Note that $\mathbf{S}_{\hat{\lambda}_1} = \mathbf{S}_{\hat{\lambda}_2}^*$, so that only the Hautus matrix associated with the first eigenvalue need be considered. Partition this $\mathbf{K}_{\hat{\lambda}_1}$ matrix into blocks $\mathbf{N}_{\hat{\lambda}_1}$ and $\mathbf{M}_{\hat{\lambda}_1}$ as described in Section II.

**F Construction-Step 3**: Construct the unstable eigenvector. Consider the mxm block of $\mathbf{N}_{\hat{\lambda}_1}$ associated with control machines; denote this block $\mathbf{N}_{\hat{\lambda}_{1c}}$. Two possibilities exist.

**Case 1**: rank$\{\mathbf{N}_{\hat{\lambda}_{1c}}\}$<m. In this case, select the vector **k** such that $\mathbf{N}_{\hat{\lambda}_{1c}}\mathbf{k}=\mathbf{0}$, and let $\hat{v}_1=\mathbf{N}_{\hat{\lambda}_1}\mathbf{k}$, $\hat{v}_2=\hat{v}_1^*$. This results in the eigenvector components for control machine frequencies *all* identically equal to zero, indicating absolutely no participation of the control machines in the unstable mode. This is unlikely in practice, and is mentioned only for completeness.

**Case 2**: rank$\{\mathbf{N}_{\hat{\lambda}_{1c}}\}$=m. For illustration, suppose the sacrificial machine has index 1. Let $e \in \mathcal{R}^m$ denote the vector $[1, 0, ...0, 0]^T$, and select **k** to satisfy $\mathbf{e}=\mathbf{N}_{\hat{\lambda}_{1c}}\mathbf{k}$. As in Case 1, $\hat{v}_1=\mathbf{N}_{\hat{\lambda}_1}\mathbf{k}$, $\hat{v}_2=\hat{v}_1^*$. This achieves objective (ii) from the Introduction. The sacrificial machine has a non-zero eigenvector component in the unstable mode, but all other control machines show zero participation. For the swing dynamic model, if a generator's frequency deviation has a zero component in the eigenvector, so too does its angle deviation.

**Important Test**: To complete the step below, one must verify that $\{\hat{v}_1, \hat{v}_2, v_3, v_4, ..., v_n\}$ form a linearly independent set.[1]

[1]Should this test fail, another machine from the control group can be selected as the sacrificial machine, the states re-ordered, and steps 1 through 3 repeated. Should all possible selections fail this test, it becomes necessary to allow more than one control machine to participate in the unstable mode, with a

**F Construction-Step 4**: Construct the feedback matrix **F**. For the **k** constructed above, let $\mathbf{w}_1 = \mathbf{M}_{\hat{\lambda}_1}\mathbf{k}$. Then construct the real matrix $\mathbf{W} \in \mathcal{R}^{mxn}$,

$$\mathbf{W} = [\mathcal{R}e\{\mathbf{w}_1\}\ Im\{\mathbf{w}_1\}\ 0 \ldots 0]$$

Also construct the real, full rank matrix $\mathbf{V} \in \mathcal{R}^{nxn}$

$$\mathbf{V} = [\mathcal{R}e\{\mathbf{v}_1\},\ Im\{\mathbf{v}_1\},\ \mathcal{R}e\{\mathbf{v}_3\},\ Im\{\mathbf{v}_3\}, \ldots, \ldots\ \mathcal{R}e\{\mathbf{v}_{n-2}\},\ Im\{\mathbf{v}_{n-2}\},\ \mathbf{v}_{n-1},\ \mathbf{v}_n]$$

For illustrative purposes, we have assumed above that the two real eigenvalues of this state representation have been indexed as n-1 and n. Then $\mathbf{F} \in \mathcal{R}^{mxn}$ is constructed as

$$\mathbf{F} = \mathbf{W}\mathbf{V}^{-1}$$

With **F** as selected above, the results of [2] guarantee that the system with feedback, whose state matrix is given by (**A** - **BF**), possesses the desired eigenvalue locations and eigenvector properties.

## IV. MALICIOUS CONTROL WITH LOCAL FREQUENCY MEASUREMENTS ONLY

A textbook interpretation of the Separation Principle is typically given as follows. Suppose one designs a state feedback control to place eigenvalues of a linear system at desired locations. Next one designs a state estimator[2] to yield the estimate $\hat{\mathbf{x}}$. Then consider the composite linear system that represents the replacement, within the state feedback, of the state measurement, **x**, with the estimate, $\hat{\mathbf{x}}$. Standard texts (see, for example, section 7-5 of [7]) prove that this composite system preserves eigenvalues at those locations produced by the placement through state feedback. The remainder of the eigenvalues are associated with the dynamics of the estimator. As a result, one is able to "separate" the task of designing state feedback to achieve pole placement, from the design of the state estimator. One simply links the two after their individual designs are finished to obtain a dynamic controller that does not need full state measurement, but only the available input and output measurements.

The question for our application is the following: linking of the state feedback and the state estimator preserves the location of the "placed" eigen*values*; does it also preserve (for those components associated with original system states) the components of the eigen*vectors*? The answer is yes, as argued below.

We direct the reader to [7], p. 367. Our desired result is an immediate consequence of the construction used there. In particular, there exists a similarity transformation that takes the composite system, with estimator, to a lower block triangular form, in which the state matrix (with feedback) appears as the upper left block. Hence, the eigenvalues predicted by the state feedback are preserved in the composite system. To see that the components of the eigenvectors associated with the physical system's states are also preserved, note that the linear transformation used is also lower block triangular, and its upper left block is simply an identity matrix. Hence, in the transformed system, the physical system states (and their eigenvector components) are preserved, untransformed.

For our application, the outputs to be measured to estimate state are the frequency deviations for all machines in the control group. For illustration, suppose the state equations are ordered so that frequencies associated with control machines appear first. With system states denoted by $\mathbf{x} \in \mathcal{R}^n$, this ordering, and measurements denoted by $\mathbf{y} \in \mathcal{R}^m$, one has $\mathbf{y}=\mathbf{C}\mathbf{x}$, with $\mathbf{C} \in \mathcal{R}^{mxn}$ given by:

$$\mathbf{C} = [\ \mathbf{I}_{mxm}\ \ \mathbf{0}\ ]$$

Given knowledge of the (uncontrolled) linear system description through the three matrices (**A**, **B**, **C**), the algorithm for construction of the linear state estimator may be found in section 7-4 of [7].

## V. IEEE 14 BUS TEST SYSTEM EXAMPLE

Our demonstrations employ a 14 bus example that is a (slightly) modified version of the IEEE 14 bus test system. A one line diagram for the system is provided in Figure 1, with generators appearing at buses 1, 2, 3, 6, and 8. No infinite bus is employed. The state dimension is ten, consisting of five generator frequency deviations (ordered consistent with the bus numbering), and five machine angle deviations. For the first case to be examined, the control group consists of the generators at buses 2 and 3. For the second case, machines 1, 2, and 3 form the control group. In both cases, generator 2 is the sacrificial machine.

---

modification of **e** above to allow two or more non-zero components. In a large number of numerical experiments conducted by the authors, in no case did the linear independence test ever fail for any initial selection of sacrificial machine.

[2]Within the power systems literature, the term "state estimator" is widely used to refer to algorithms that recover an estimate of the solution to a set of nonlinear algebraic equations that relate the *steady state* of the power system to a set of measurements. Our use of this term here refers to a *dynamic* state estimator.

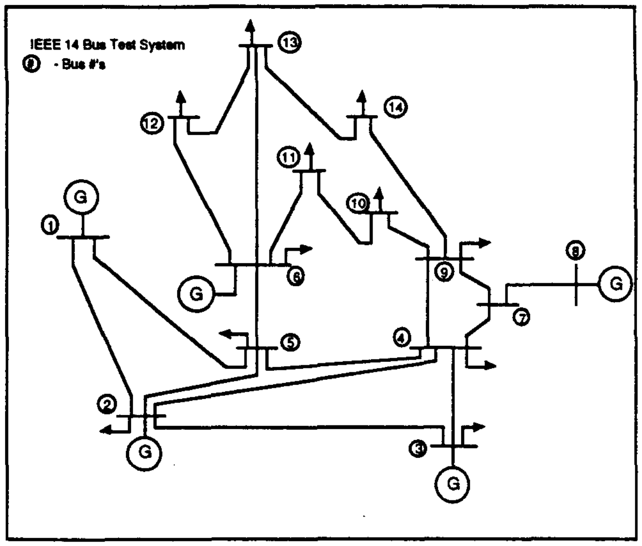


**Figure 1: 14 Bus System One-Line Diagram**

For the operating point and system data employed, the (**A**, **B**, **C**) matrices describing the linearized dynamics are given in appendix A. Note that **A** is the same for each case, while **B** and **C** change according to the control group.

For Case 1, the eigenvalue/eigenvector pair of **A** chosen to be destabilized by feedback is given by:

$\lambda$ = -0.1187 +13.9015i; **v** =

| |
|---|
| 0.1154+0.0167i |
| 0.0475+0.0060i |
| 0.0614+0.0105i |
| -0.9407-0.1504i |
| 0.2568+0.0400i |
| 0.0011-0.0083i |
| 0.0004-0.0034i |
| 0.0007-0.0044i |
| -0.0102+0.0678i |
| 0.0027-0.0185i |

This mode is chosen because the components of its eigenvector corresponding to generators 2 and 3 are relatively small, suggesting that this is primarily an intra-area mode for machines 1, 6, and 8. Following the steps of Section III, we seek to move this eigenvalue to +0.1187 +13.9015i, with an associated eigenvector that has zero entries in those components corresponding to frequency deviation for machine 3. The resulting state feedback matrix is:

$\mathbf{F}^T$ =

| | |
|---|---|
| -1.4640 | -0.0570 |
| -0.4562 | -0.0174 |
| -0.4520 | -0.0187 |
| 3.7479 | 0.1497 |
| -1.3789 | -0.0547 |
| 1.6709 | -1.1668 |
| 0.4127 | -0.3673 |
| 0.7642 | -0.3518 |
| -4.5014 | 2.9829 |
| 1.6537 | -1.0970 |

*

| | |
|---|---|
| 0.0743 | 0 |
| 0 | 0.0743 |

To confirm the design goal, eigenvector and the absolute values of participation factors [9] associated with the new unstable mode are:

| state | eigenvector | lparticipation facsl |
|---|---|---|
| ω1 | -0.2171-0.1727i | 0.0502 |
| ω2 | 0.0017-0.0554i | 0.0033 |
| ω3 | 0.0000-0.0000i | 0 |
| ω4 | -0.1142+0.9145i | 0.4261 |
| ω5 | 0.0631-0.2480i | 0.0435 |
| δ1 | -0.0126+0.0155i | 0.0497 |
| δ2 | -0.0040-0.0002i | 0.0033 |
| δ3 | 0.0000+0.0000i | 0 |
| δ4 | 0.0657+0.0088i | 0.4268 |
| δ5 | -0.0178-0.0047i | 0.0435 |

For Case 2, we select a mode of **A** given by:

$\lambda$ = -0.0884 + 9.1444i; **v** =

| |
|---|
| 0.2193-0.0198i |
| 0.1619-0.0154i |
| 0.1561-0.0159i |
| -0.2125+0.0164i |
| -0.9151+0.0763i |
| -0.0024-0.0240i |
| -0.0019-0.0177i |
| -0.0019-0.0170i |
| 0.0020+0.0232i |
| 0.0093+0.1000i |

and move this eigenvalue to +0.0884 + 9.1444i, while forcing components of the eigenvector associated with machines 1 and 3 to zero. The feedback that accomplishes this goal is given by:

$\mathbf{F}^T$ =

| | | |
|---|---|---|
| -0.6793 | 0.9747 | -0.3600 |
| -0.3866 | 0.5607 | -0.2034 |
| -0.4496 | 0.6577 | -0.2352 |
| 0.2035 | -0.2817 | 0.1103 |
| 1.2316 | -1.7357 | 0.6604 |
| 27.8282 | -60.9042 | 9.6548 |
| 15.4815 | -33.9040 | 5.3660 |
| 17.6838 | -38.7385 | 6.1264 |
| -8.8832 | 19.4494 | -3.0800 |
| -52.1104 | 114.0973 | -18.0672 |

*

| | | |
|---|---|---|
| 0.1061 | 0 | 0 |
| 0 | 0.0743 | 0 |
| 0 | 0 | 0.0743 |

The new eigenvector and associated magnitudes of participation factors are given by:

| state | eigenvector | lparticipation facsl |
|---|---|---|
| ω1 | 0.0000+0.0000i | 0 |
| ω2 | 0.0787-0.3243i | 0.0588 |
| ω3 | 0.0000-0.0000i | 0 |
| ω4 | -0.1289+0.1766i | 0.0504 |
| ω5 | -0.3879+0.8238i | 0.9052 |
| δ1 | 0.0000-0.0000i | 0 |
| δ2 | -0.0354-0.0089i | 0.0007 |
| δ3 | 0.0000+0.0000i | 0 |
| δ4 | 0.0192+0.0143i | 0.0006 |
| δ5 | 0.0897+0.0433i | 0.0108 |

As described in the preceding section, the control may be made dependent only on measurements of inputs and machine frequency deviations within the control group, provided the state is observable from the control group frequency measurements. Straightforward numeric algorithms (available, for example, in [8]) confirm this observability condition.

## VI. CONCLUSIONS

This paper has demonstrated the possibility for a potentially subtle form of anti-competitive behavior exercised by a group of generators through their dynamic control. An algorithm has been demonstrated for constructing a linear feedback control that destabilizes one mode of the system in such a way that all but one of the control group generators have zero participation in this mode. Numerical examples have demonstrated that it is also possible that the one control group generator that does participate in the unstable mode can have a participation factor considerably smaller than machines outside the control group. As a result, machines outside the control group would experience larger magnitude growing oscillations, and protective mechanisms might force the removal of these machines from the network before any machines within the control group were so affected.

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## BIOGRAPHICAL SKETCHES


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## APPENDIX

**A =**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| -0.0754 | 0 | 0 | 0 | 0 | -199.4235 | 180.4162 | 6.3879 | 9.5592 | 3.0602 |
| 0 | -0.1077 | 0 | 0 | 0 | 243.5090 | -386.0927 | 97.6856 | 30.4570 | 14.4410 |
| 0 | 0 | -0.1077 | 0 | 0 | 7.6282 | 90.0311 | -119.1232 | 12.6939 | 8.7700 |
| 0 | 0 | 0 | -0.2513 | 0 | 30.9971 | 67.2109 | 28.9808 | -171.9813 | 44.7925 |
| 0 | 0 | 0 | 0 | -0.1946 | 7.9334 | 25.4575 | 15.4448 | 33.5227 | -82.3584 |
| 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1.0000 | 0 | 0 | 0 | 0 | 0 |

**Case 1: $B^T$ =**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 13.46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 13.46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**C =**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Case 2: $B^T$ =**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9.43 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 13.46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 13.46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**C =**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |